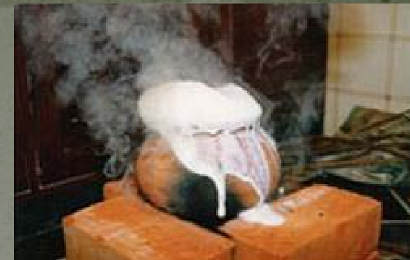

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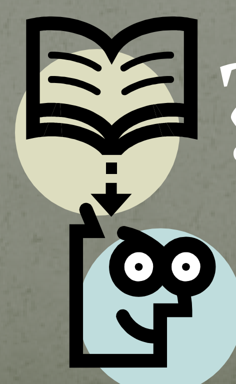

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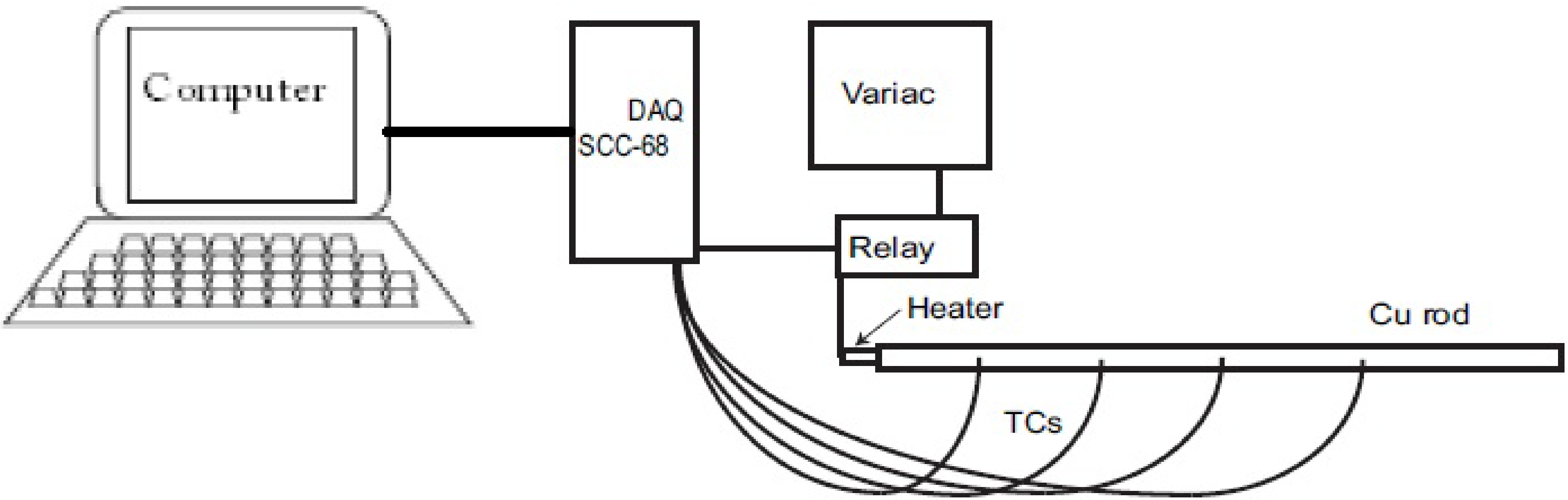
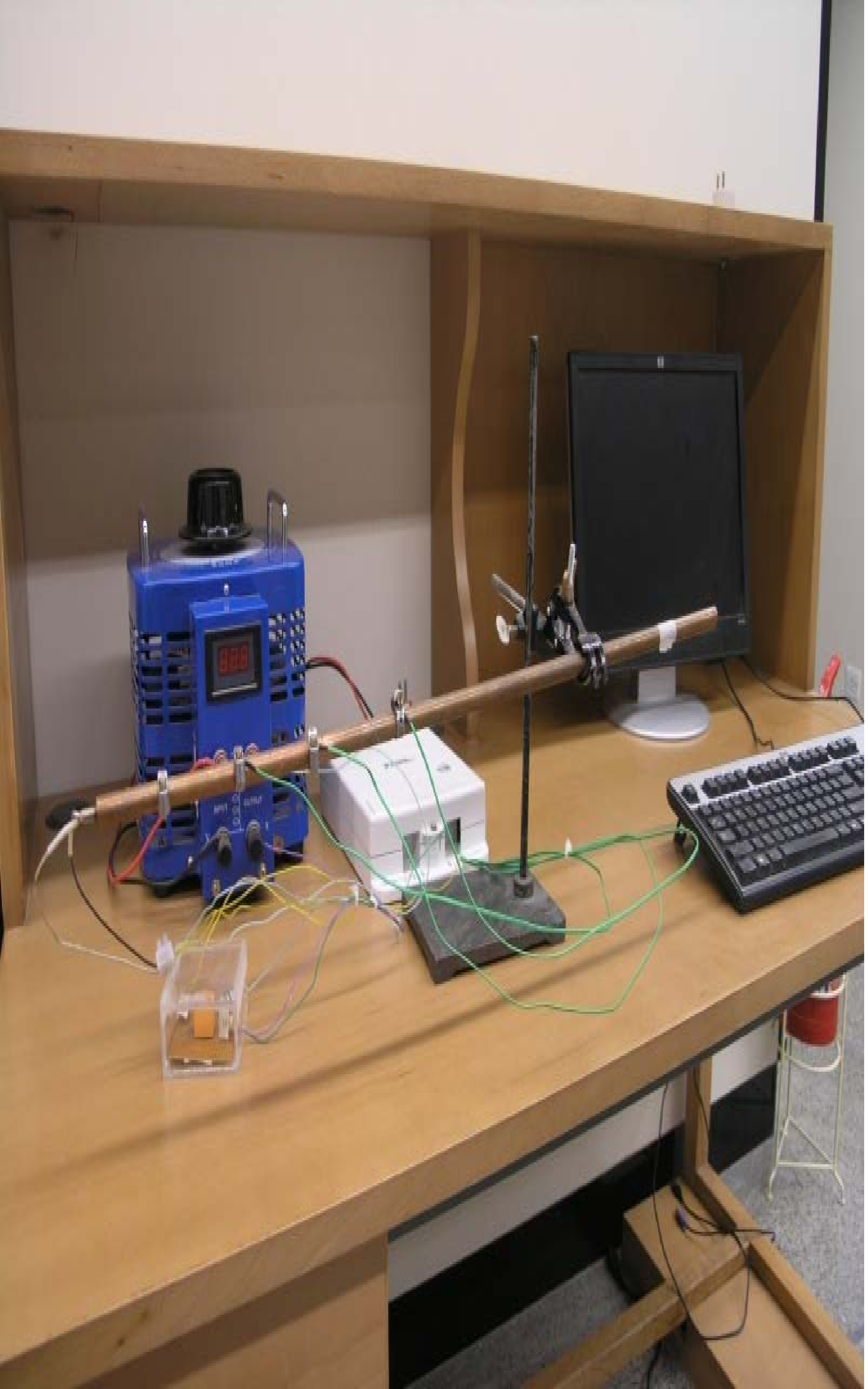
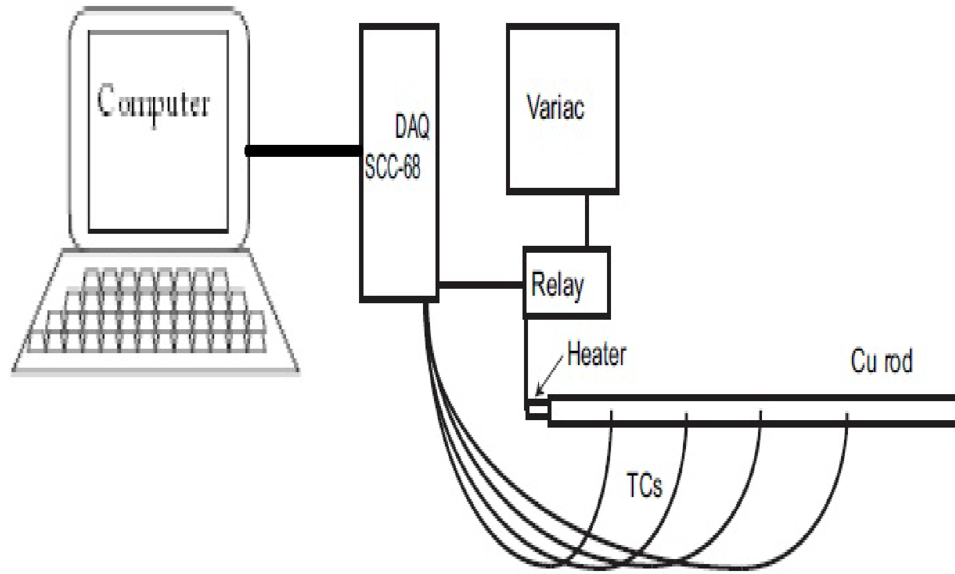
Temperature oscillations in a metal: Probing aspects of Fourier analysis

