Phenomenon# 18

Estimation, Uncertainty, Confidence and all that

Murtaza Shahzad, Jamal Liaqat, Muhammad Sabieh Anwar

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Every measurement you will ever make in the Phenomenon Lab ends the same way: a list of numbers and a question.

What do these numbers actually tell us?

In this first module you will learn the craft of answering that question.

The apparatus is deliberately simple: a resistor and a capacitor in series, driven by a sine wave (Fig. 1). While the capacitance is fixed at $C = 0.1\ \mu\text{F}$, the resistance R is unknown, and your task is to estimate its magnitude by *extracting it from data*. Data is generated by changing, step-by-step, the frequency f of the sine wave and measuring the amplitude of the output sine wave measured across the resistor, V_R. The input voltage V_0 is kept fixed. Along the way you will meet many facets of understanding data and harnessing to make *estimations*.

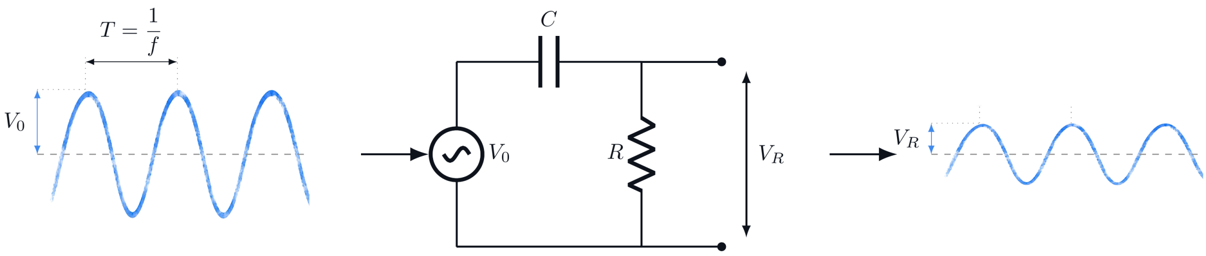


Figure 1: The circuit. A sine wave of amplitude V_0 drives a capacitor C and a resistor R in series; the output V_R is the voltage across the resistor.

The following relationship holds between all of these variables,

$$V_R = \frac{V_0\, R}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}, \tag{1}$$

with the unknown R. The only quantity you control is f. Notice the nonlinear nature of the relationship, the quantity we wish to estimate, R, sits *inside* the square root, tangled with f and C.

[Q 1]. **Set up and measure.** Following the Appendix, drive the circuit with a sine of amplitude V_0 and record V_R at each frequency in the provided list, capturing many cycles per reading.

[Q 2]. **Predict before you plot.** Sketch how you expect V_R to behave as f increases. Is your sketch one of the shapes in Fig. 2? Something else?

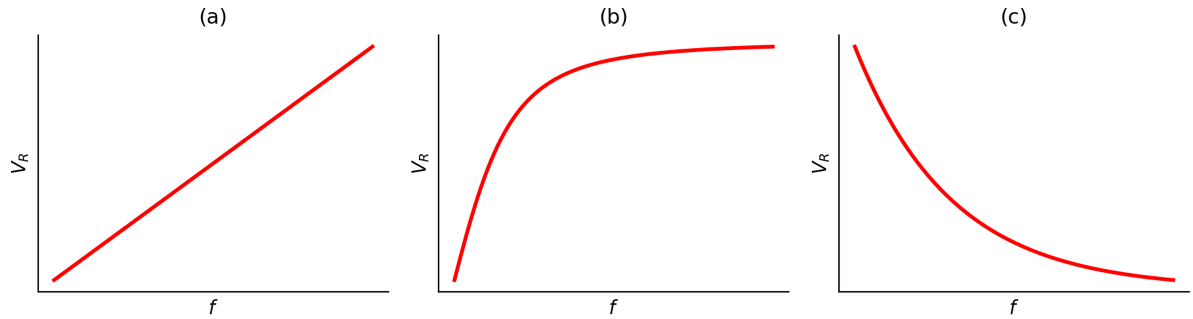


Figure 2: Three candidate behaviours for the raw data.

