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Investigating properties of white noise in the undergraduate laboratory

Umer Hassan, Sohaib Shamim and M Sabieh Anwar

School of Science and Engineering, Lahore University of Management Sciences (LUMS),
Opposite Sector U, D.H.A, Lahore 54792, Pakistan

E-mail: umersiddiqui@lums.edu.pk, sohaibshamim@lums.edu.pk and sabieh@lums.edu.pk

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Abstract

This paper describes a simple noise circuit for the undergraduate physics laboratory. Students use this circuit to study the properties of electrical noise on a personal computer. This is made possible by using a data acquisition system that allows the experimenters to obtain large amounts of data on the computer, suitable for subsequent mathematical computations. Various properties such as mean, noise power, noise power density and the probability distribution of noise voltages are also explored.

1. Introduction

Noise is inevitably present in electric circuits. Therefore, topics like noise analysis and its reduction are popular candidates for inclusion in the undergraduate laboratory curriculum. For example, this concept has already been elegantly introduced in [1]. The concept of the Fourier transform is also extensively explored in laboratory experiments in areas as diverse as optics, quantum mechanics and signal processing [1–4]. The present paper describes an experiment which simultaneously investigates various properties of electrical noise and its frequency composition.

The experiment starts with students designing passive RC filters with various cut-off frequencies. They then use a home-built noise generator based on the design presented by Horowitz and Hill [5]. These authors use a commercial noise generator that is replaced by our modular design, providing students with a deeper understanding of the noise generation process. Finally, data acquisition enables students to acquire large amounts of data within the limited time allotted to their experimental sessions (in our case, 3–4 h). The stored data are easily quantified by our students who, in the process, learn about the various statistical properties of noise and random processes. For example, they calculate the standard deviation and root mean square (rms) of the noise voltages, estimate the probability distribution functions and directly observe Gaussian distributions.

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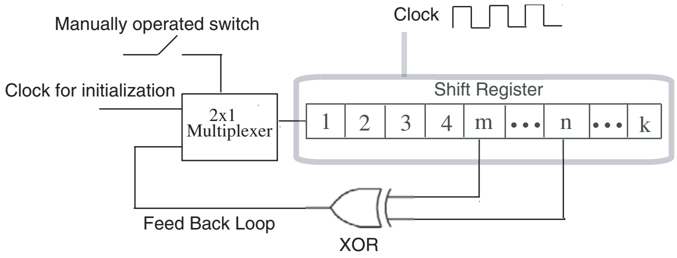

Figure 1. Conceptual diagram of the shift register-based pseudorandom sequence generator.

The noise generator is based on the use of shift registers and logic gates. The design is modular, so students with a more sophisticated background in electronics can also build the circuit as part of a design project. With the help of this circuit, students have a ready source of band-limited noise that they can use for measuring its various statistical properties [6]. The circuit uses readily available off-the-shelf components. A neat experiment investigating the autocorrelation of white noise is discussed in [1] and the current paper is a straightforward extension of the ideas presented therein.

2. Details of the noise generator

A digital circuit can generate periodic sequences consisting of an (almost) random pattern of bits, called a pseudorandom bit sequence (PRBS). Our PRBS circuit employs serial-in-parallel-out (SIPO) shift registers. For such registers, inputs are entered serially and the outputs are extracted in parallel fashion. An exclusive OR operation of the mth and nth bits is then performed, where m and n are appropriately selected [5]. The overall concept is shown in figure 1.

There is also the need to provide a periodic pulse train, called a clock to the shift registers. The bits move forward within the register when a clock transition occurs. An input sequence is also required to initialize the registers. In our case, both purposes are served by separate clock signals that are generated from stable multivibrator circuits [5, 9]. These multivibrators are based on the popular 555 timer.

Suitable outputs from the SIPO shift registers are then fed into XOR gates. The resultant output is a sequence of bits which shows randomness within a period T, before the output deterministically repeats itself. The maximum number of states that can be generated through this approach is 2^k, k being the overall number of output bits in the shift registers.

The complete schematic of the PRBS generator (drawn in Proteus, a commercial circuit visualization tool) is shown in figure 2. The circuit comprises the following modules.

(i) Multivibrator for clocking the shift registers.
(ii) Multivibrator for initializing the shift registers.
(iii) SIPO shift registers.
(iv) XOR gates for implementing the maximal length shift register sequence.
(v) Level shifter.