- Apparatus of Experiment.
- Heat Flow and Heat Equation.
- Comparison of Heat Equation and Wave Equation.
- How we heat the rod in experiment?
- Fourier Analysis of our Heating Technique.
- Particular Solution of our Heating Material.
- Deducing the Damping of Temperature (Damping Coefficient).
- Calculate the Thermal Wave Velocity.
- Finding the Thermal Diffusivity of Material.
- Thermal Conductivity of Material.


Computer
DAQ
SCC-68
Variac
Relay
Heater
Cu rod
TCs

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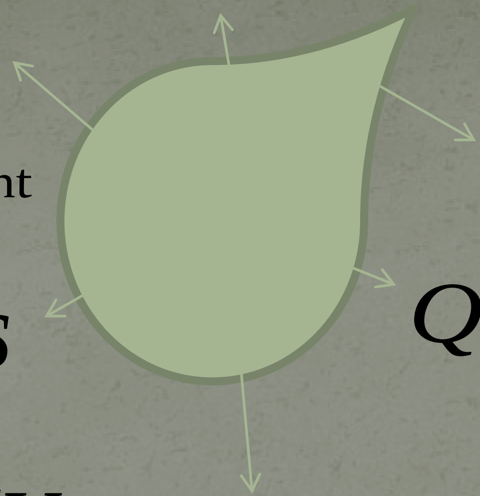
Heat Flow

Heat flows from region of higher temperature to those of lower temperature and experiments show that the rate of heat flow is proportional to the temperature gradient.

$$\boldsymbol{j} = -\kappa \nabla \boldsymbol{T}$$

$$\boldsymbol{j} = -\kappa \nabla \boldsymbol{T}$$

V=volume
S =surface
dS =surface element
n =unit vector



ρ =mass density
σ =mass heat capacity

$$H = \oiint_S \boldsymbol{j}.d\boldsymbol{S} = \oiint_S \boldsymbol{j}.\boldsymbol{n}d\boldsymbol{S}$$

$$Q = \oiiint_V \rho \sigma T dV$$

$$H = -\kappa \oiiint_V \nabla^2 T dV$$

$$H = \frac{\partial Q}{\partial t} = \oiiint_V \rho \sigma \frac{\partial T}{\partial t} dV$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{D} \frac{\partial T}{\partial t}$$

Where D =Thermal Diffusivity =κ/σρ

This equation is called ***Heat Equation*** or ***Diffusion Equation***.

Comperison Between Heat and Wave Equations

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{D}\frac{\partial T}{\partial t}$$

Heat equation or Diffusion equation

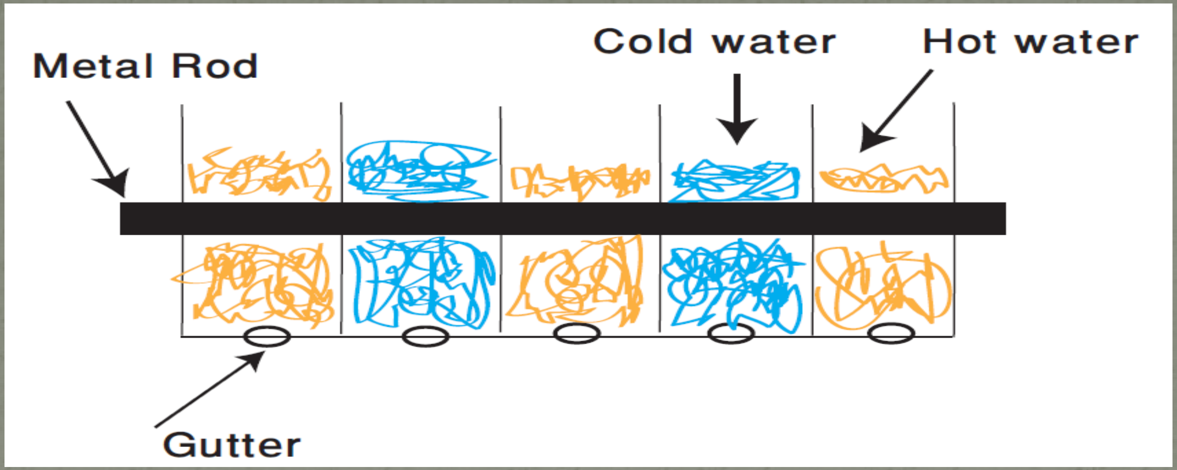
$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

