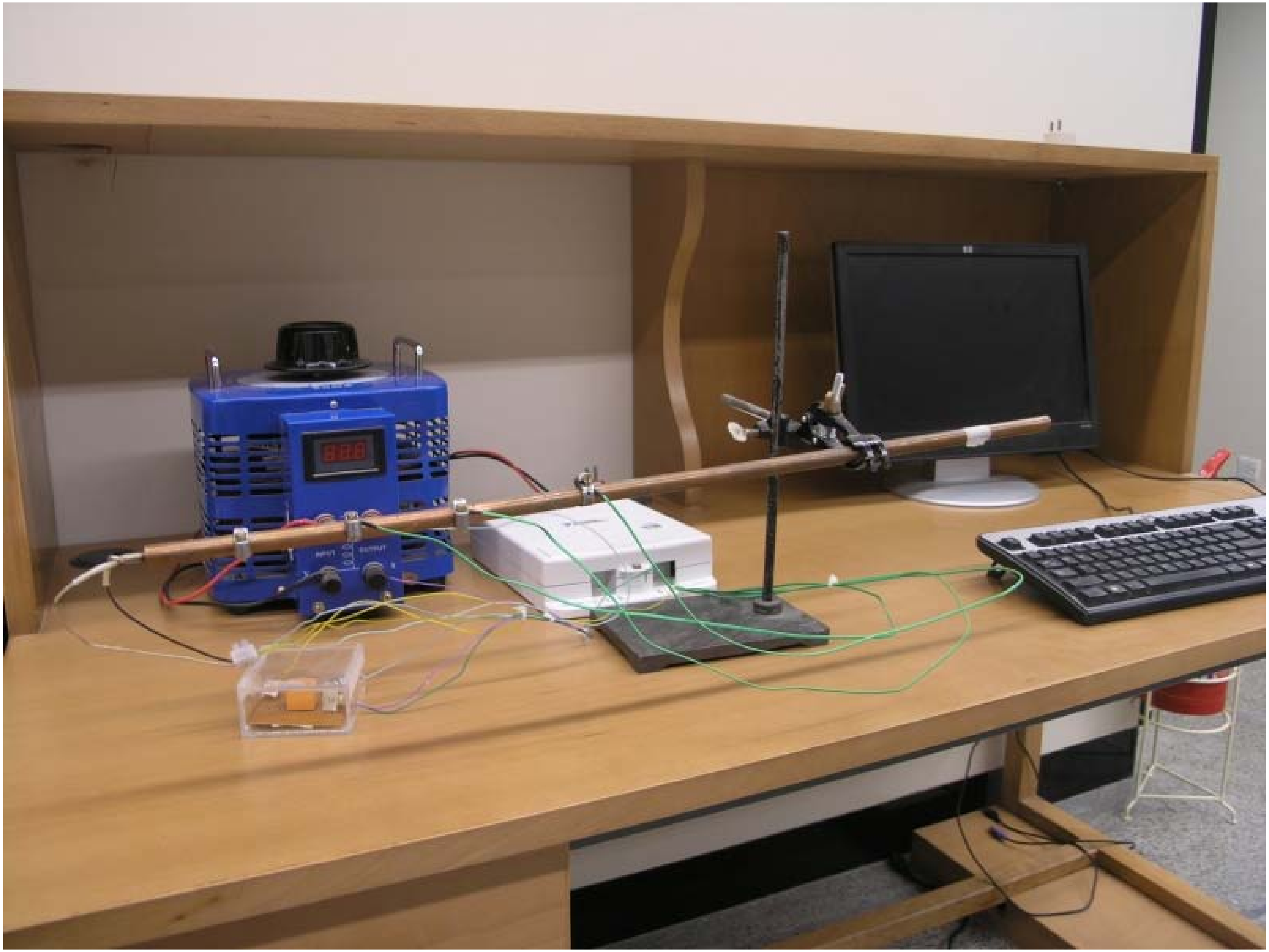
Temperature Oscillations as a Result of Pulsed Heating

