Hunerkada

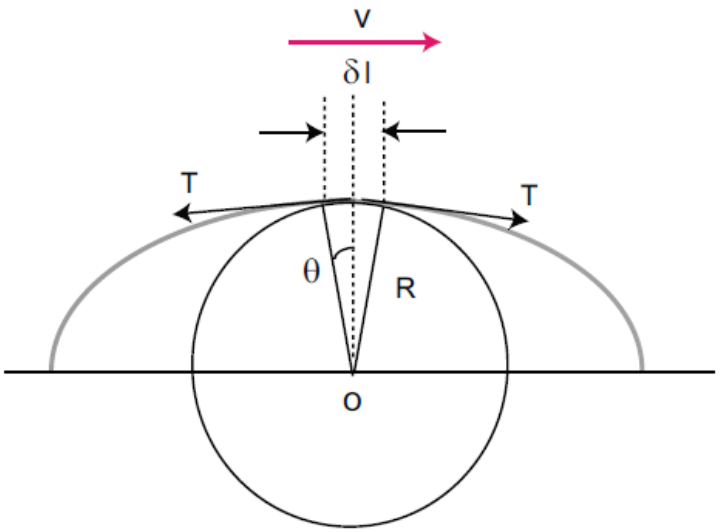
Video based investigations of a nonlinear vibrating string

Contents

- Experiment verification of resonance foldover, elliptic and circular trajectories, hysteresis and jumps as observed in strings

Circular Modes in vibrating string

When stretched string is driven in one plane, the vertical components of tension remain un balanced and constitute a centripetal force which causes circular modes



Critical Amplitude

- Linear wave equation $\omega_n = \frac{n\pi}{L}\sqrt{T_o/\rho},$
- Euler Lagrange Equations reduce to :

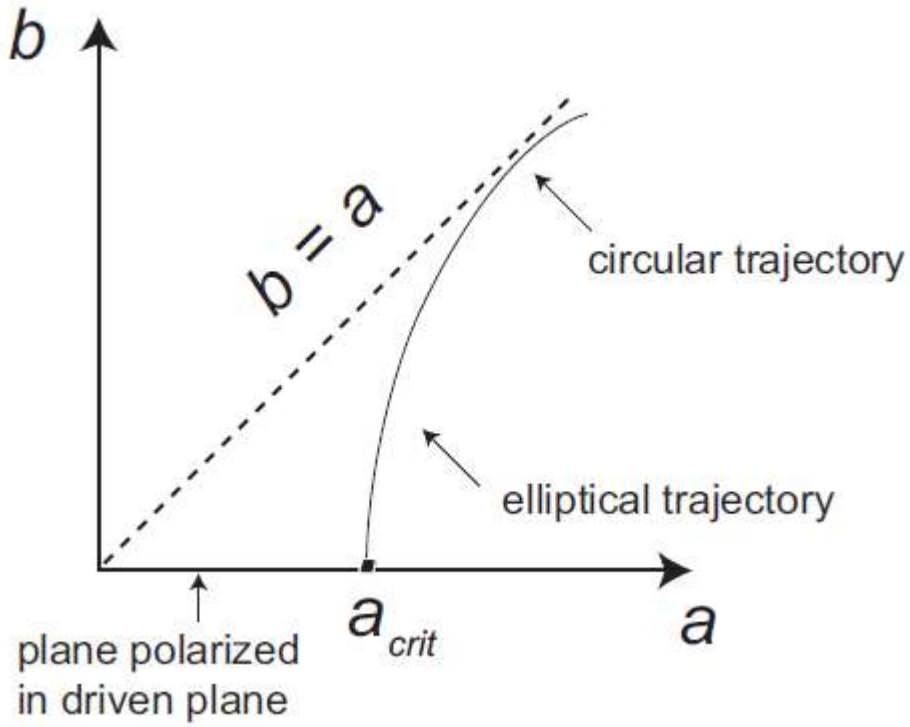
$$\ddot{X} + \omega^2 X[1 + \sigma(X^2 + Y^2)] = A\cos(pt),$$
$$\ddot{Y} + \omega^2 Y[1 + \sigma(X^2 + Y^2)] = 0.$$