For the PRBS generation, we have used two 8 bit shift registers (part number 74 164). This is more economical than using a single 16 bit shift register. The sequence required to initialize the shift register is also produced by using a 555 timer as shown by the shaded

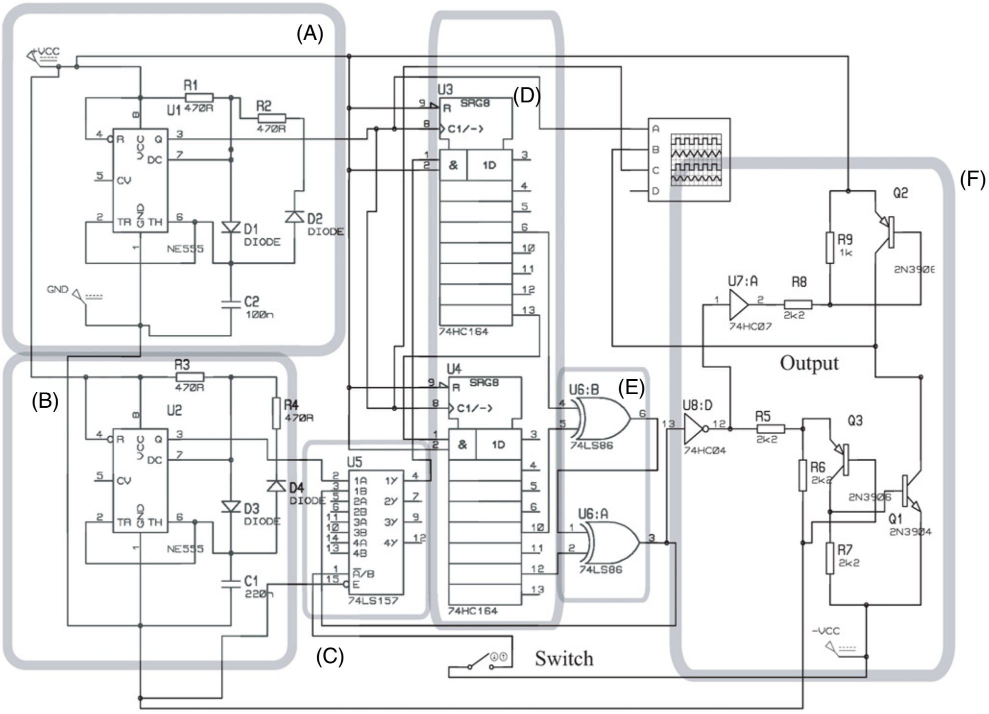

Figure 2. Schematic for the noise generator: (A) multivibrator for clock, (B) multivibrator for initialization, (C) multiplexer, (D) shift registers, (E) XOR gates, (F) level shifter.

box B. The clocks for the shift registers are provided by another timer circuit shown in the shaded box A.

The attainment of the maximal length sequence depends on selecting suitable taps, feeding them into XOR gates and routing the output through the feedback path into the shift register input. Thus, if we number the parallel outputs of the first shift register as 1, 2, 3, . . . , 8 and 9, 10, 11, . . . , 16 for the second shift register, the taps must be taken from the pin numbers 4, 13, and 15. As a consequence, we obtain a pseudorandom bit sequence that repeats itself every 4.37 s. Finally, the PRBS output is available at pin 3 of the XOR labelled U6:A in the circuit diagram.

At the start of the experiment, the input bit (pin 1 of U3) is connected to the initialization multivibrator. To initiate the generation of the PRBS, this pin must be connected to the feedback path instead. This switching is performed manually through an ON/OFF switch and a multiplexer (part number 74 157).

The output of the noise generator is unipolar which is then fed into the level shifter to convert it into a symmetric bipolar signal. This circuit is shown by the shaded box F. A typical output of the circuit after the level shifting operation is shown in figure 3.

The noise generator can be used in a variety of applications such as the generation of band-limited Gaussian white noise [5, 7]. It can also be used to encode or encrypt messages for secure communication [7, 10]. If we use an identical PRBS generator at the receiver, it provides the decryption key, making it useful for error detection and correction of codes. It can also be used as a modulo-*n* divider.

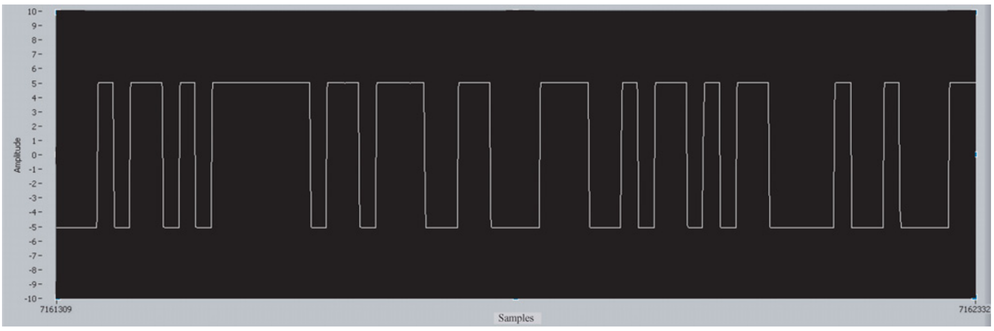

Figure 3. Pseudorandom bit sequence generator output after the level shifter.