Wave equation

Diffusion Wave equation yields to infinite speed of field propagation. Thus equations yield no traveling waves, no wavefronts, and no phase velocity. Rather, the entire domain "breathes" in phase with the oscillating source.

Diffusion Waves transport, obeys a linear la [j = - κ$\nabla T(x , t)$], rather than a square law. That affects they neither show reflection–refraction law of normal waves nor they transfer energy.

Energy Transfer by Heat Wave



The total heat transfer in metal rod is zero.

$$\boldsymbol{j} = -\kappa \nabla \boldsymbol{T} = \frac{A\kappa\omega}{2\pi D}\sum_{n=0}^{\infty}\exp\left(-\sqrt{\frac{(2n+1)\omega}{2D}}x\right)\cos\left(\sqrt{\frac{(2n+1)\omega}{2D}}x-(2n+1)\omega t\right)$$

$$\langle \boldsymbol{j} \rangle = 0$$

Solution of Heat Equation

We can solve the linear Differential equation with the help of *Separation of Variable Method* and by applying boundary conditions on all frequencies by summation them.

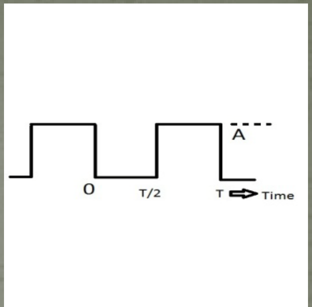
The solution of Heat equation

$$T(x,t) = \sum_{\omega} F(\omega) \exp(-\sqrt{\frac{\omega}{2D}}x) \exp(i(\sqrt{\frac{\omega}{2D}}x - \omega t))$$

At origin x=0

$$T(0,t) = \sum_{\omega} F(\omega) \exp(-i\omega t)$$

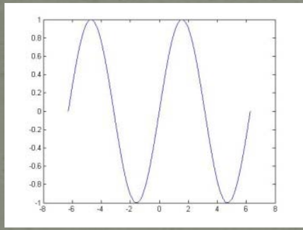
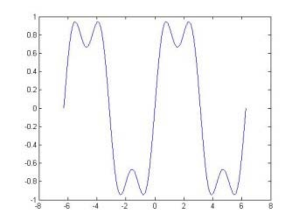
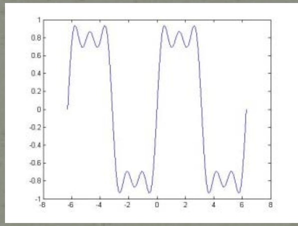
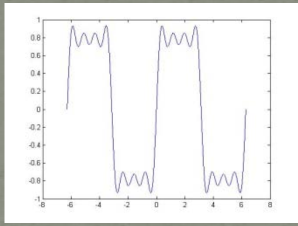
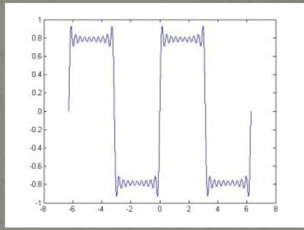
How we heat the rod in experiment?



In the present experiment we apply a periodic square pulse for 200 sec intervals from heater attached at one end of copper (Cu) rod.

Fourier showed that a periodic signal can always be represented as a sum of sinusoids (sines and cosines, or sines with angles). That representation is now called a Fourier Series in his honor.

