Verifying Gas Laws *

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This experiment provides a vivid introduction to the famous gas laws governing the behaviour of a gas under different conditions. Using modern sensors and data acquisition techniques we will demonstrate and understand the quantitative prediction of these laws.

KEYWORDS

Ideal Gas · Charles's Law · Boyle's Law · Pressure · Work · Internal Energy · Data Acquisition

1 Experimental Objectives

In this experiment, we will,

1. interpret the laws of thermodynamics,
2. learn and understand ideal gas laws by verifying Amontons's, Boyle's and Charles's laws,
3. correlate proportionalities between pressure, volume and temperature by equations and graphs,
4. learn the use of transducers to measure pressure, temperature and volume,
5. apply data acquisition techniques for scientific experimentation.

2 Theory

2.1 First law of thermodynamics

The basic gas laws we learn from school are all special cases of the first law of thermodynamics that we now describe. We are all aware that energy is always conserved. Applying the same principle to thermodynamics, we can write an equation that relates different variables such as heat entering a system, work done on the system and the system's internal energy. Heat is defined as the energy transferred from a warm to a cooler body as a result of temperature difference between them.

Consider a gas contained in a cylinder. We call the gas 'our system'. If we heat it, this energy may appear in two forms. First, it can increase the internal energy U of the system. The change in the internal energy is represented by,

$$U_B - U_A = \Delta U \tag{1}$$

where U_B and U_A represent the final and initial internal energies. Second, the heat input to the system can also expand the gas. This increases the volume and work is done by the system on its surroundings, reducing the internal energy content of the system itself. Therefore, heat entering the system Q manifests as an increase in internal energy and work done by the system on its surroundings. Expressing this mathematically, we obtain,

$$Q = \Delta U - W \tag{2}$$

where $-W$ is the work done *by the system on surroundings*. Alternatively, $+W$ is the work done *by the surroundings on the system*. The equation above can be recast as,

$$\Delta U = Q + W, \tag{3}$$

Implying that the change in internal energy can be attributed to the heat energy entering the system as well as the work done by the surroundings on the system.

Equation (3) defines the first law of thermodynamics that is merely a statement of energy conservation. Writing equation (3) in differential form we obtain the useful expression,

$$dU = dQ + dW \tag{4}$$

We will now see how the first law leads to the famous gas laws.

2.2 Amontons's Law

A French physicist named Guillaume Amontons, in the early 17th century, discovered the fact that the pressure of a gas is directly proportional to its temperature when kept in a constant volume. This relationship between the pressure and the temperature of a gas goes by the relatively less known name, the **Amontons's law**. It is mathematically expressed as:

$$P \propto T. \tag{5}$$

For comparing the same substance under two different sets of conditions, the law can also be written as:

$$P_1 T_2 = P_2 T_1. \tag{6}$$

Now, temperature is a measure of the average kinetic energy of molecules in a system. From the perspective of kinetic theory of gases, this relationship is simple to understand. As the kinetic energy of a gas increases, its particles collide with the container walls more frequently, thereby exerting extra pressure. Remember that pressure is the average force exerted by the molecules on the wall per unit area.

2.3 Isochoric (constant-volume) process

In Amontons's law, the volume of the gas is assumed to remain constant. Let's see how to apply the first law to such constant-volume process, also called an isochoric process.

Suppose we have n moles of a gas inside a sealed container with rigid and fixed boundaries. This implies that the gas cannot expand, hence $dW = 0$. If we supply heat energy to this container, the thermal energy input will only increase the internal energy of the system, since there is no change in volume nor any work Equation (4) becomes,

$$dQ = dU \tag{7}$$

meaning that all the head added brings a change in the internal energy of the system. The heat required to raise the temperature of n moles of gas at constant volume is given by,

$$dQ = nC_v dT \tag{8}$$

where C_v is the molar specific heat capacity of the gas at constant volume, and dT is a small change in temperature. On a PV diagram, such a process is shown as Figure (1).