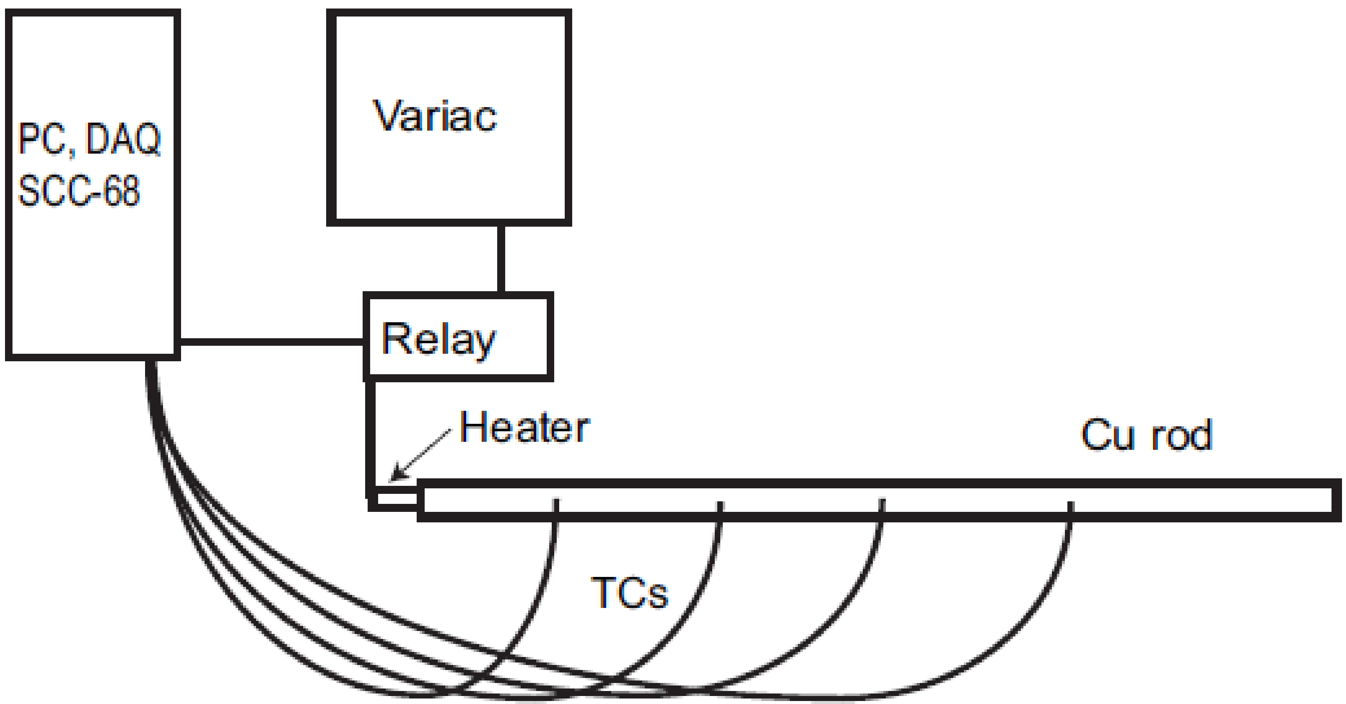
Rafi Ullah

ہنرکدہ

۱۵اپریل۲۰۱۱



Schematics



Heat Equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{D}\frac{\partial T}{\partial t}$$

where

$$D = \frac{\kappa}{\sigma\rho}$$ (Diffusivity of the material)