In our case if we represent the square wave in the form of Fourier series then graphically it looks like:

And mathematically,

$$T(0,t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=0}^{\infty}\frac{\sin\left((2n+1)\omega t\right)}{2n+1}$$

Where $\omega = 2\pi/T$ is the fundamental frequency of the wave

$$T(0,t) = \sum_{\omega} F(\omega)\exp(-i\omega t)$$

General solution of heat equation

$$T(0,t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=0}^{\infty}\frac{\sin\left((2n+1)\omega t\right)}{2n+1}$$

Fourier series of our case

$$T(x,t) = \frac{A}{2} + \frac{A}{\pi}\sum_{n=0}^{\infty}\left(\frac{1}{2n+1}\right)\exp\left(-\sqrt{\frac{(2n+1)\omega}{2D}}x\right)Sin\left(\sqrt{\frac{(2n+1)\omega}{2D}}x - (2n+1)\omega t\right)$$

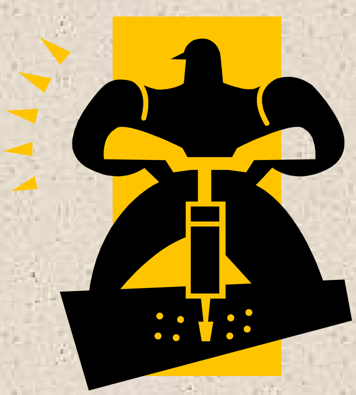
$$\varepsilon = \sqrt{\frac{(2n+1)\omega}{2D}}$$