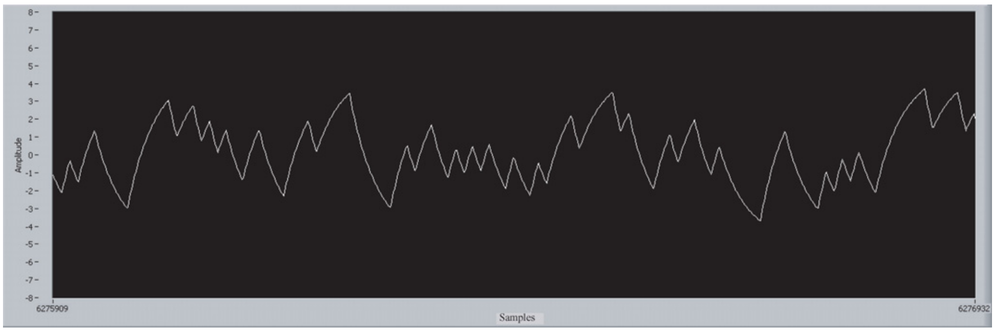

Figure 4. White noise taken after an RC filter with cut-off frequency of 500 Hz.

3. The experiment

The experimental setup includes the home designed printed circuit board, a data acquisition system consisting of the SCC-68 signal conditioning module and a PCI 6221 DAQ card (National Instruments). Students are also provided with an assortment of resistors and capacitors for designing the low-pass filters. The commercially available data acquisition software LabVIEW 8.5.1 (National Instruments) is used to observe the acquired data, although code written in any other language may also be used. Our class use Matlab for data analysis, a choice stemming from their prior training. The acquisition and data analysis script files have been placed on our website [8] and are free to use. Students acquire 1000 samples and repeat the experiment ten times. Each time, they start at a different random point in time, ensuring that they get 1000 × 10 points that are pseudorandom. Students then use Matlab to compute the autocorrelation of the acquired samples, and investigate various statistical properties, as described in the following section.

4. Observations and results

4.1. Autocorrelation of filtered white noise

When the output of the pseudorandom bit sequence generator is fed into a low-pass RC filter, the output approximates band limited white noise. Figure 4 shows a typical output obtained after a low-pass filter with a cut-off frequency of 500 Hz. We expect that using the low-pass

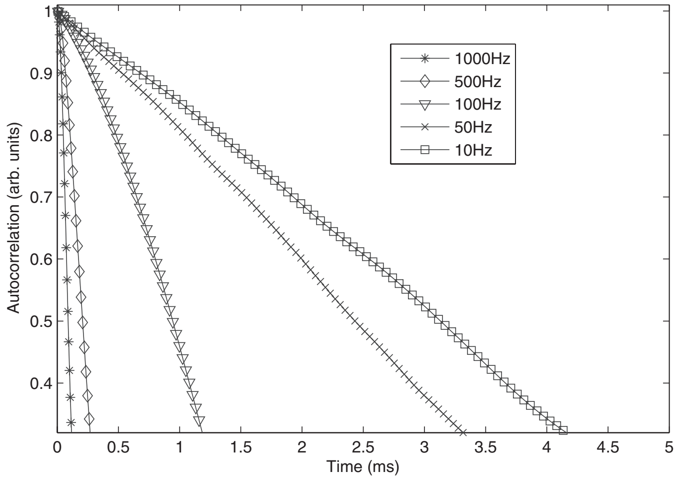

Figure 5. Normalized autocorrelation plot for the cut-off frequencies, 1000, 500, 100, 50 and 10 Hz.

filter of lower cut-off frequencies, the high frequencies present in white noise are eliminated, and the peak width of the autocorrelation function increases. Students are set to anticipate this behaviour.

Let $g(t)$ be any real signal; the autocorrelation $S(\tau)$ is given by the expression

$$S(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)\,\mathrm{d}t. \tag{1}$$

In our experiment, the sampled data are not continuous, but discrete. For a discrete time signal $f(n)$ comprising of N sample points, students use the discrete autocorrelation $S(k)$,

$$S(k) = (1/N)\sum_{n=0}^{N-1} f(n)f(n+k). \tag{2}$$

For each sample width or scan of 1000 points, students compute the autocorrelation function and then compute the average over ten scans. This procedure is repeated for low-pass filters of different cut-off frequencies. The autocorrelation plots are shown in figures 5 and 6. These plots confirm the expectations about the dependence of the autocorrelation peak width on the cut-off frequency.