2.4 Charles's Law

When air in a hot-air balloon is heated and it expands, the net weight becomes less than the weight of an equivalent volume of cold air, and the balloon starts to rise. When the gas is allowed to cool, the balloon returns to the ground. This was first demonstrated by Joseph and Etienne Montgolfier in the late 17th century when they used a fire to inflate a spherical balloon about 30 feet in diameter that traveled about a mile and one-half before coming back to earth. Jacques Charles replicated this work and noticed that the volume of a gas is

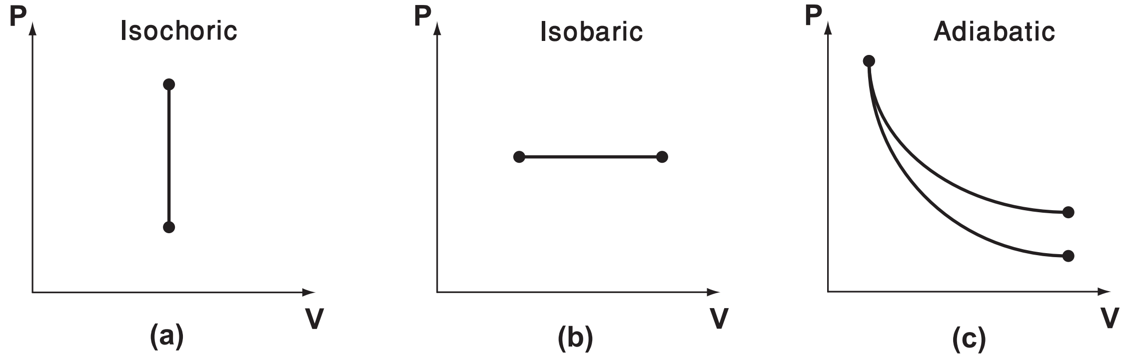

Figure 1: PV diagrams representing constant volume (a), constant pressure (b) and constant temperature (c) process.

directly proportional to its temperature which was later formulated as **Charles's law**. This is mathematically written as,

$$V \propto T. \tag{9}$$

2.5 Isobaric process - Charles's law assumes a constant pressure.

Let's now work out how the first law deals with this situation.

Suppose the gas is in a container fitted with a movable piston. The piston is movable causing the volume of the gas to change. The weight of the piston is constant and is balanced by an equal and opposite force due to the gas pressure. If we now heat this container, the gas molecules will gain energy and will perform work to push the piston out causing the volume of the gas to change by dV, but the pressure p remains unchanged. The heat absorbed by 'n' moles of gas at constant pressure is given by:

$$dQ = nC_p dT \tag{10}$$

where C_p is the molar specific heat capacity at constant pressure. From Equation (4) we obtain for this isobaric process,

$$nC_p dT = dU + pdV. \tag{11}$$

In an ideal gas, the internal energy is manifest solely by the temperature. Therefore, if the temperature increment at constant pressure has the same value as the temperature increment at constant volume, the increase dU of the energy must also be the same. Hence, using Equations (7), (8) into Equation (11) yields the relation,

$$nC_p dT = nC_v dT + pdV. \tag{12}$$

2.6 Boyle's law

A British scientist, regarded as one of the founders of modern chemistry and a pioneer of the experimental method, Robert Boyle is best known for his work referred to as the "Boyle's law". It was inspired by the prior work of an Italian scientist Torricelli who performed the first measurement of barometric pressure and exhibited that air has weight. Boyle referred to his experimental work on gases as "spring of air". These experiments were based on the observation that gases are elastic which means they return to their original size and shape after being stretched or squeezed.

Boyle studied the elasticity of gases in a J-tube, see Figure 2. By adding mercury to the open end of the tube, he trapped a small volume of air in the sealed end and then noted what happened to the volume of the gas as he added mercury to the open end.