$$k = \sqrt{\frac{(2n+1)\omega}{2D}}$$

$$k = (wave - number) = \frac{2\pi}{\lambda} = \frac{2\pi}{v\tau}$$

$$D_{\varepsilon} = \frac{\pi}{\tau\varepsilon^{2}}$$

$$D_{v} = \frac{v^{2}\tau}{4\pi}$$

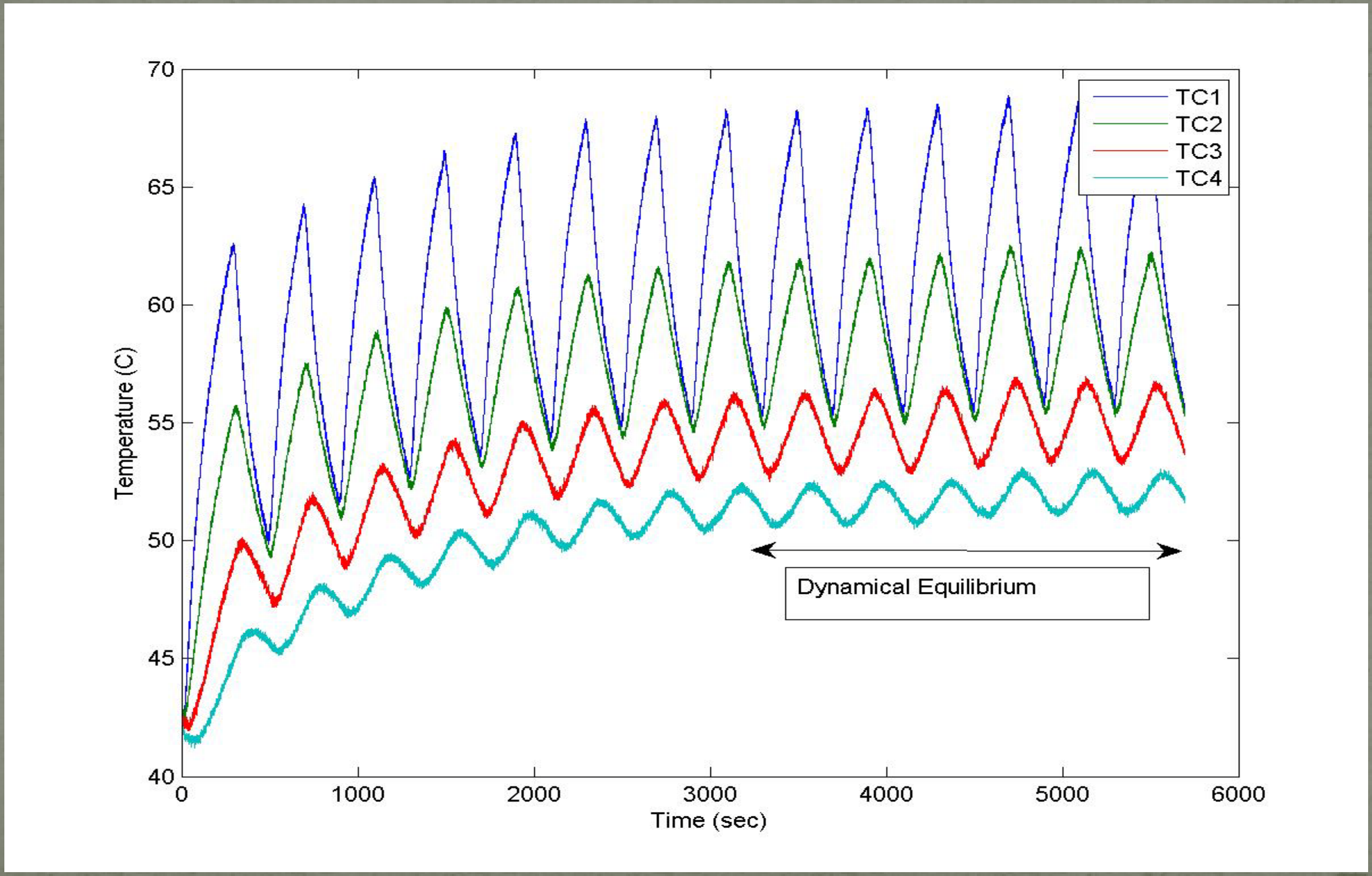


Expeimental Working

Experimental Data:

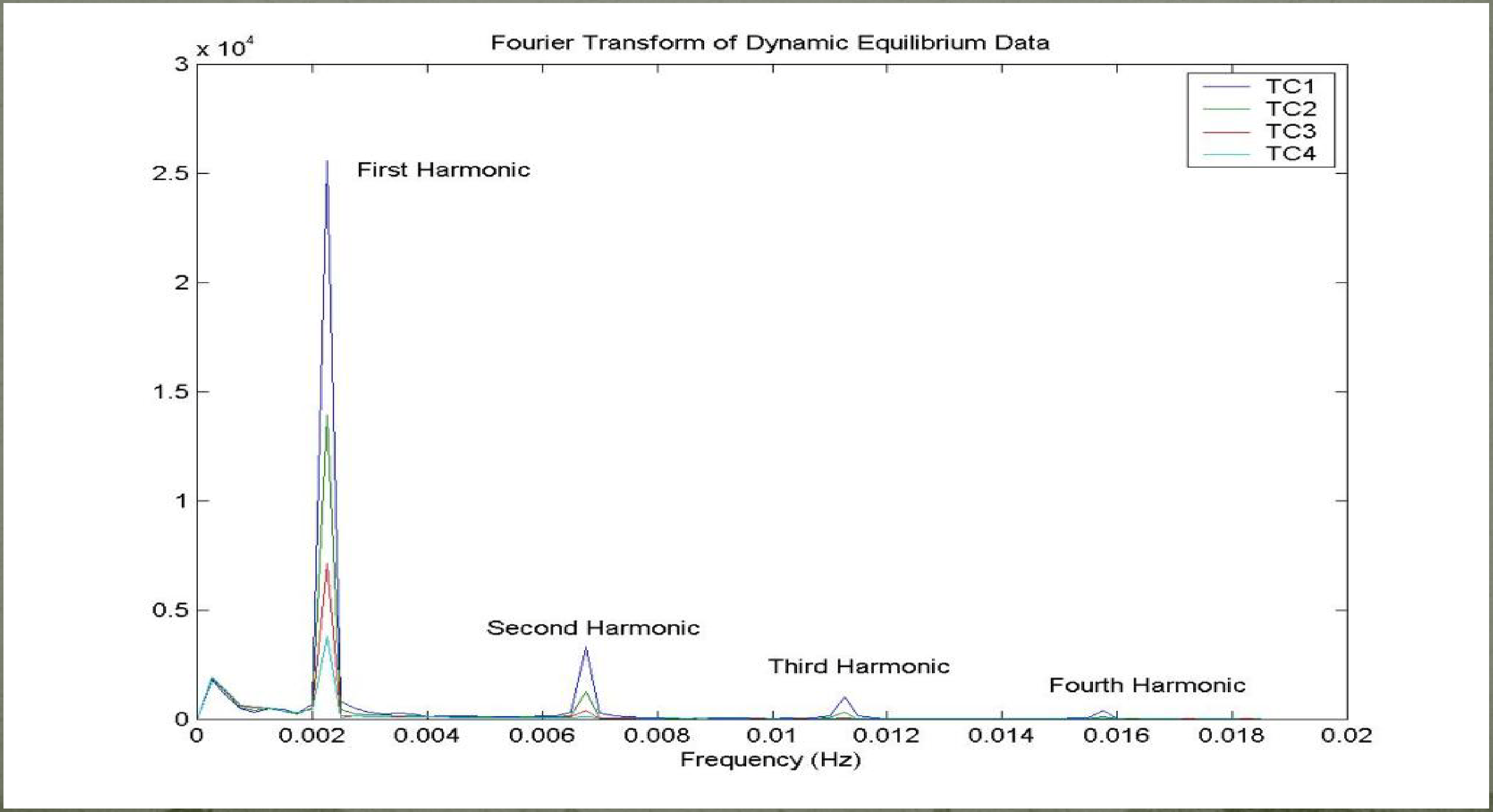
The metal bar of copper (Cu) is heated at 200-s intervals, until a *dynamical equilibrium state is achieved.*