4.2. Probability density function

White noise has many interesting statistical properties in addition to autocorrelation. With the large amounts of data available on the computer, thanks to the data acquisition system, students can easily investigate these properties. For example, they can confirm that the noise voltages are distributed around a mean that is nearly zero. Students can also find the probability density function (pdf), compute the standard deviation of the probability density function and compare it with the rms of the noise voltage data points.

A histogram is a good approximation to a pdf for a large number of data points. If we empirically sample enough values of a continuous random variable and make a histogram showing the relative frequencies of output ranges, the histogram will approximate the random variable's probability density. For example, the probability of finding any particular noise voltage $P(V)$ is given by the probability density function shown in figure 7. Clearly, it

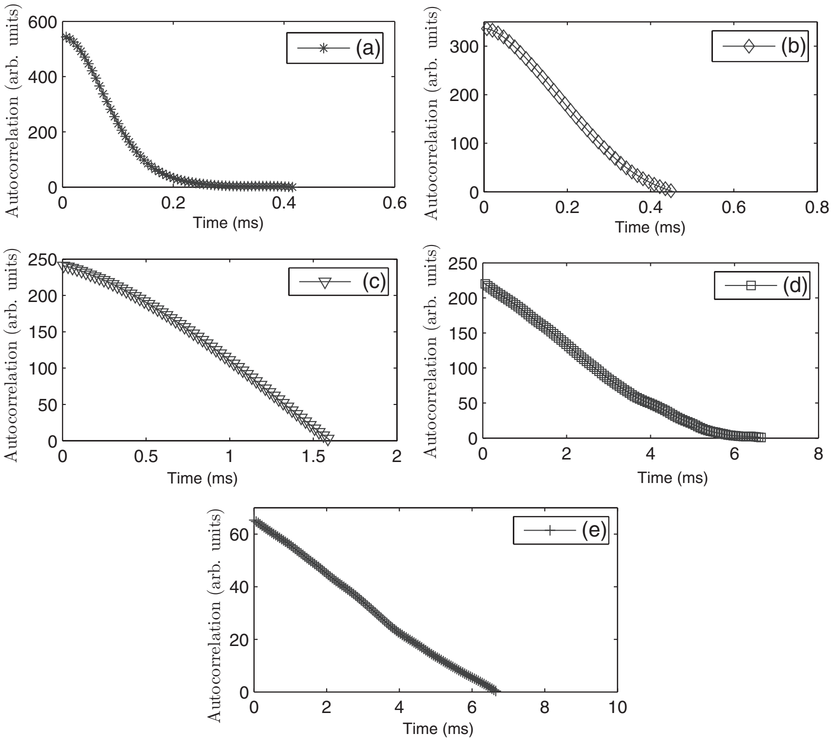

Figure 6. Unnormalized autocorrelation plots with low-pass cut-off frequencies of (a) 1000 Hz, (b) 500 Hz, (c) 100 Hz, (d) 50 Hz and (e) 10 Hz.

Table 1. Statistical properties of the noise filtered at different cut-off frequencies.

Cut-off frequency (Hz)	Mean (V)	Standard deviation (V)	$S(0)$ $(V)^2$	rms noise voltage (V)	Effective noise power density $\mathcal{N}$ (V^2 Hz^{-1})	$\mathcal{N}B$ $(V)^2$
1000	0.0477	1.1050	1.2232	1.1060	0.0007	1.0996
500	0.0793	0.9355	0.8815	0.9389	0.0011	0.8639
100	0.2477	0.5198	0.3314	0.5757	0.0021	0.3299
50	0.1617	0.4197	0.2008	0.4481	0.0026	0.2042
10	0.5673	0.7330	0.8590	0.9268	0.0547	0.8592

resembles the Gaussian distribution. This is quite an illuminating observation for students, as they see a well-defined bell-shaped curve for time domain data that are random and ‘featureless’. Students are asked to fit their histograms to the Gaussian distribution function. In the process, they estimate the mean and standard deviation of the distributions and compare the latter with the rms noise voltage. Typical results are presented in table 1.