κ Thermal Conductivity

σ Heat Capacity

ρ Mass Density

Solution of Heat Equation

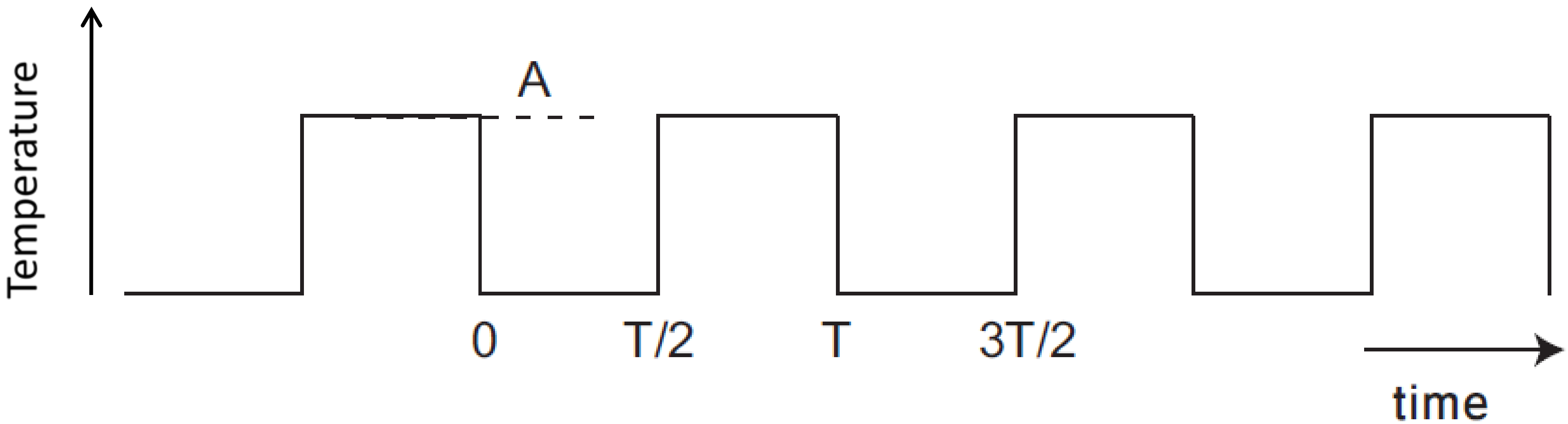
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{D}\frac{\partial T}{\partial t}$$

$$T(x,t) \propto \exp\left(i\left(kx - \omega t\right)\right)$$

For $\quad x \geq 0$

$$T(x,t) = \sum_{\omega} A(\omega)\exp\left(-\sqrt{\frac{\omega}{2D}}x\right)\exp\left(i\left(\sqrt{\frac{\omega}{2D}}x - \omega t\right)\right)$$

Boundary Conditions (pulsed heating)



$$T(0,t) = \frac{A}{2} - \frac{A}{\pi}\left(\sin(\omega_0 t) + \frac{\sin(3\omega_0 t)}{3} + \frac{\sin(5\omega_0 t)}{5} + \cdots\right.$$