This state is reached when temperature at the measuring points oscillate around their respective mean values.



Fourier Transform

For the analysis of frequencies lie in heat pulse we can transfer the data from *Time domain* to *Frequency domain* by taking Fourier transform of dynamic equilibrium portion of data.



Damping Coefficient

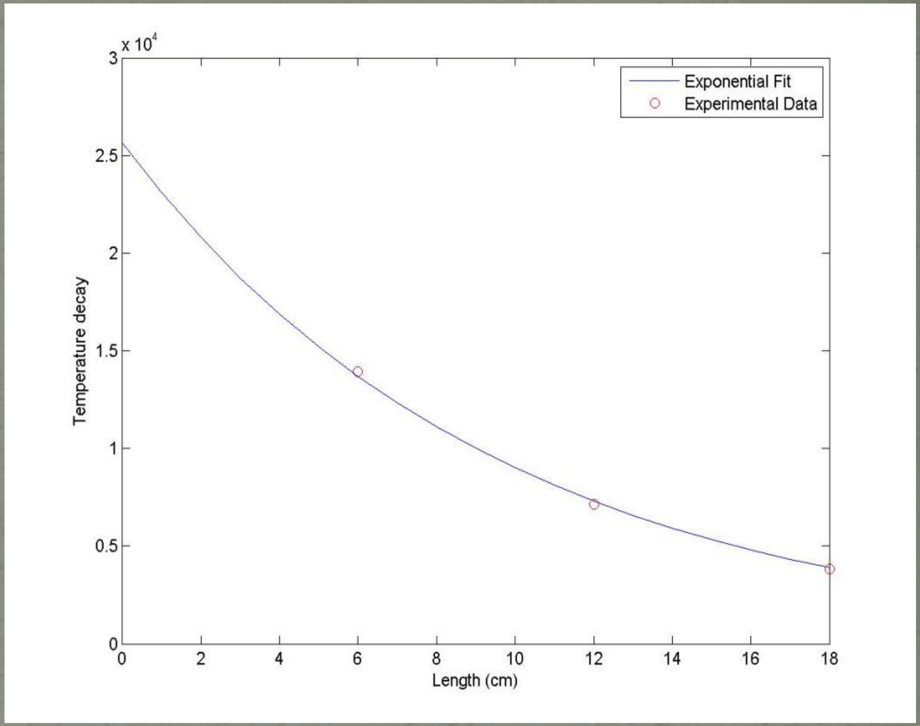
Graphical representation of the peak values of thermocouples show that "decay of temperature is exponential".