4.3. Measurement of statistical properties

Let the mean of the pdf be denoted by μ and its standard deviation by σ. The mean square value $\overline{x^2}$ is given by the expression

$$\overline{x^2} = \mu^2 + \sigma^2, \qquad (3)$$

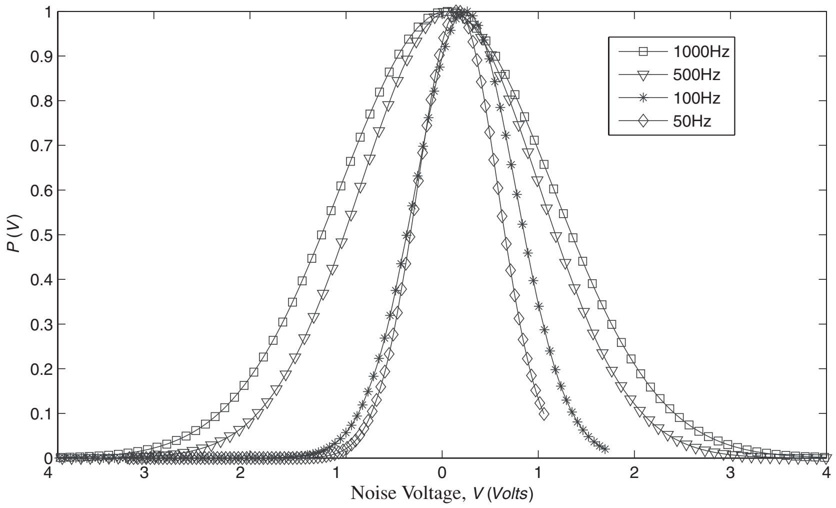

Figure 7. Probability distribution functions at different cut-off frequencies. The points represent experimentally determined values and the continuous lines are the Gaussian curve fits.

and the root mean square (rms) value is evaluated by taking the square root,

$$x_{\mathrm{rms}} = \sqrt{\overline{x^2}}. \tag{4}$$

Ideally, if the mean is zero, the standard deviation is equal to the rms value of the noise voltage V. Table 1 shows the experimental values of the mean, standard deviation and rms values; the former two values emerge from the curve-fitting procedure and the rms noise voltage is determined from applying the standard definition to the sampled data points.

The effective noise bandwidth, B, is another important metric to characterize the noise generator. For a real filter the effective noise bandwidth is defined as the bandwidth of an ideal brick-wall filter which will pass the same noise power as the real filter with a real gradual fall-off. For a single pole RC filter it is approximated as $\pi/2$ times the -3 dB bandwidth of the filter [9]. For example, the effective noise bandwidth for the filter of cut-off frequency 500 Hz is 785 Hz. Using the effective noise bandwidth, students also calculate another important property known as the effective noise power density, denoted as $\mathcal{N}$ and given by the expression

$$S(0) = \overline{x^2} = \mathcal{N} \times B, \tag{5}$$

where $S(0)$ is the (unnormalized) autocorrelation function evaluated at $\tau = 0$, also equal to the total noise power. Table 1 shows the effective noise power densities for the noise generator calculated for different cut-off frequencies. We require the students to pay particular attention to the units of $\mathcal{N}$ as $\mathrm{V}^2\ \mathrm{Hz}^{-1}$. We also expect them to compare the different columns inside the table and verify relations (3) and (5).

5. Conclusions

The experiment provides the students with an introduction to data acquisition systems and how data is stored and manipulated on the personal computer. Building the noise generator exposes students to the design of combined analog–digital circuits. They also learn the basics of simple analog filter design. Furthermore, the experiment gives a detailed introduction to various statistical properties of white noise like autocorrelation, probability density function, and effective noise power density. These are terms they frequently encounter in an undergraduate physics curriculum, but they do not get, in many cases, the proper orientation towards measuring and interpreting these quantities. In particular, our students are quite interested in observing the Gaussian probability distribution function of white noise which seems counter-intuitive when they first look at the random time-domain data of the noise voltages.

Acknowledgment

We thank Wasif Zia (LUMS School of Science and Engineering) for help with the data acquisition.

Appendix. Matlab code for plotting the autocorrelation

```
%loading the saved file
a = load (‘samples’);
%Equating t with number of samples
t = -999:1:999;
%To adjust t with the sampling interval
t1 = t.*0.0067;
k = 1;
for i = 1:1000:10000
    z = 1;
    for j = i:(i+1000-1)
        one (k,z) = a(j);
        z = z+1;
    end
    k = k+1;
end
%averaging the samples
for s = 1:1000
    two(s) = (sum(one(:,s))) / 10;
end
size(two);
%finding autocorrelation
auto = xcorr2(two, two);
%now normalizing
maxauto = max(auto);
norm = auto.  / maxauto;
%plotting autocorrelation
plot (t1,norm,’:’);
```

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