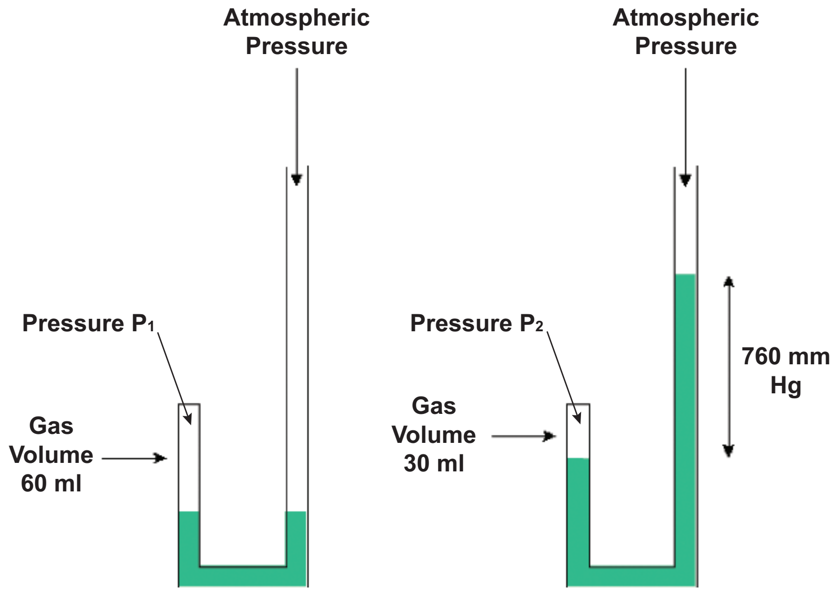

Figure 2: An illustration of the J Tube used in Boyle's experiment

Boyle observed through this demonstration that the product of the pressure times the volume for any measurement in this tube was equal to the product of the pressure times the volume for any other measurement. Mathematically,

$$P \propto \frac{1}{V} \tag{13}$$

or alternatively,

$$P_1V_1 = P_2V_2. \tag{14}$$

This is came to be known as **Boyle's Law**.

Q 1. What is the ratio of pressures P_1/P_2 of the gas inside the sealed end?

2.7 Adiabatic process

Suppose the gas contained in a piston fitted container is insulated from the surroundings so that no heat exchange takes place between the gas and the surroundings. If the gas does some work, it will cause a change in the temperature of the gas. Hence, from Equation (4) we have $dU = dW$. Any such process in which there is no heat absorbed or given away is known as an adiabatic process.

Now imagine the gas doing work on the piston as a result of which the volume of the gas changes by an amount dV. The work done is,

$$dU = dW = -pdV. \tag{15}$$

Since the change in internal energy is $nC_v dT$, we obtain,

$$nC_v dT + pdV = 0. \tag{16}$$

Using Equation (16) and the idea gas law $pV = nRT$, one can prove that in an adiabatic process.

$$pV^\gamma = \text{constant}. \tag{17}$$

where $\gamma = C_p/C_v$ In the following sections, we will apply the above principles and observe them in the real world.

Q 2. Derive Equation (17). It will help to use $d(pV) = d(nRT)$ which means $pdV + Vdp = nRdT$ as well as the result in Equation (11).

3 Introduction to the Apparatus

The overall experimental setup is shown in Figure (4). It consists of sensors that are connected to a data acquisition card and are programmed to collect data through a Labview program. A hot plate is used as the source to provide heat energy to the air molecules contained inside a small 25 ml conical flask sealed using a cork and teflon tape. Using a rubber tube, the flask is connected to either a pressure sensor or a sealed syringe depending upon the law you are asked to verify. The flask also has a thermocouple retrofitted inside to measure the temperature change of the air. You may heat the flask directly or use a secondary beaker filled with water to provide a uniform temperature bath using the hot plate. Hence, we can measure pressure using a pressure transducer, temperature using a thermocouple and volume of syringe using a linear potentiometer.

A more detailed description of the components used in the apparatus are,

1. **Beakers and Hot plate** for heating water. The heating is achieved by the provided hot plate. The maximum temperature of the hot plate is around 400 $°C$.

