HW1 Solution.

Gyromagnetic ratio is given by

$$\gamma = gq/2m$$

For an electron

$$\gamma = \frac{+2.002 \times 1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31}} = 1.76 \times 10^{11} \text{ C/Kg}$$

$$\omega = +\gamma B$$

$$= 1.76 \times 10^{11} \times 1T = 1.76 \times 10^{11} \text{ rad s}^{-1} \quad \text{Hz}$$

$$2.80 \times 10^{10} \text{ Hz}.$$

For Muon

$$\gamma = \frac{(-2.002) \times (-1.6 \times 10^{-19})}{2 \times 1.88 \times 10^{-28}} = 8.51 \times 10^{8} \text{ C/Kg}$$

$$\omega = 8.51 \times 10^{8} \text{ rad/s} = 1.35 \times 10^{8} \text{ Hz}. \quad \text{Hz}$$

For proton

$$\gamma = -2.66 \times 10^{8} \text{ C/Kg}$$

$$\omega = 2.66 \times 10^{8} \text{ rad/s} = 4.2 \times 10^{7} \text{ Hz}$$

For Neutron

$$\gamma = gq/2m_p$$

$$= \frac{-3.83 \times 1.6 \times 10^{-19}}{2 \times 1.675 \times 10^{-27}} = -1.82 \times 10^{8}$$

$$\omega = \gamma B = 1.82 \times 10^{8} \text{ rad/s} = 2.8 \times 10^{7} \text{ Hz}$$

Note electron's higher Larmor frequency, $\simeq 660$ times higher than a proton.

part (b)

— will not be graded

Mag. moment is

$$\vec{\mu}_j = g_j \mu_B J$$

where $\vec{J}$ is total ang. momentum.

$$\vec{J} = \vec{L} + \vec{S}$$

$$\vec{\mu}_j = \vec{\mu}_L + \vec{\mu}_s \qquad \vec{\mu}_L = g_L \mu_B \vec{L}$$

$$g_j \mu_B \vec{J} = g_L \mu_B \vec{L} + g_s \mu_B \vec{S} \qquad \vec{\mu}_s = g_s \mu_B \vec{S}$$

$$g_j \vec{J} = g_L \vec{L} + g_s \vec{S}$$

Taking Dot product with J.

$$g_j \vec{J} \cdot \vec{J} = g_L \vec{L} \cdot \vec{J} + g_s \vec{S} \cdot \vec{J}$$

$$g_j J^2 = g_L (L^2 + \vec{L} \cdot \vec{S}) + g_s (\vec{L} \cdot \vec{S} + S^2) \quad \text{—— (1)}$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$J^2 = L^2 + S^2 + 2\vec{L} \cdot \vec{S}$$

$$\vec{L} \cdot \vec{S} = \tfrac{1}{2} (J^2 - L^2 - S^2)$$

$$g_j J^2 = g_L \left(L^2 + \tfrac{1}{2}(J^2 - L^2 - S^2) \right) + g_s \left(\tfrac{1}{2}(J^2 - L^2 - S^2) + S^2 \right)$$

$$= g_L \left(\tfrac{1}{2} L^2 + \tfrac{1}{2} J^2 - \tfrac{1}{2} S^2 \right) + g_s \tfrac{1}{2} \left(S^2 + J^2 - L^2 \right)$$

eigenvalues of J^2, L^2 and S^2 are given as ($\hbar = 1$)

$$J^2 = j(j+1), \quad L^2 = l(l+1)$$

$$S^2 = s(s+1)$$

$$g_j = \frac{\overset{1}{g_l}}{2j(j+1)}\left[l(l+1) + j(j+1) - s(s+1) \right] + \frac{\overset{2}{g_s}}{2j(j+1)}\left[j(j+1) - l(l+1) - s(s+1) \right]$$

Problem # 2

partition fn for a system of spin j is given by

$$Z = \sum_{-J}^{J} e^{-\beta(-g_j \mu_B m_j B)}$$

let $x = g_j \mu_B B \beta$ where $\beta = 1/k_B T$

$$Z = \sum_{-J}^{J} e^{m_j x}$$

This simplifies to (as done in class)

$$Z = \frac{\sinh\left((J + 1/2)\, x \right)}{\sinh(x/2)}$$

$$M = \frac{1}{z} \sum_j g_j \mu_B \exp(m_j x)\, m_j$$

$$= \frac{g_j \mu_B}{z} \left(\sum_j m_j \exp(m_j x) \right) \Longrightarrow \partial z / \partial x$$

$$\frac{1}{z} \partial z / \partial x = \frac{\cosh((j+\frac{1}{2})x)(j+\frac{1}{2})}{\sinh((j+\frac{1}{2})x)} - \frac{1}{2}\frac{\cosh(x/2)}{\sinh(x/2)}$$

$$\frac{1}{z} \partial z / \partial x = (j+\tfrac{1}{2}) \coth((j+\tfrac{1}{2})x) - \tfrac{1}{2} \coth(x/2)$$

Now

$$M = g_j \mu_B \left[\left(\frac{2j+1}{2}\right) \coth((j+\tfrac{1}{2})x) - \tfrac{1}{2} \coth(x/2) \right]$$

* Multiply and divide by j
* Muliply with N for total Magnetization.

$$M = \underbrace{N g_j \mu_B j}_{M_s} \underbrace{\left[\frac{2j+1}{2j} \coth\left((j+\tfrac{1}{2})\frac{xj}{j}\right) - \frac{1}{2j} \coth(jx/2j) \right]}_{B_j(y)} \quad \text{where } y = xj$$

$$M/M_s = B_j(y) \quad \text{Brillouin fn.}$$

For limit, $x \ll 1$, $\mu_B B \ll k_B T$ (high temp limit)

$$B_j(x) = \frac{(j+1)y}{3j} + O(y^3)$$

$$M = M_s \frac{j(j+1)x}{3j}$$ (Neglecting higher order terms.)