- Above Equations are solved simultaneously to get:

$$|b| = +a\sqrt{1 - \frac{2A}{a^3\omega^2\sigma}}.$$

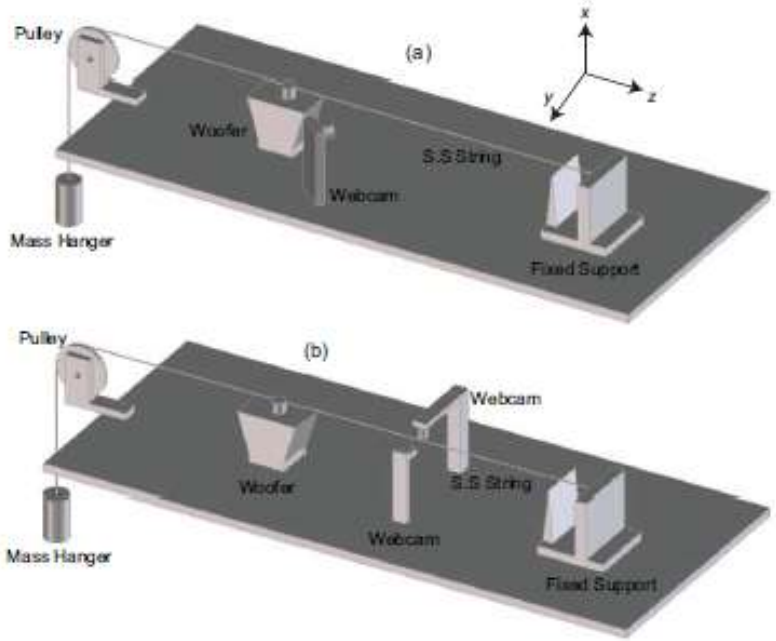
- Critical amplitude: $a_{crit} = (2A/\sigma\omega^2)^{\frac{1}{3}}$
- No real roots of b exist for a < critical amplitude

