Mach-Zehnder Interferometry and Erasure of 'Which-Path' Information*

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One of the most astounding and bewildering depictions of quantum mechanics is the single-particle interference experiment. If a source that emits only one particle (such as photons or electrons) at a time, is used in a classic double slit experiment, an interference patterns will emerge. This pattern is built up over multiple passes of identical particles. Interestingly, the pattern could be switched on and off even after light has emerged from the interferometer.

The Mach-Zehnder interferometer is a useful device for exploiting the wave nature of light and observing various aspects of interference. The goal of this experiment is to familiarize students with this kind of an interferometer. We will observe interference of light, and see conditions under which interference can be washed out, and ultimately recovered.

Although we will not use single photons and so we cannot truly observe the quantum behavior of single-photon interference, but still the experiment brings home fundamental ideas about interference, the uncertainty principle, delayed choice, the foundations of quantum mechnics as well as the nature of light. An excellent article that we highly encourage reading before embarking on this experiment is Kwiat's "*A Do-it Yourself Quantum Eraser*" [1]. The Mach-Zehnder interferometer also provides a popular route for implementing quantum computers operating with photons [2]. A similar experiment was described in the reference [3].

KEYWORDS: Interferometer · optical path length · polarization · Jones calculus · information erasure · uncertainty principle

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1 List of Equipment

1. Optical breadboard

2. HeNe laser with mount, such as HR020, Thorlabs

3. Kinematic mirror mounts, KM100-E02, Thorlabs

4. Silver mirrors, ME1-P01, Thorlabs

5. Non polarising beam splitters, BS013, Thorlabs

6. Linear polarizers, LPVISE100-A, Thorlabs

7. Plano-convex lens $f = 25.4$ mm, LA1027, Thorlabs

8. Photo detector, DET36A/M, Thorlabs

9. Post holders, PH3/M, PH3-ST, Thorlabs

10. Stainless steel posts

11. Oscilloscope, BK Precision 2534

12. Irises

13. IV converter (homemade)

14. Multimeter for voltage measurement

2 Experimental Objectives

The experimental objectives include:

1. optical alignment of an interferometer,

2. recording and interpreting intensity patterns with and without the presence of polarizers in different orientations,

3. exploring the effect of obtaining, and subsequently erasing 'which-path' information,

4. understanding the quantum erasure experiment itself and how this relates to the uncertainty principle.

3 The Mach-Zehnder Interferometer

The Mach-Zehnder interferometer is an amplitude splitting interferometer which divides the incoming beam into two paths. The path differences result in an interference pattern. This contrasts with a waveform splitting device such as the classic Young's double slit interferometer. We are going to use a Mach-Zehnder interferometer to verify the basic principles of polarization, obtainment and eventually the erasure of 'which-path' information. This so called 'erasure' experiment underpins deep questions about quantum mechanics as we will highlight later.

The first step of course is setting up and aligning the basic interferometer (see next section). The interferometer's layout is shown in Figure 1.
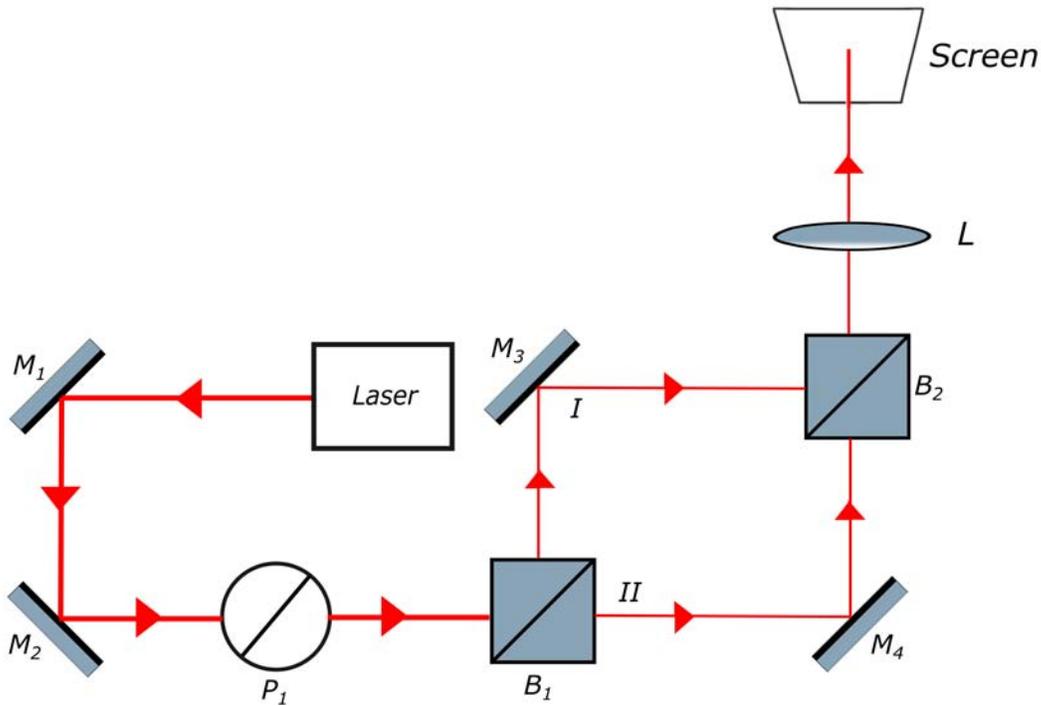