[Q 3]. **Plot the raw data:** V_R versus f, using the associated Jupyter notebook. Which candidate does your plot resemble? Does it agree with your sketch and with the reading of Eq. (1) above?

[Q 4]. **Now try to extract R.** Look at your curve. Keep looking. Can you determine R from it directly, with any confidence? What, exactly, is stopping you?

Here is the difficulty in plain terms. For a *straight line*, $y = mx + c$, a graph surrenders its parameters immediately: the gradient m and intercept c can be read off, and any constant hiding inside m or c is then yours. For a *curve* the parameter is spread along its whole shape. The remedy is creating new variables: if we can *rearrange* Eq. (1) so that some function of V_R is a straight-line function of some function of f, then R will be forced into a gradient or an intercept, where we can read it. This act is called **linearization**.

[Q 5]. **Linearize Eq. (1) yourself.** Rearrange it into the form $y = mx + c$, where y is built only from measured quantities (V_R, V_0) and x only from f. State clearly: what is your y, your x, and where does R appear, in the gradient, the intercept, or both?

[Q 6]. **Plot your linearized variables.** Does the plot resemble candidate (a) of Fig. 3 — a straight line? If it is still curved, revisit your algebra. Fit a straight line with the notebook, and extract R from it.

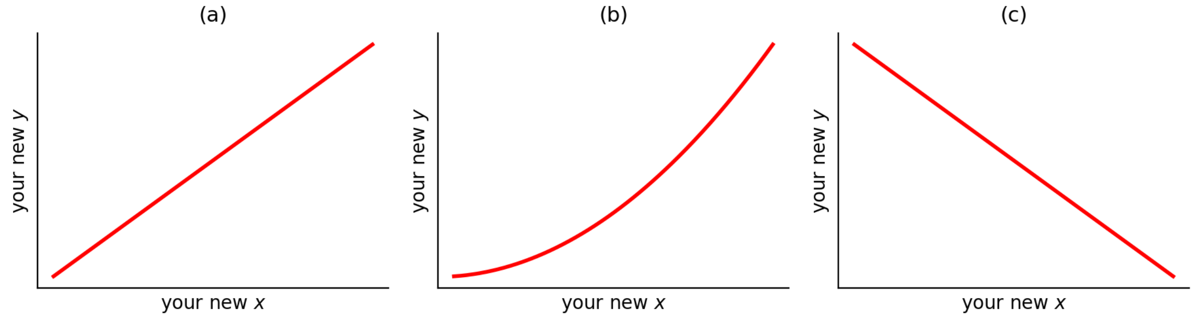


Figure 3: Candidate outcomes for a linearized plot. Only a straight line means the linearization has worked.

[Q 7]. **Find a *different* linearization.** Eq. (1) can be rearranged into a straight line in more than one way. Find a second one. hint: think about logarithms.

[Q 8]. **Compare.** You now hold two values of R from the *same* measurements. Are they identical? Should they be?

How trustworthy is your estimate of R?

Your V_R values carry noise, and that noise must travel through every transformation you applied — squaring, reciprocating or logging.

[Q 9]. **Hypothesize first, in writing.** Your two linearizations transformed V_R differently. Will a small error in V_R matter *equally* in both plots? Ot more in one case than the other? At which end of the frequency range? Commit to a prediction and a reason before computing anything.

[Q 10]. **Guess or measure your uncertainty.** Let's assume that all the voltage readings are off by 10%. From the many cycles in each recording, compute the amplitude of every individual cycle. The scatter (standard deviation) of these per-cycle amplitudes can be considered as uncertainty u_{V_R} of your amplitude. Tabulate u_{V_R} for each frequency.

[Q 11]. **Propagate.** For each linearization, carry u_{V_R} through your transformation into an uncertainty u_y on your plotted variable (the notebook walks through the derivative rule). Plot both linearizations *with error bars*. Which prediction from your hypothesis survived contact with the data? What happened to the error bars at small V_R under a reciprocal? Under a logarithm near $V_R \to V_0$?

[Q 12]. **Propagate to R.** Carry the uncertainty of the fitted gradient (or intercept) through to u_R, and state your result as $R \pm u_R$ for each route. Do the two routes now agree *within uncertainties*?