Critical Amplitude

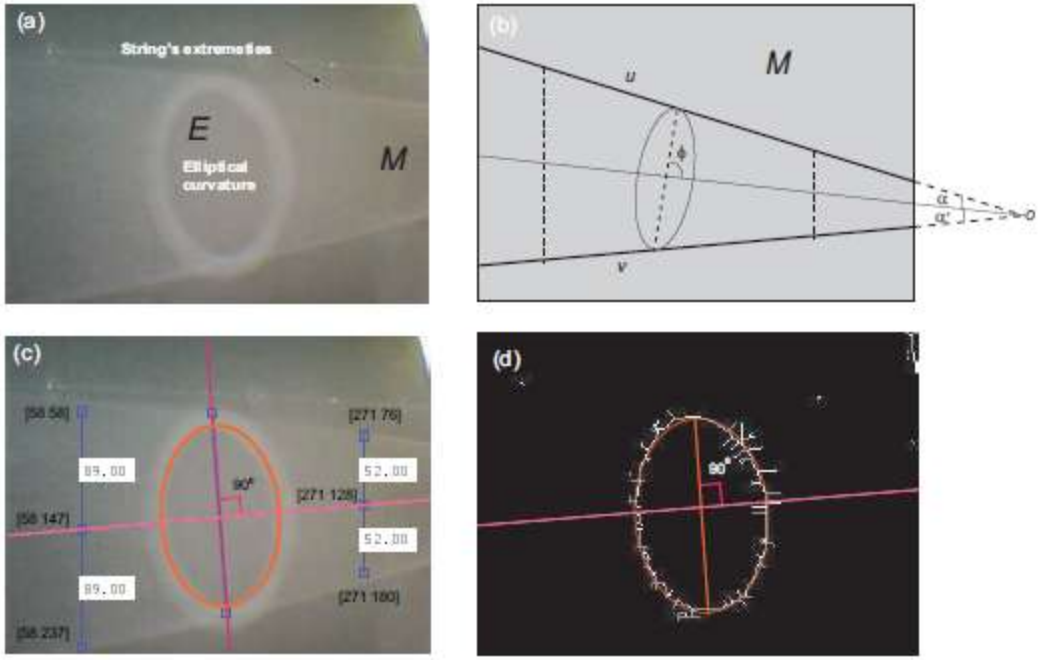


Experimental setup

As seen on vibrations experiment in experimental physics lab

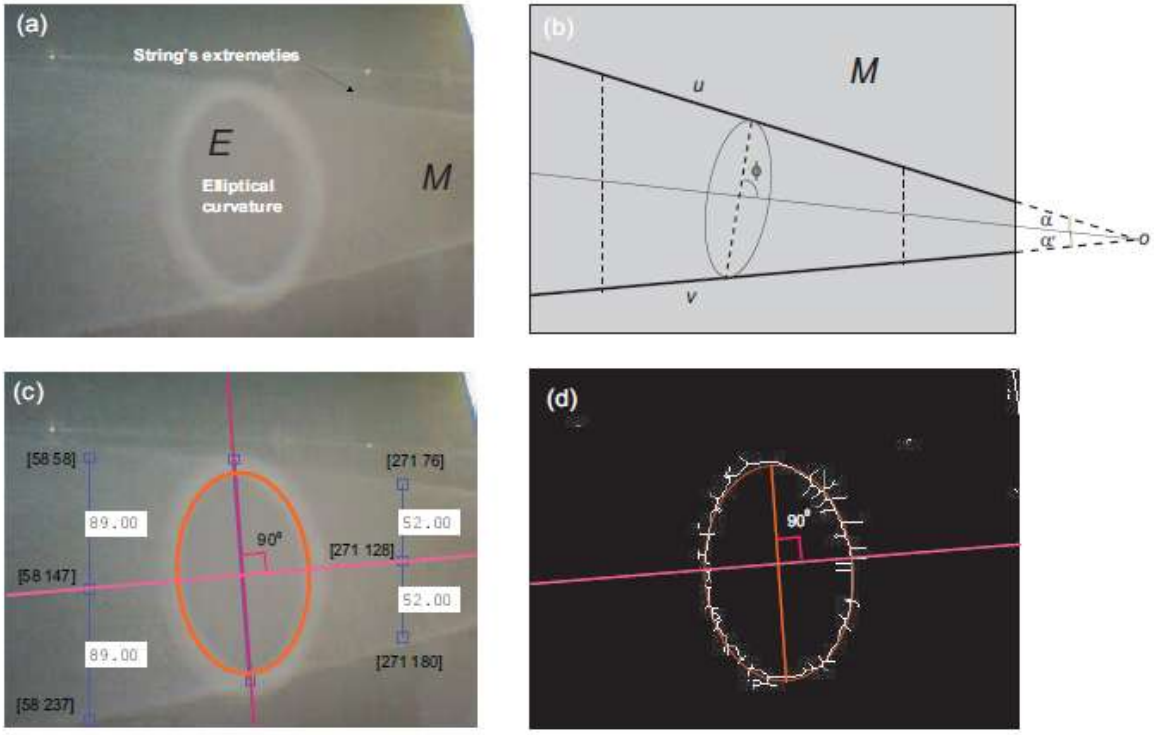


Phase Angle Measurement



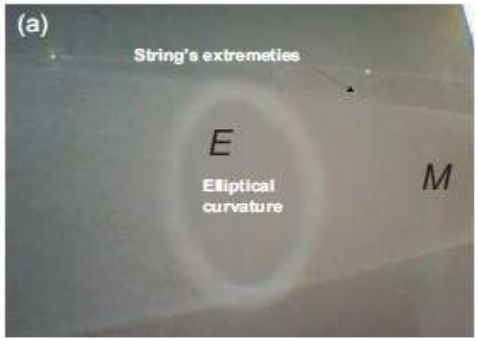
Tilt angle measurement

- Attached a small white tag and verified that the tilt angle is 90 degrees by observing it



Circular and Elliptical trajectories

- Little white tag is attached again and a picture is taken from the camera at 45 degrees.