Figure 1: Schematic of the Mach-Zehnder interferometer. Herein M_1, M_2, M_3, M_4 are highly reflecting mirrors, B_1, B_2 are non-polarizing beamsplitters and L is a convex lens of focal length 25.4 mm. The red line shows the conceived path of the laser beam.

As the HeNe's output laser beam is unpolarized, we use a linear polarizer labelled P_1 set at 45° to polarize the input beam. Mirrors M_1 and M_2 steer the path, B_1 produces two beam paths labelled I and II; M_3 and M_4 merely redirect these beams and finally B_2 interferes the

paths I and II. The resulting pattern is finally observable on a screen.

We start the experiment by placing the laser in its mount and aligning the beam by a procedure that is generally called 'walking' the beam.

3.1 Procedure for Optical Alignment

'Walking the beam' is a process of aligning a laser beam using two adjustable mirrors in such a way that it will reach a specific point in space called the target.

It can be a difficult and frustrating task to properly adjust the knobs that control the horizontal and vertical angles of the mirrors. Without making the correct sequence of adjustments it is even possible to move further away from the target. Fortunately there is a specific procedure that can be followed that would eventually work or at at least, in most cases [4].
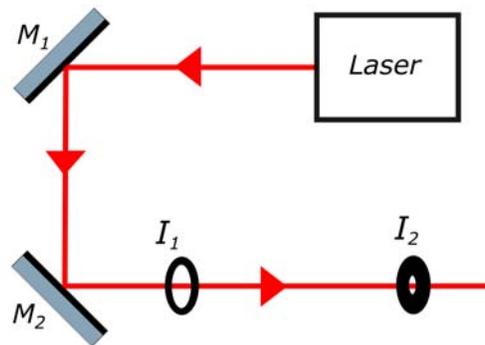

Figure 2: Schematic for beam walking. The red line shows the conceived path of the laser.

1. Place two silvered mirrors, M_1 and M_2 in kinematic mounts, such that the laser hits the center point of the mirrors, and is reflected by 90° by each of the mirrors. Place two irises, I_1 and I_2, further apart on the optical breadboard in the path of the beam after being reflected from M_2. Both irises must be at the same height and in line. You can use the provided scale for height adjustments to the beam.

2. Now keeping the second iris closed and first iris open, adjust the beam to fall on the center of the second iris by adjusting M_1.

3. Now close the first iris. Adjust the beam to fall on the center of the first iris by adjusting M_2.

4. You will notice that adjusting M_2 may now cause the beam to deviate from falling on the center of I_2. So repeat the process back-and-forth until no further adjustments are required and the beam passes through the center of both irises. This method of aligning the laser is called 'beam walking'.

3.2 Setting up the interferometer

Once the beam has been aligned, we can set up the other parts of the interferometer. Beam walking, however, only ensures that the beam is aligned between the irises and not beyond them. Therefore, we need to set up the interferometer in the location spanned by the two irises. So remove the irises, I_1 and I_2, and place the remainder of the components on the breadboard as described below. Use Figure 1 as reference.

1. Place the beam splitters B_1, B_2 and the mirrors M_3 and M_4 at 45° as shown in Figure 1. All these components are placed in kinematic mounts, which are equipped with two adjustment screws for vertical and horizontal alignment.

2. Make sure all the beams fall on the center of all the optical components.

3. Place a screen some distance away on one side of B_2. You may block the other side.

4. We suspect that at this stage you will see two dots on the screen coming from the two paths in the interferometer. Adjust the mirrors M_3 and M_4 using their adjustment knobs till you see one unified dot.