Do all points deserve an equal say? (Weighted fitting)

Your error bars are not all the same size, yet an ordinary straight-line fit treats every point as an equal witness.

[Q 13]. **Hypothesize first.** Should a point with a large error bar count as much as a point with a small one? If not, which of *your* points should dominate the fit, and in which direction would you expect R to move once they do?

[Q 14]. **Fit with weights.** Repeat the straight-line fit weighting each point by the inverse of its uncertainty, and extract $R \pm u_R$ again. Compare against the unweighted result: did R move the way you predicted? Did u_R shrink? Which value would you report, and why?

Summary

It is worth stepping back. You took one set of noisy measurements and, from it, produced a defensible number: $R \pm u_R$. To get there you (i) re-plotted the same data until an invisible parameter became a readable gradient or intercept; (ii) let the scatter of your own data tell you its uncertainty, and watched that uncertainty be reshaped by your own transformations; and (iii) gave your most reliable points the loudest voice in the fit. Every experiment you meet from now on ends with exactly these three moves.

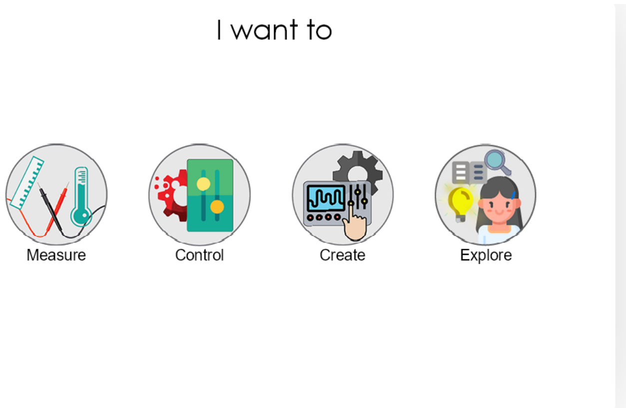
Appendix

Driving and recording the RC circuit with PhysLogger

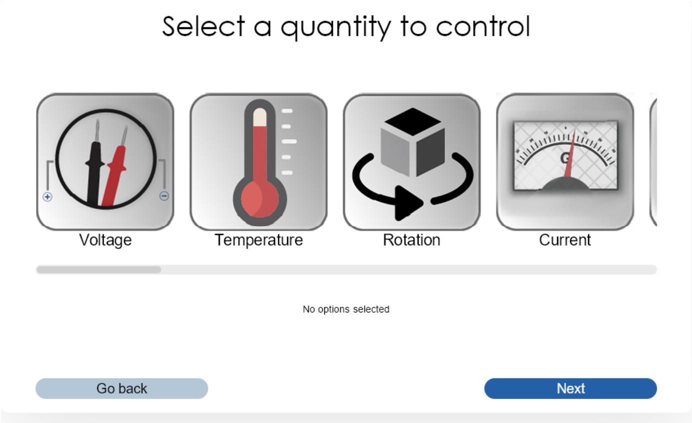
Make sure the PhysLogger is connected to the computer with the USB type-C cable, and that the RC circuit is wired to one of the analog channels. The setup has two halves: first we *generate* the input sine wave (**Control**), then we *record* the output voltage across the resistor (**Measure**). Follow the two sequences below in order.

A. Generating the input sine wave (Control)

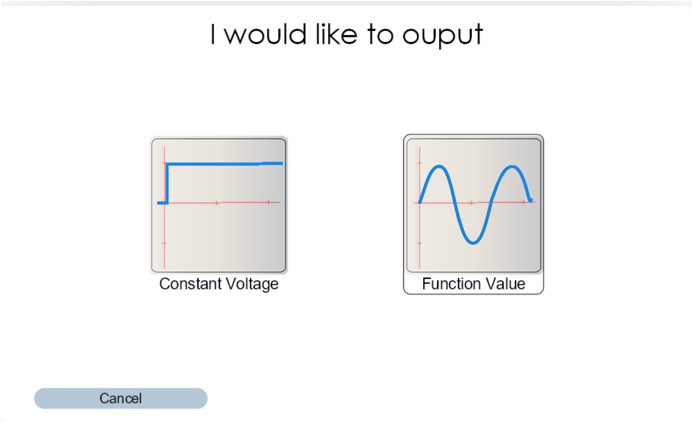
Step 1. On the opening **I want to** screen, select **Control**.