- Some coordinates lying in the circle are identified manually and after scaling, the images are retraced

Circular and Elliptical trajectories

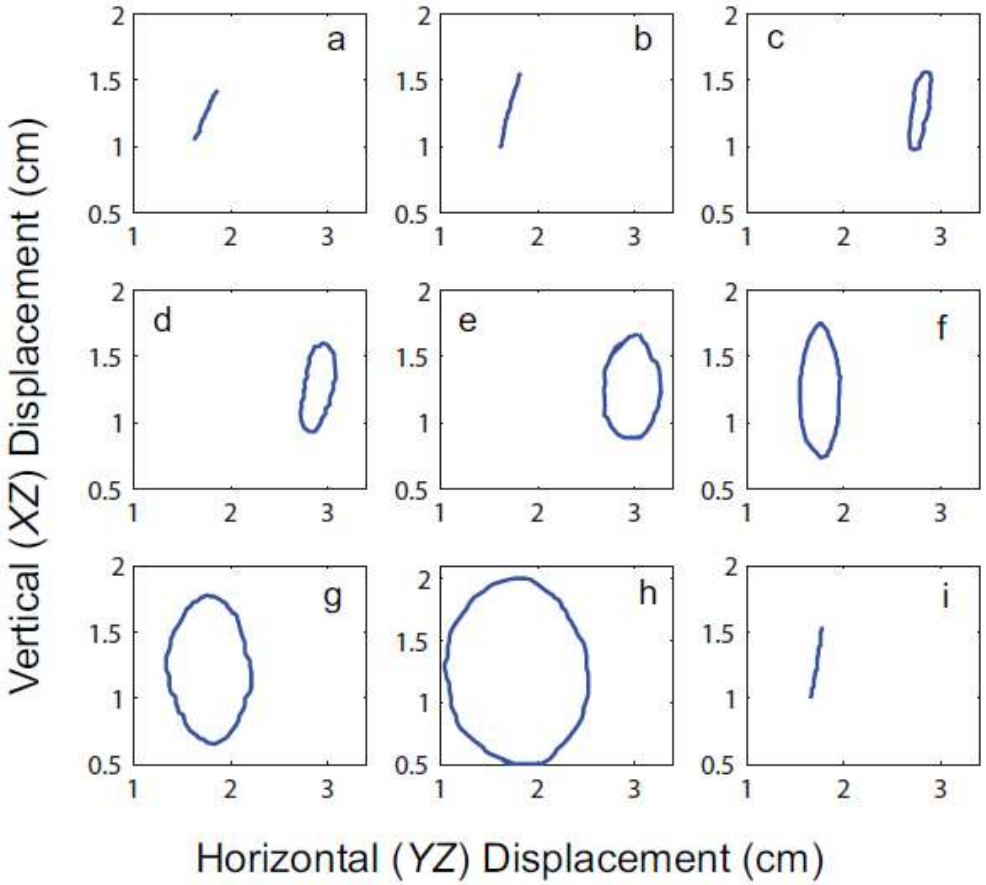
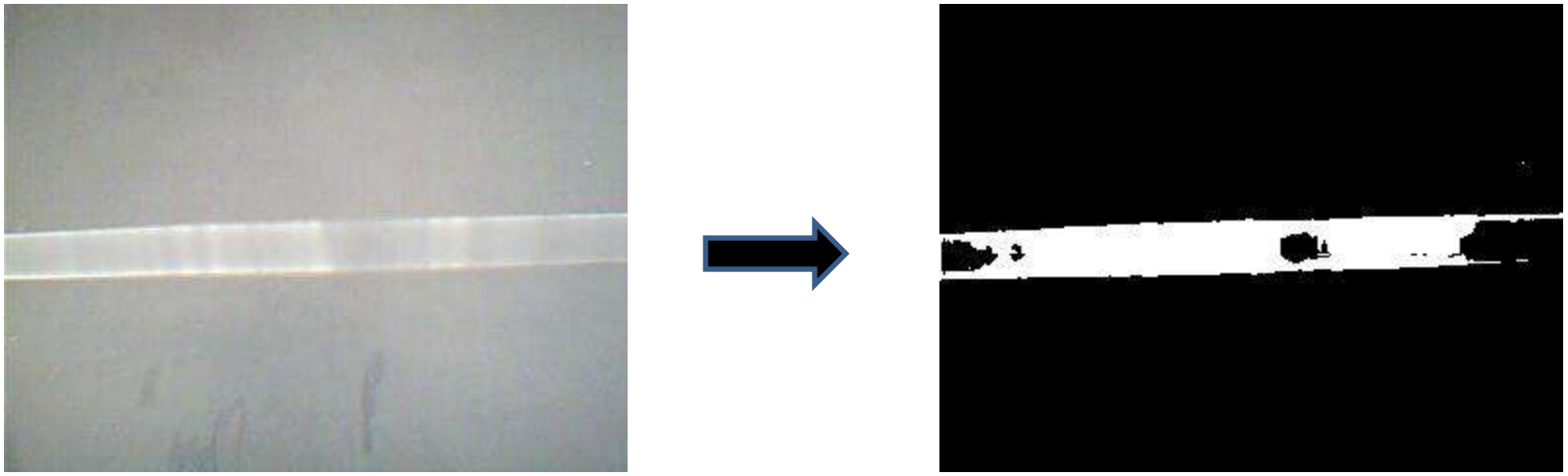