5. Now place the plano-convex lens between the screen and B_2 at a distance such that the one unified dot falls as a focused point on the lens. You can use a laser card reader to see this.

6. The lens helps magnify the pattern. Hopefully, now you will see fringes on the screen.

Note that *extremely good alignment* of the laser and the beams coming from the two optical paths is essential to obtaining good fringes. So spend ample time on this initial part of the experiment until you see a sharp interference pattern.

Describe your observations and carefully note down your experiment on your notebook. Interference between the two paths, labelled I and II, of the interferometer results in the appearance of fringes. The two paths carry light with parallel polarization, allowing these

paths to interfere. This implies that after the second beamsplitter, which recombines these paths, it is impossible to tell which path the light has actually taken. It has taken both paths simultaneously. This will be true for classical light from a laser (as is the present case) as well as for single photons. Thus there is complete absence of 'which-path' information, which is necessary for the appearance of interference fringes.

4 Interference, Erasure and All that!

If we set up the Mach-Zehnder interferometer above with a single photon source whose emission rate is so low that there exists only one photon in the whole setup at one point in time, we still observe an interference pattern that builds up over time. This pattern is due to the wave nature of quantum particles. Now if we try to measure which path the photon has actually taken, the interference pattern disappears. This is because in quantum mechanics, measuring a system leads to irreversible changes to the system.

One way of spying on the photon's path could be through labelling or marking the paths. For example, we may label one path (I) by placing a vertical and the other (II) by a horizontal polarizer. The polarization state of the light emerging from the second beamsplitter (B_2) will then provide the 'which-path' information. In such a case, the interference pattern disappears. 'Which-path' information and interference, therefore, are mutually exclusive; The moment you distinguish paths I and II, interference eludes itself and disappears. This phenomenon holds equally true for single or multiple photons. So let's first observe this effect.

Remember, we can also view these results from the perspective of the uncertainty principle. An insightful introduction to this ideas can be found in Hobson's article [5]. If we have no knowledge about the path the light has taken, we are in essence uncertain about it. This allows the momentum (and hence wavelength) to be precise, resulting in the emergence of a visible interference pattern. On the other hand upon obtaining the path information the interference eventually disappears due to the resulting imprecision in the momentum degree of freedom. Interference experiments, therefore, are a nice manifestation of the uncertainty principle.

4.1 Obtaining 'which-path' information and collapse of fringes

In this part of the experiment, we will place P_2 and P_3 in the paths I and II respectively. The mutual angle between the polarizers P_2 and P_3 will be adjustable. If the polarizers P_2 and P_3 are crossed, 'which-path' information is obtained, and the interference pattern collapses while if P_2 and P_3 are parallel, 'which-path' information is absent and interference fringes are observable. These scenarios are depicted in Figure 3. Are you able to observe this effect?

Q 1. Take two polarizers, P_2 and P_3, and place them in each arm of the interferometer. Vary P_3 and observe how the fringes change. Describe your results.
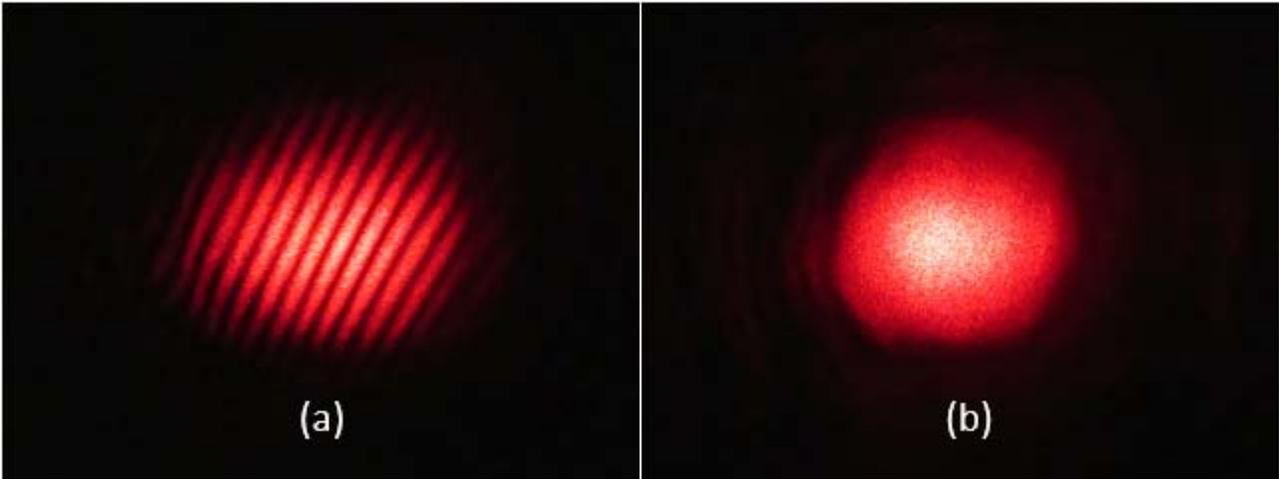

