INDIVIDUAL ASSIGNMENT

Assignment 2: Exchange Interaction

1. (a) A tiny magnetic moment $\vec{\mu}_1$, is placed at the origin, its magnetic vector potential at a postion $\vec{r}$ is

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{\mu}_1 \times \vec{r}}{|\vec{r}|^3} \tag{1}$$

Use $\vec{B} = \vec{\nabla} \times \vec{A}$ and a vector identity to verify that two magnetic dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$ separated by $\vec{r}$ have a magnetic dipolar energy equal to

$$E = \frac{\mu_0}{4\pi r^3} [\vec{\mu}_1 \cdot \vec{\mu}_2 - \frac{3}{r^2} (\mu_1 \vec{\cdot} \vec{r})(\vec{\mu}_2 \cdot \vec{r})] \tag{2}$$

(b) Calculate the dipolar energy between two protons separated at 1Å and 10Å when their spin are (i) parallel (ii) antiparallel

(c) Estimate the ratio of exchange and dipolar coupling of two adjacent Fe atoms in metallic Fe. The exchange constant in Fe can be crudely estimated by setting it equal to $k_B T_c$ where T_c is curie temperature for Fe, T_c= 1043 K.

2. Consider four spins connected by a tetrahedron, each $S = \frac{1}{2}$. Find the energy eigenvalues of the system. The Heisenberg Hamiltonian for a pair of spins is given by

$$\hat{H} = -2J\hat{S}_1.\hat{S}_2 \tag{3}$$

3. Show that Hund's rules for a shell of angular momentum l and containing n electrons can be summarized by

$$S = \frac{2l + 1 - |2l + 1 - n|}{2}, L = S|2l + 1 - n|, J = S|2l - n| \tag{4}$$

4. Find the term symbols for the ground states of the ions $Ho^{3+}(4f^{10}), Er^{3+}(4f^{11}), Tm^{3+}(4f^{12}), Lu^{3+}(4f^{14})$.

5. Consider the case of two interacting spin-$\frac{1}{2}$ electrons. The good quantum numbers are $S = 0$ and 1 so that there is a triplet state and a singlet state which will be separated by an energy gap Δ. We define the sign of Δ so that when Δ >0, the singlet state

($S = 0$) is the lower state and when Δ <0, the triplet state is the lower state. Show that the susceptibility in this model is given by

$$\chi = \frac{2Ng\mu_B^2}{k_BT(3 + e^{\frac{\Delta}{k_BT}})} \tag{5}$$

Plot the susceptibility for $\Delta = 0$, Δ >0, Δ <0