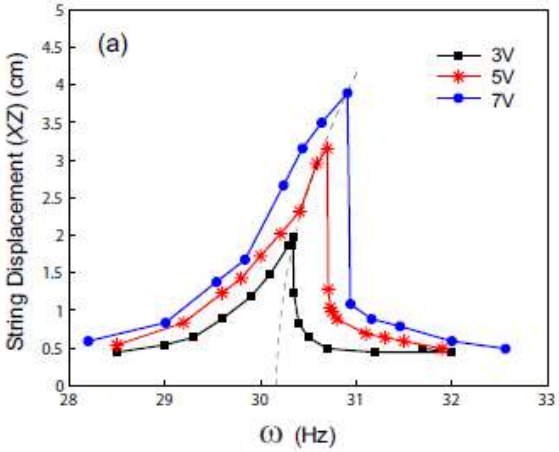
Image processing adapted



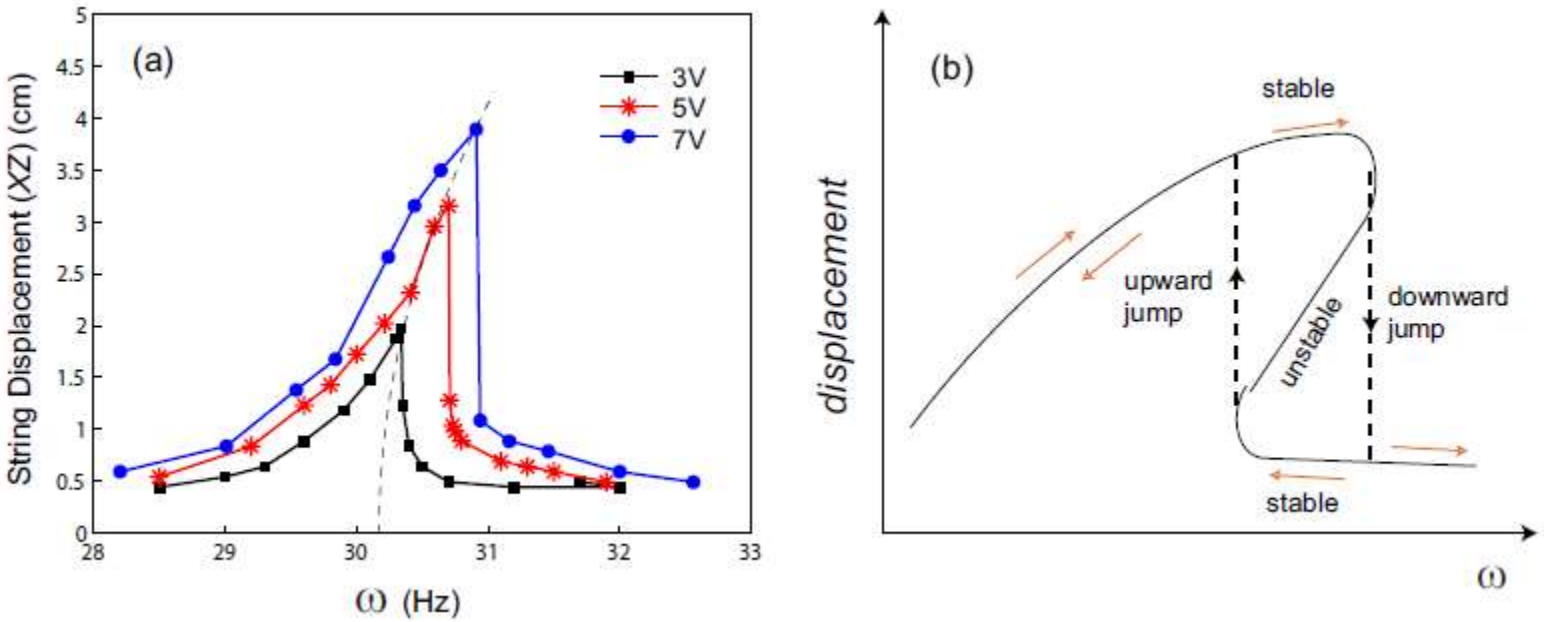
Fold over effect

- The system resembles a quartic oscillator with general form : $\ddot{X} + \omega^2 X + \alpha X^3 + k\dot{X} = A\cos pt.$
- α is the non linearity term and *k* is the damping coefficient
- ω_o differs from ω as it depends on amplitude^2

$\omega_o^2 \simeq \omega^2 + \frac{3}{4}\alpha a^2$ and results in an fold over effect