Figure 3: Pattern observed on the screen for (a) parallel polarizers, P_2 and P_3 (b) crossed polarizers.

4.2 Variation of output intensity as a function of polarizer angles

If the mutual angle between P_2 and P_3 is gradually varied between $0°$ and $90°$, fringes of variable visibility are observable. The visibility is defined as,

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{1}$$

where I_{max} (I_{min}) corresponds to the brightest (darkest) intensities in the pattern. An approach alternative to viewing the complete fringe pattern, is to scan just a single fixed point on the screen and observing how the intensity varies as a function of the angle between P_2 and P_3.

Let's first derive the purported intensity variation theoretically. In our experiment, we will keep P_2 (in path II) vertical and P_3 (in path I) at a variable angle θ with the horizontal. For the derivation, we will use the machinery of Jones' calculus [6]. Also see Figure 4.

In Jones' calculus, polarizers act as a transformation and hence are represented by matrices, whereas electric fields are written as column vectors. The light entering B_1 is polarized at $45°$ with respect to the lab frame. It's Jones vector (ignoring the absolute intensity) is:
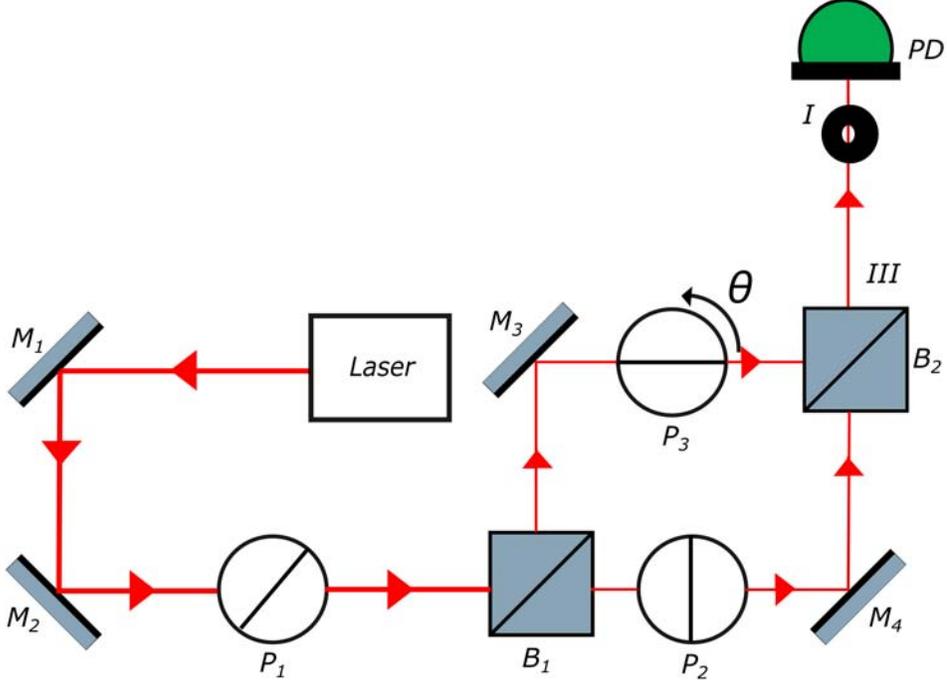
Figure 4: Schematic of the setup that labels the photons. Note that for picking a fixed point, we use an iris (I) in front of the photo-detector.

$$E_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{2}$$

When the beam passes through a perfect beamsplitter, fraction r of it is reflected and fraction t is transmitted. For a 50:50 non-polarizing beamsplitter, $t = r = \frac{1}{2}$. Hence after B_2, the Jones vector for the output electric field that is seen at location III (in Figure 4) is given by,

$$E_{out} = rP_3 r_m r E_{in} + t r_m P_2 t E_{in}, \tag{3}$$

where r_m is the reflection coefficient of each mirror, which we assume to be 1, i.e. each mirror is perfectly reflecting; and P_2, P_3 are the transformation matrices of the respective polarizers. The field in Eq. (3) is a termwise superposition of the fields emerging from paths I and II inside the interferometer. The matrix for a polarizer whose optic axis is at an angle θ with the horizontal is given by,

$$P(\theta) = \begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin^2(\theta) \end{pmatrix}. \tag{4}$$