2. **Conical flasks** We will use a 25 mL conical flask which is connected to a glass syringe through a rubber tube.

3. **Glass Syringe** The experiment uses a 10 mL glass syringe (Cole Palmer). Glass syringes are easily breakable, therefore they must be handled with care . If you feel friction in the piston, place your finger at the tip and press or pull the piston back and forth. Until the the piston moves smoothly.

Characteristics	Symbol	Min	Typ	Max	Unit
Pressure range	P_{OP}	20	-	400	kPa
Supply voltage	V_S	4.64	5.0	5.36	V_{dc}

Table 1: Specifications of the pressure sensor.

4. **Piezoresistive pressure transducer** A pressure sensor (MPXH6400, Freescale Semiconductor) is used to monitor the pressure variation inside the conical flask. The sensor is connected to the flask through a pipe and luer connectors. Three wires labeled V_{IN}, GND and V_{OUT} are attached to the terminals 2, 3 and 4 respectively, see Figure (3). A fixed voltage V_{IN} of +5 volts is applied at pin 2 and the output of the sensor is read from pin number 4 through the DAQ system. Some important characteristics of the pressure transducer are given in Table 1.

   Transfer function:

$$V_{OUT} = V_{IN} \times (0.002421 \times P - 0.00842) \tag{18}$$

$$V_{IN} = 5.0 \pm 0.36 V_{DC} \tag{19}$$

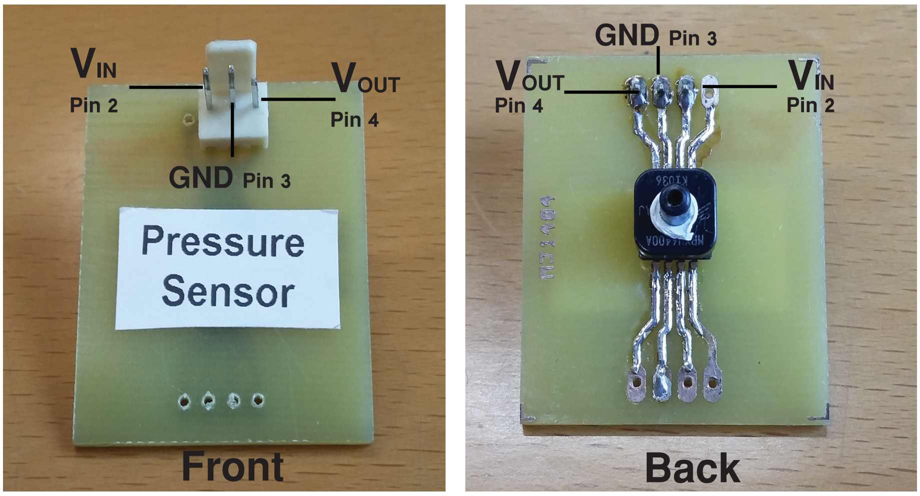

Figure 3: Pressure transducer's pin numbers

5. **Thermocouple (J type)** Two thermocouples connected to the DAQ are used in this experiment. One is placed inside the beaker containing water to record its temperature. The second thermocouple is suspended inside the conical flask and records the temperature variation of the air.

6. **Potentiometer** A linear potentiometer is used to measure the volume change inside the glass syringe. It has two pairs of terminals at opposite ends. They are numbered as 1, 2, 2 and 3 respectively. The terminals numbered 2 are internally connected with the potential divider pin. Using DAQ card we provide +5 volts to pin number 1 and ground to the pin number 3.