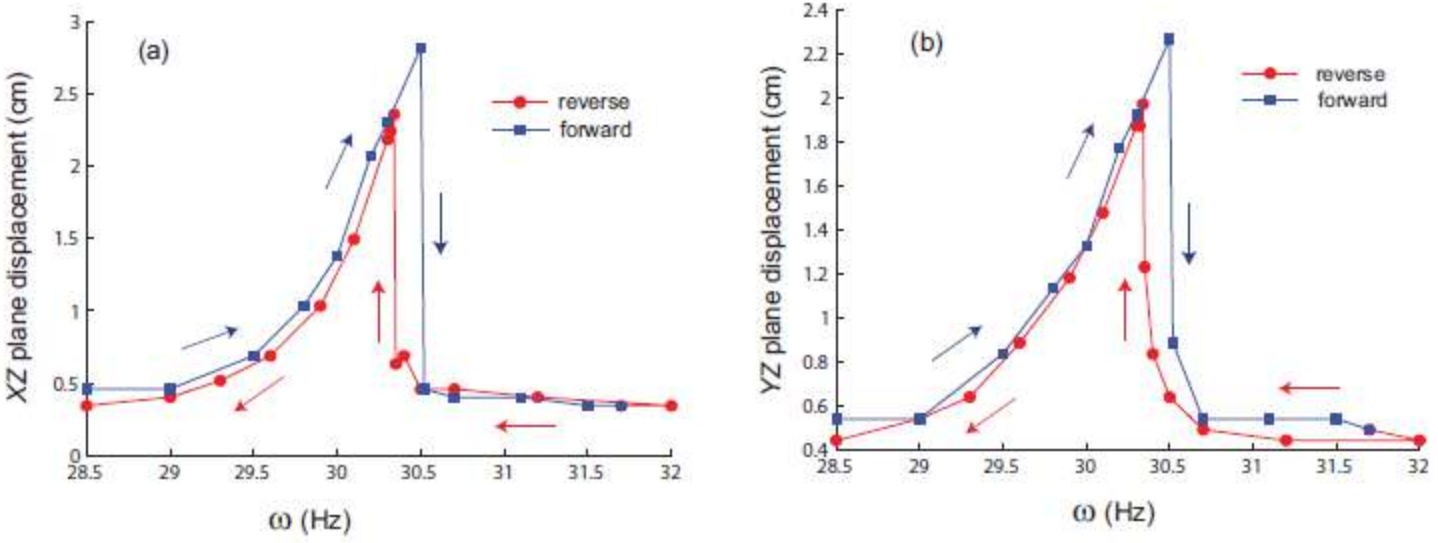


Fold over effect



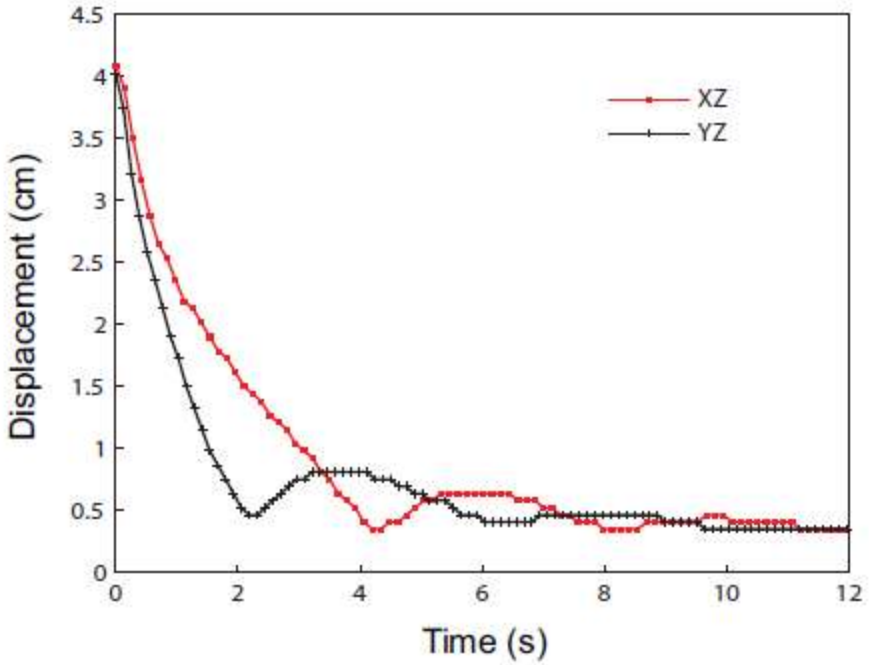
Hysteresis

- The path followed by increasing the frequency is not the same as the path retraced by decreasing the frequency.



Decay of vibrations

- Movement in both of the axis shows different decay patterns.



Sub harmonics

- When the string is vibrated at the second harmonic, we observe that the string is also vibrating with fundamental mode due to parallel superposition of first and second harmonic oscillations in both of the planes

$$\begin{aligned} X(t) &= A\cos(\omega t) + B\cos(2\omega t) \\ Y(t) &= C\cos(\omega t + \frac{\pi}{2}) + D\cos(2\omega t + \frac{\pi}{2}), \end{aligned}$$

Questions