$$= N g_j \mu_B j \cdot \frac{j(j+1)x}{3j}$$

$$x = g_j \mu_B B m_j \beta$$

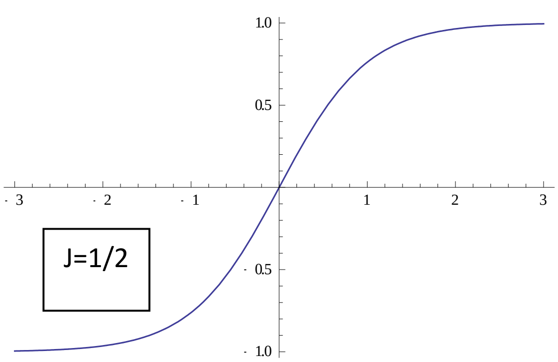
$$M = \frac{N g_j^2 \mu_B^2 \, j(j+1) B}{3 K_B T}$$

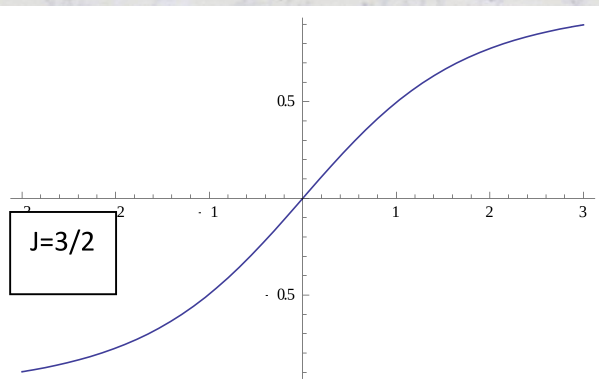
Hence

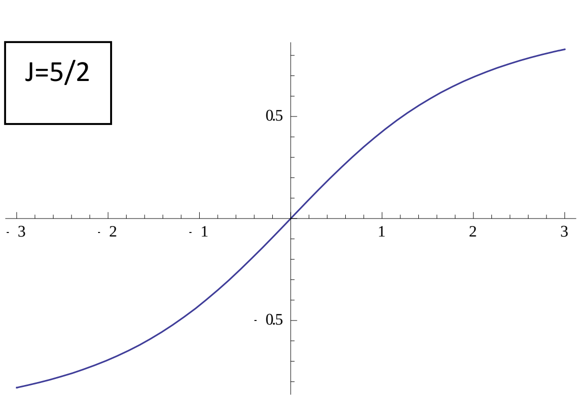
$$\chi = M/H = \frac{\mu_0 M}{B} = \frac{N \mu_0 \mu_{eff}}{3 K_B T}$$

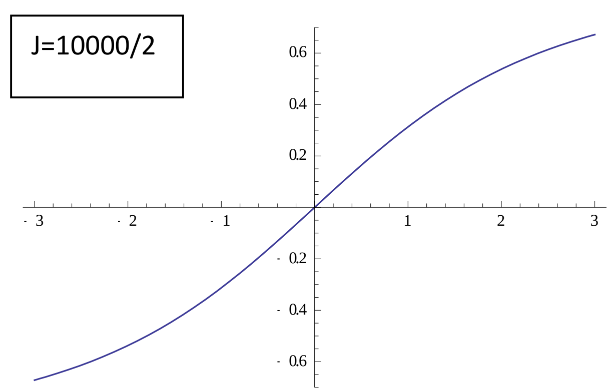
where $\mu_{eff} = g_j \mu_B \sqrt{j(j+1)}$

$\therefore$ $\chi \propto 1/T$ (Curie law holds).









Problem # 3

$$d\vec{\mu}/dt = \gamma(\vec{\mu}\times\vec{B}) + \alpha/\mu\,(\vec{\mu}\times d\vec{\mu}/dt)$$

taking cross product with μ

$$\vec{\mu}\times d\vec{\mu}/dt = \vec{\mu}\times\gamma(\vec{\mu}\times\vec{B}) + \alpha/\mu\;\vec{\mu}\times(\vec{\mu}\times d\vec{\mu}/dt)$$

using vector identity as $a\cdot(a\times b) = 0$. $\vec{b}\cdot\vec{a}\times\vec{a}$

$$\vec{\mu}\times(\vec{\mu}\times d\vec{\mu}/dt) = \vec{\mu}(\underbrace{\vec{\mu}\cdot d\vec{\mu}/dt}_{0}) - d\vec{\mu}/dt\,(\vec{\mu}\cdot\vec{\mu})$$

$$= -\mu^2\, d\vec{\mu}/dt$$

$$\mu\times d\vec{\mu}/dt = \gamma\,\vec{\mu}\times(\vec{\mu}\times\vec{B}) + \alpha/\mu\,(-\mu^2 d\mu^2/dt)$$

$$\vec{\mu}\times d\mu/dt = \gamma\vec{\mu}\times(\vec{\mu}\times\vec{B}) - \alpha\mu\, d\vec{\mu}/dt$$

$$d\vec{\mu}/dt = \gamma(\vec{\mu}\times\vec{B}) + \alpha/\mu\left(\gamma\,\vec{\mu}\times(\vec{\mu}\times\vec{B}) - \alpha\mu\, d\mu/dt\right)$$

$$\boxed{(1+\alpha^2)\,d\vec{\mu}/dt = \gamma(\vec{\mu}\times\vec{B}) + \alpha\gamma/\mu\,(\vec{\mu}\times(\vec{\mu}\times\vec{B}))}$$