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In our case, $P_2 = P(\frac{\pi}{2})$ and $P_3 = P(\theta)$. Furthermore, the two beam splitters are assumed to be identical. Inserting these terms into Equation (3) we finally obtain,

$$E_{out} = \frac{1}{4} \begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin^2(\theta) \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{4\sqrt{2}} \begin{pmatrix} \cos^2(\theta) + \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) + \sin^2(\theta) + 1 \end{pmatrix}.$$

Since the intensity is given by the modulus square of the electric field, $I_{out} = E_{out}^* E_{out}$, we obtain for the output intensity observable at the detector,

$$I_{out} = 2 + 2\sin(2\theta) + 2\sin^2(\theta), \tag{5}$$

indicating how the output intensity is expected to vary in relation to the angle θ of P_3.

Q 2. Make sure you understand the derivation given above and can finally reproduce Equation (5).

Q 3. Rotate P_3 through 360°, by manually rotating the rotation mount that holds the polarizers. You can read the angles on the scale provided on the rim of the rotation mount. What changes do you see in the interference pattern as P_3 rotates?

Q 4. Can you explain why when both the polarizers are at the same angle, the interference pattern comes back.

Q 5. Capture and save images of the interference pattern for iterations performed in steps of 15°.

Q 6. Using ImageJ, perform line profile analysis of the captured images. A line profile displays a graph of the intensities of pixels along a line or rectangular selection. The x-axis represents distance along the line and the y-axis is the pixel intensity respectively as gray values [7]. A typical line profile for fringes is shown in Figure 5. ImageJ is a popular image processing program designed in scientific imaging. We use it in several experiments in the Physlab, LUMS. The software can be downloaded from `https://imagej.nih.gov/ij/` if not already installed on the computer.

Q 7. In this step we will remove the screen and place a photo-detector and an iris after B_2 as shown in Figure 4. The photo-detector produces a current proportional to the optical
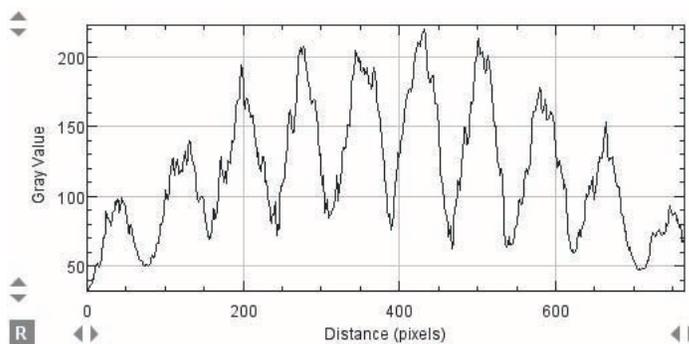
Figure 5: A typical line profile, when P_3 is at an angle of 30°
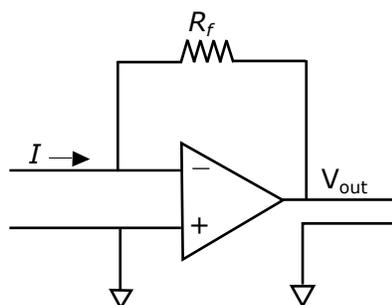with respect to the horizontal.

Figure 6: Circuit diagram for the IV converter, where I represents the photocurrent from the phot-detector. In our case $R_f = 5.6$ MΩ and $V_{out} = -IR_f$ volts.

intensity. The current is converted to a voltage by an IV-converter (a transimpedance amplifier) whose circuit diagram is shown in Figure 6. The output voltage V_{out} is a measure of the optical intensity and is read off using a digital multimeter.

Q 8.   Now one more time, we will vary θ, but instead of recording images of the pattern, we will note down the optical intensities as a function of θ. Does the resulting plot appear to be a good representative of the expected variation determined in Equation (5)? Fit your data to the mathematical model and discuss the results.