Mathematical representation:

$$\varepsilon = -\frac{1}{\Delta x}\ln\frac{A_2}{A_1}$$

Where A1 and A2 are peaks of temperature at thermocouples .

$$\varepsilon = (10.57 \pm 0.18)m^{-1}$$



Thermal Diffusivity

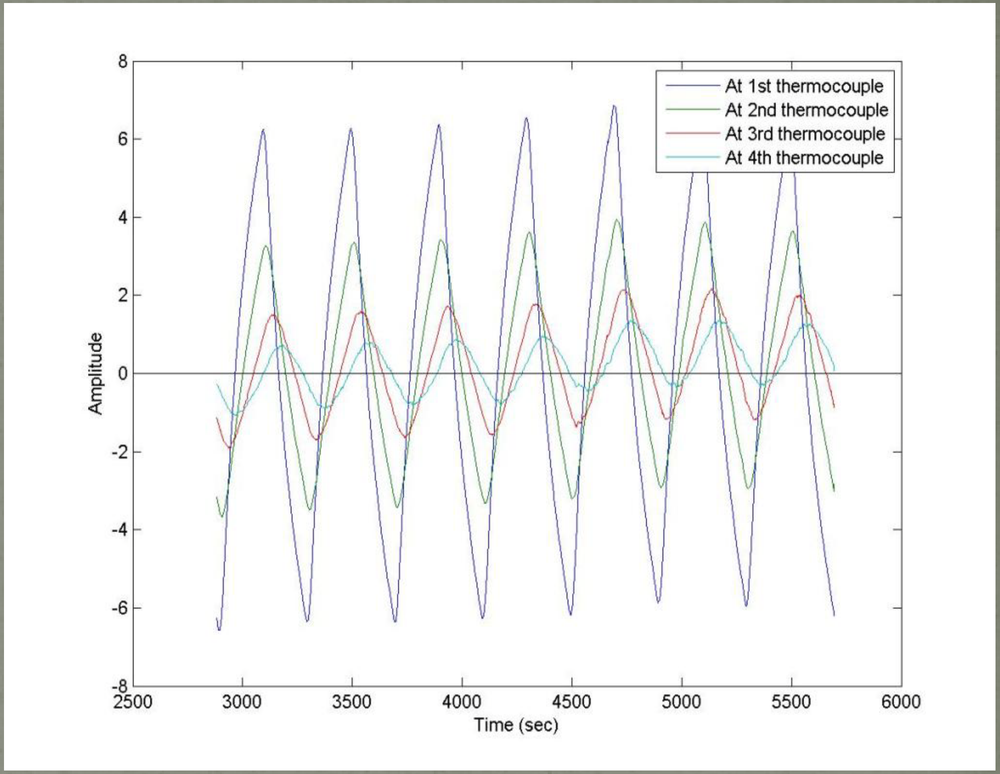
$$D_\varepsilon = \frac{\pi}{\tau\varepsilon^2} = \frac{\pi}{400\times(10.57\pm.18)^2} = (0.70\pm.02)\times10^{-4}m^2s^{-1}$$

Thermal Wave Velocity

We calculate thermal wave velocity by measuring the time difference between two equal-phase points on graph . The wave velocity will then be

$$v = \frac{\Delta x}{\Delta t}$$

$$v = (2.50 \pm .04) \times 10^{-3} ms^{-1}$$



Thermal Diffusivity

$$D_v = \frac{v^2 \tau}{4\pi} = \frac{(2.50 \pm .04)^2 \times 10^{-3} \times 400}{4\pi} = (2.47 \pm .07) \times 10^{-4} m^2 s^{-1}$$

$$D_\varepsilon = (0.70 \pm .02) \times 10^{-4} m^2 s^{-1} \qquad D_v = (2.47 \pm .07) \times 10^{-4} m^2 s^{-1}$$

Average of both values

Thermal Diffusivity $D = (1.58 \pm .04) \times 10^{-4} m^2 s^{-1}$

Thermal Conductivity

We can also calculate the thermal conductivity of Cu from its Diffusivity through:

$$\kappa = \rho \sigma D$$

Where $\rho = 8940 kgm^{-3}$ and $\sigma = 385 Jkg^{-1} K^{-1}$

$$\kappa = (545.61 \pm 15.04) Wm^{-1} K^{-1}$$

The reference value of thermal diffusivity is $D = 1.02 \times 10^{-4} m^2 s^{-1}$

The reference value of thermal diffusivity is $\kappa = 401 Wm^{-1} K^{-1}$

THANK YOU

Acknowledgments
This work has been supported ***by Dr Sabieh Anwar*** and ***Mr. Rafiullah***.