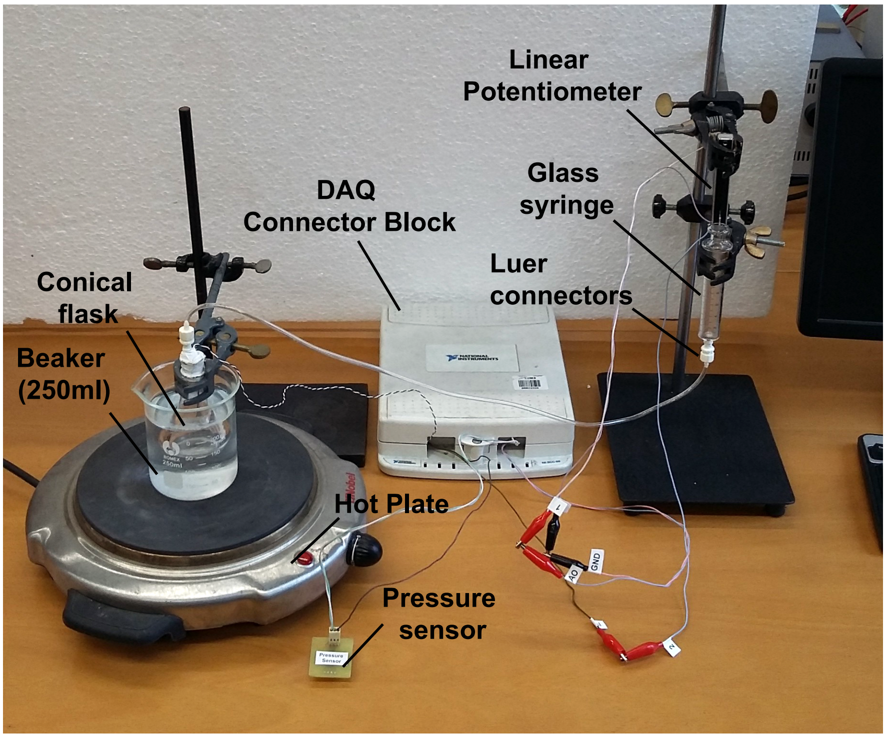

Figure 4: The complete apparatus

4 The Experiment

4.1 Calibration of the potentiometer

In our experiment, we will use a linear potentiometer to measure the volume change of air in the glass syringe. The change is recorded in terms of voltage that is recorded electronically through the DAQ. We subsequently convert the linear displacement into volume using an equation. The purpose of calibration is to find the functional relationship between voltage output of the potentiometer and volume.

- Make connections to the DAQ card according to Figure (5).
- Use the LabVIEW program to see the voltage readings from the potentiometer.
- Note down the initial reading of the voltage when the volume of syringe is zero.
- Now start increasing the volume and measure the corresponding voltage. Try to keep the increments small to get a large number of readings.
- Plot voltage versus volume and fit the data to obtain a calibration equation for the potentiometer. You will use this equation as an input to LabView codes in the subsequent steps.