Step 2. Select **Voltage** as the quantity to control, then press **Next**.

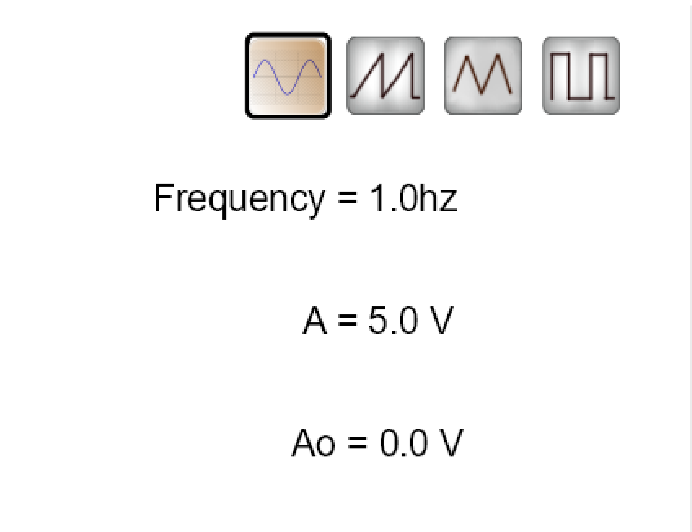


Step 3. When asked what to output, select **Function Value**.

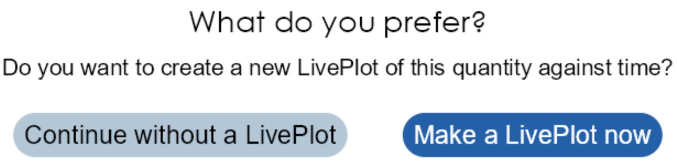


Step 4. Select the analog output channel that your RC filter is connected to, then press **Next**.

Step 5. Choose the **sine** waveform (first icon), and set the **Frequency** and amplitude **A** to the values you want to drive the circuit with.

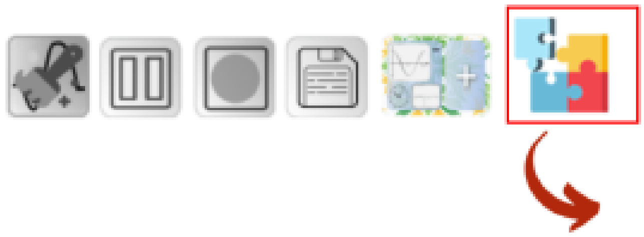


Step 6. When prompted, press **Make a LivePlot now**, so that the generated signal is visible on screen.

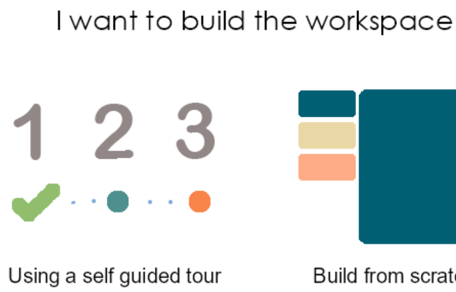


B. Recording the output voltage (Measure)

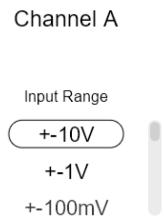
Step 1. Go to the **Extensions** menu.

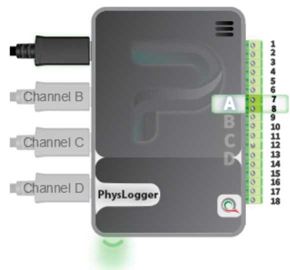


Step 2. Under **I want to build the workspace**, choose **Using a self guided tour**. This brings back the **I want to** screen: this time select **Measure**.



Step 3. Select **Voltage**, and then the analog input channel that the RC filter's output (across the resistor) is connected to, with a suitable input range.





Step 4. On the live plotting window, add a graph using the graph icon (the one marked with a +). Then open the **Quantities** menu using the icon at the bottom of the toolbar, and **drag** the measured voltage from **Soft Quantities** onto the y-axis of the new graph. The recorded signal now appears on its own graph.