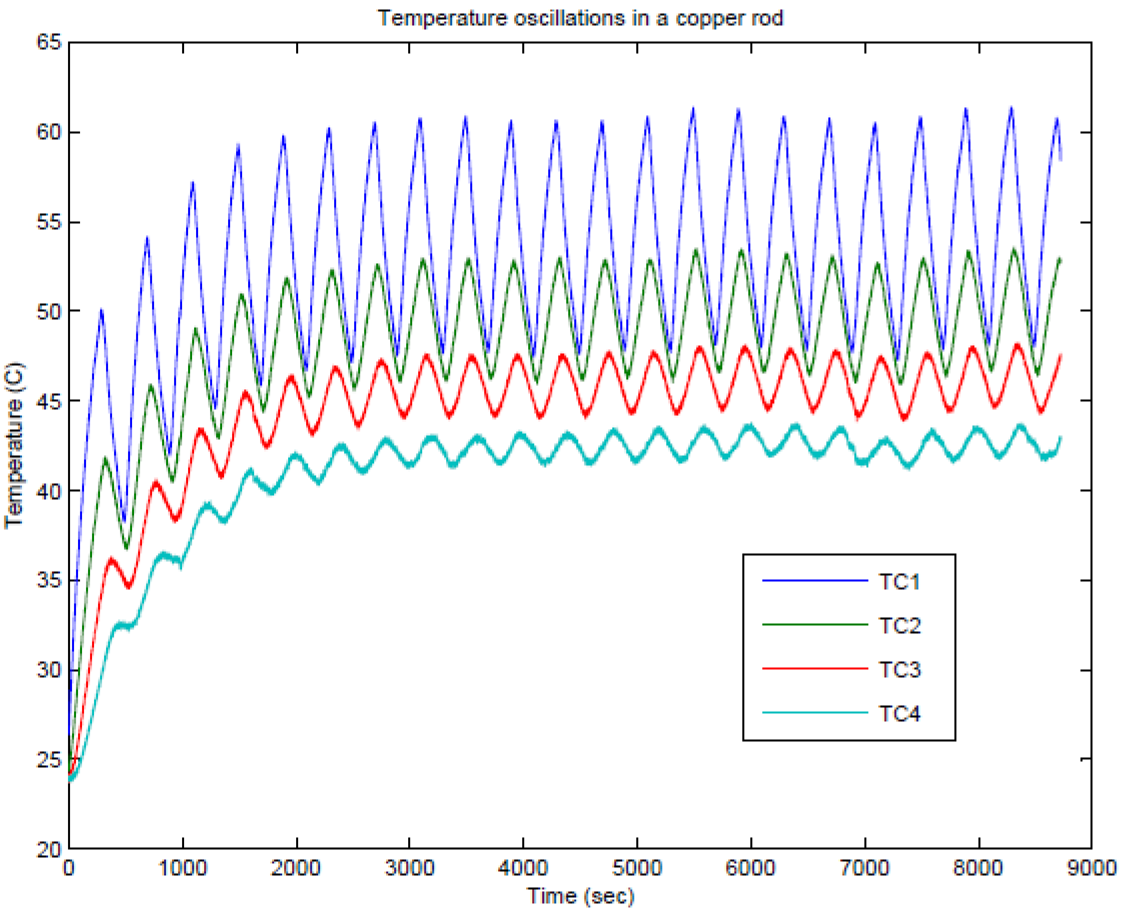
Particular Solution

$$T(x,t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\exp(-\sqrt{\frac{\omega_{2n+1}}{2D}}x)\sin(\sqrt{\frac{\omega_{2n+1}}{2D}}x - \omega_{2n+1}t)$$