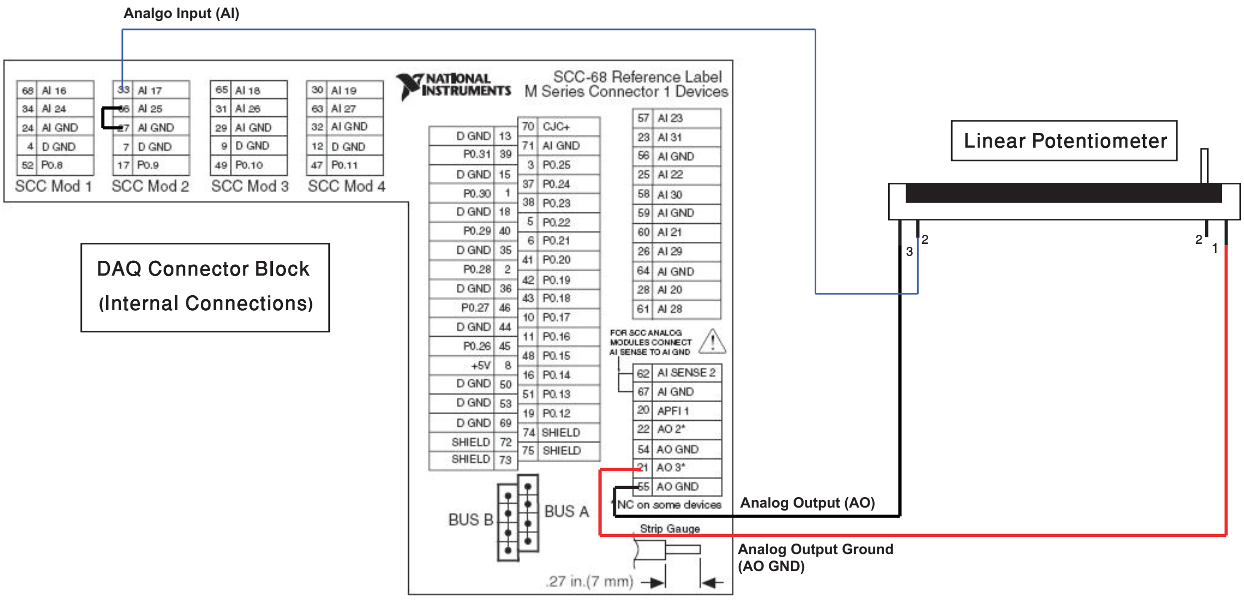

Figure 5: Schematics of the Potentiometer with DAQ card

4.2 Verifying Charles's law

In the present section, we aim at verifying the Charles's law and predicting the value of the absolute zero of temperature. The experimental arrangement is shown in Figure (4). Complete and verify the assembly.

- Connect the 10 mL glass syringe to 25 mL flask with the help of the provided plastic tubing. Make sure there is no droplet of water inside the flask. This may produce extra pressure due to the evaporation of the water molecule and not due to the expansion of air.
- To properly seal the flask, use silicone or teflon tape. Place the flask in a beaker containing water and heat it on a hot plate.
- Download and open the LabView code **temperatureversusvolume.vi** and run the file.
- As the water is heated, you will notice a change in the volume of the gas, making the piston of the syringe to move upwards. This change in volume is recorded using the linear potentiometer whose output is a variable voltage connected to the DAQ.
- The change in temperature is monitored using the thermocouple placed inside the flask and connected to the DAQ.
- Using the recorded data, plot the curve between volume and temperature and calculate the value of the absolute zero by extrapolation. The predicted value is $-273°$C.
- Repeat the same experiment by directly heating the ask on a hot plate and calculate the absolute zero temperature.