4.3   Erasure of 'which-path' information

Now comes the surprising part! Hopefully, you've observed that whenever P_2 and P_3 are crossed, we've obtained 'which-path' information and interference disappears. The light (photons) have crossed the interferometer and emerged out of B_2. However, surprisingly, interference can still be recovered. This can be observed by introducing P_4 as shown in Figure 7, and crossing P_2 and P_3 (by setting them at 0° and 90°). The polarizers, P_2 and P_3,
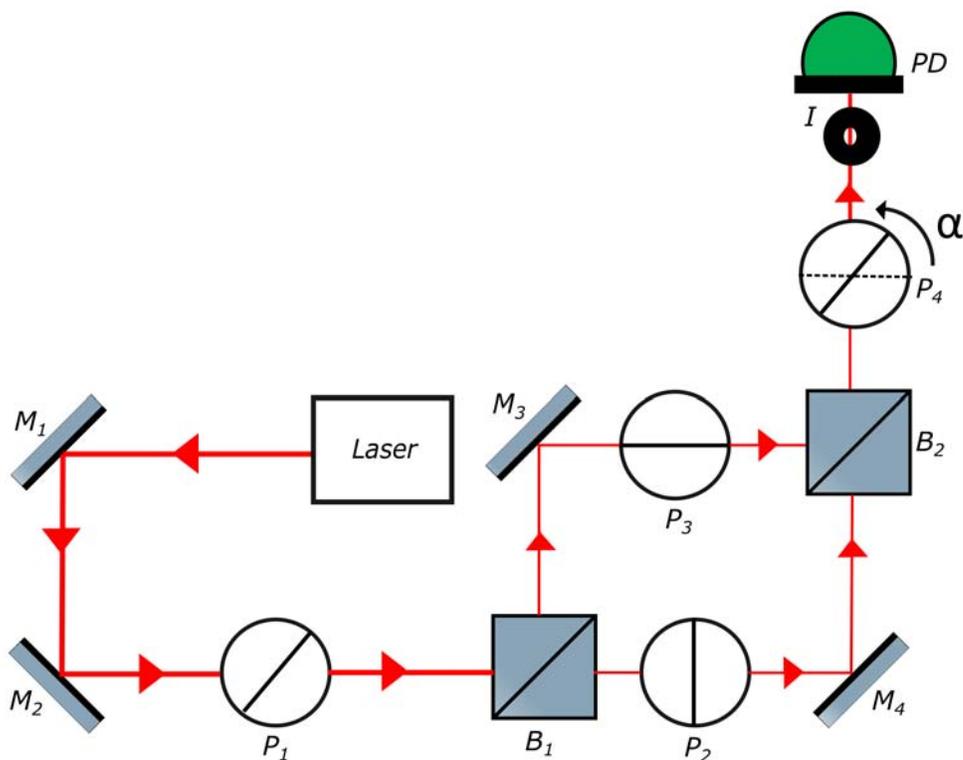
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Figure 7: Schematic of the setup that demonstrates erasure of the classical laser light. At what angle α is the 'which-path' information erased?

therefore, respectively polarize the light horizontally and vertically, whereas P_4 can possibly erase the 'which-path' information recovering interference. What is the angle of P_4 at which interference fully reappears?

Q 9. Rotate P_4 between $0°$ and $360°$ (angle α). What changes do you see in the interference pattern as P_4 rotates? Can you explain why at a certain angle of P_4 the interference pattern re-emerges? What is this angle?

Q 10. Capture images of the interference pattern for iterations of $15°$ and using ImageJ, perform line profile analysis of the captured images

Q 11. Now place a photo-detector after P_4 and find the values of the output voltage using an IV converter.

Q 12. Use Jones' calculus to derive the variation of intensity as a function of the angle α.

Q 13. Plot the voltages against their respective angles in Matlab and perform curve fitting. What is your fitting function?

4.4 Discussion

The erasure experiment is a classic example of a delayed choice experiment that apparently instigates a paradox [8]. The counter-intuitive aspect of the experiment is really the timing. The interference, naively, happens before the beamsplitter B_2. Once light, which is no doubt traveling at finite speeds, emerges out of B_2 we should have no control over the experiment, for the choice has already been made between interference or no-interference. This does not seem to be the case, however. We've convinced ourselves that the choice can still be made through the insertion of a polarizer even *after* B_2.

This puzzling behavior has equally profound conclusions. We learn that the wave nature of light really implicates a nonlocal existence. Light is a quantum field that permeates all of space [9]. Second, an experiment dealing with waves and fields, must be considered holistically. An arrangement with the polarizer P_4 in place is markedly different from an arrangement without P_4: one cannot analyze such an experiment piecemeal, i.e., through the concatenation of one component after the other.

References

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