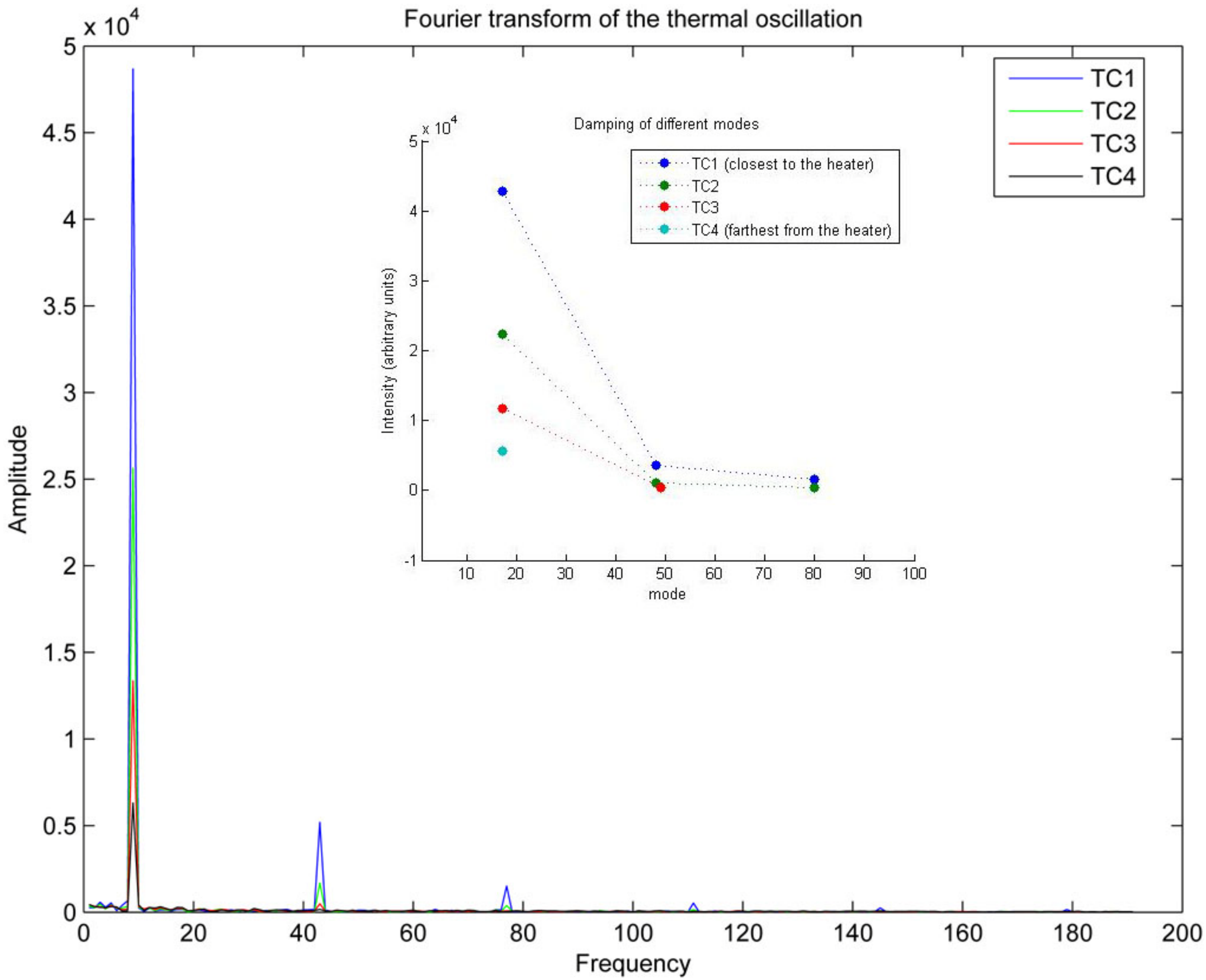
$$\delta \equiv \sqrt{\frac{\omega_{2n+1}}{2D}}$$

Damping Coefficient

Experiment


Temperature oscillations in a copper rod
65
60
55
50
45
40
35
30
25
20
Temperature (C)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
Time (sec)
TC1
TC2
TC3
TC4

Damping of different modes



- Damping Coefficient

$$\delta(\omega_1) = 11.19 \pm 0.04\, m^{-1}$$

$$\delta(\omega_3) = 20.59 \pm 0.04\, m^{-1}$$

$$\delta(\omega_5) = 22.93 \pm 0.04\, m^{-1}$$

measured

$$\delta(\omega_1) = 13.2\, m^{-1}$$

Theoretically estimated

Diffusivity

$$D_v = \frac{v^2 T}{4\pi}, \qquad \mathrm{D}_\delta = \frac{\pi}{T\delta^2}$$

$$\overline{D} = 1.30 \pm 0.16 \times 10^{-4}\, m^2 s^{-1}$$

Thermal Conductivity

$$\kappa = \sigma\rho D$$

$$\kappa = 450 \pm 50\ \mathrm{W\ m^{-1}K^{-1}}\ \text{(measured)}$$

$$\kappa = 401\ \ \mathrm{W\ m^{-1}K^{-1}}\quad \text{(reported)}$$

The bottom line

- The experiment helps understand heat equation, its practical applicability and the limitations imposed by heat-losses.
- Heat losses which haven't been accommodated in the heat equation are responsible for off the mark results
- Currently we are trying to upgrade this experiment to measure the thermo-power