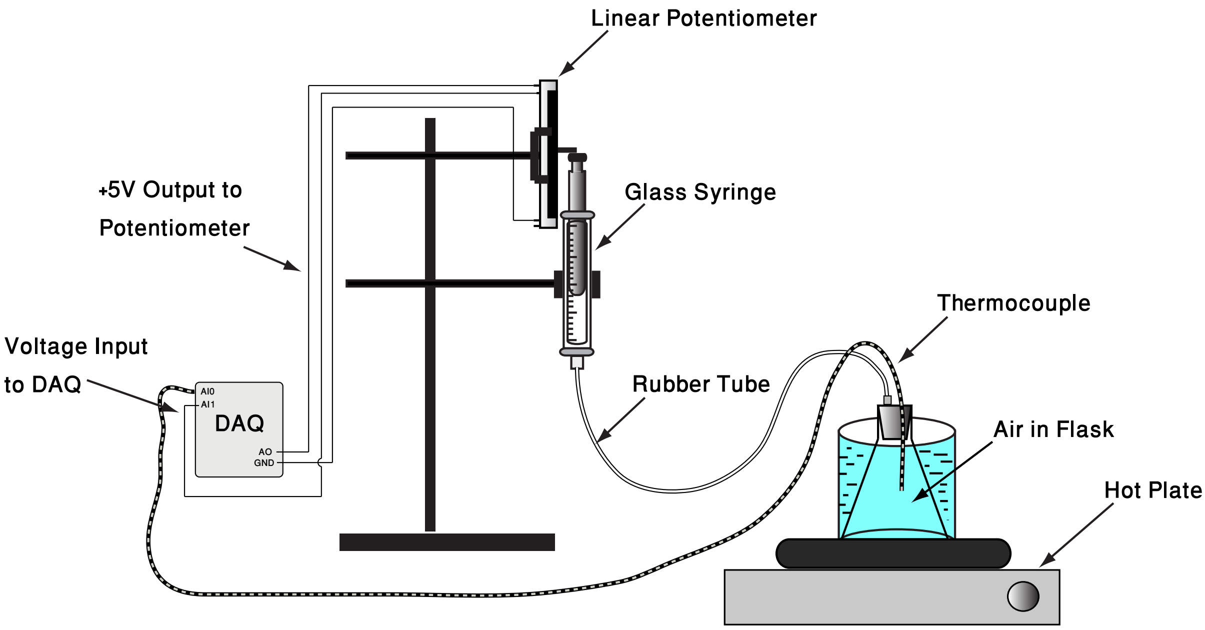

Figure 6: Experimental setup for verifying Charles's law, keeping pressure constant. Ideally, this is an isobaric process.

4.3 Verifying Amontons's Law

- Connect the 25 mL flask to the pressure sensor as shown in Figure (7). Make sure all connections are sealed properly.
- Place the flask in the hot water reservoir.
- Download and open the LabView file named as **pressureversustemperature.vi** and click Run. The values for temperature and pressure will be recorded and displayed in real-time on screen.
- Import data in MATLAB and Plot a curve for pressure versus temperature.
- Estimate the value of absolute zero temperature.

  Note: Do not heat the flask for a long time. The maximum temperature should not exceed 90°C.

4.4 Verifying Boyle's Law

- Connect the syringe directly to the pressure sensor. How would you change the volume of air in the syringe while keeping the temperature constant?
- Take at least 10 readings and plot a graph of pressure against volume.
- Calculate the value for γ.
- Since work is done, therefore the temperature must change. However, the experimental observations contradict this. Explain why?

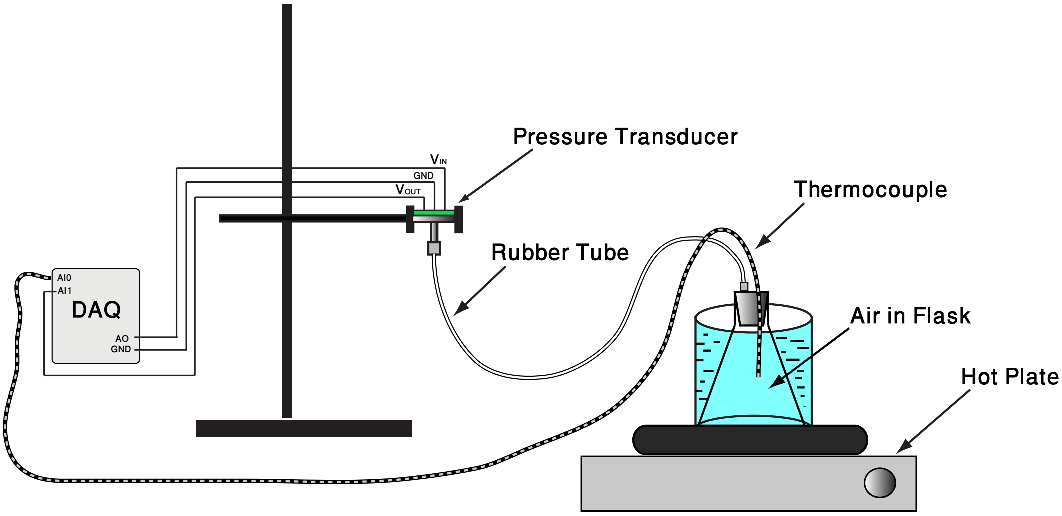

Figure 7: Experimental setup for Pressure Vs Temperature, keeping volume constant.

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