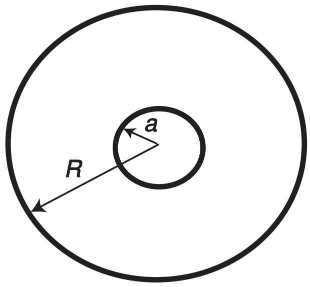
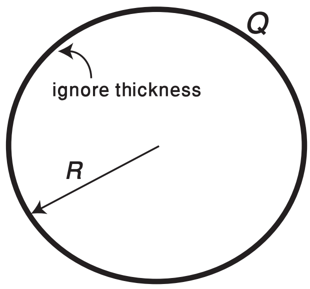
HW 3: Electrostatics

You are welcome to directly use the vector identities given on the inside flap of Zangwill's book. Draw neat sketches to assist your explanations.

1. A disk of negligible thickness and radius R has a hole drilled at its centre. The hole has radius a. What's the electrostatic potential at the hole's centre?



2. (a) A thin spherical shell of radius R carries a uniformly distributed charge Q. What is the electrostatic potential energy of this system. Compute this potential energy by assembling this body of charge.



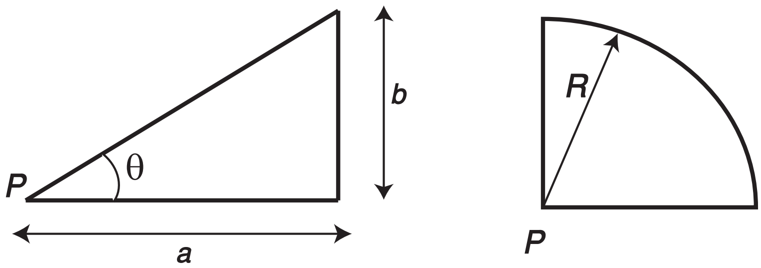
   (b) What is the system's electrostatic potential energy if the charge on the shell is doubled?

   (c) Here is anther way to look at the charge doubling problem. You've calculated the energy for a *single* shell in part (a). Now consider a second identical shell of charge Q being placed directly on top of the first shell. This would effectively double the charge. However, simply doubling the energy computed in (a) does not give the result in (b). How do we explain this discrepancy? Which answer is correct?

3. Repeat the calculation in Section II. of the article [J.M. Kalotas, A.R. Lee and J. Liesegang, *Amer. Journ. Phys.* **64**, 272 (1996)] and use a computer program to make

a contour plot of the electrostatic potential for a symmetric dipole. The article is uploaded on the website.

4. The accompanying figure shows a right angled triangular sheet and a quadrant of a circle. Ignoring the thickness, find the electrostatic potential at the point P in each of the cases. These objects carry a uniform charge σ per unit area.

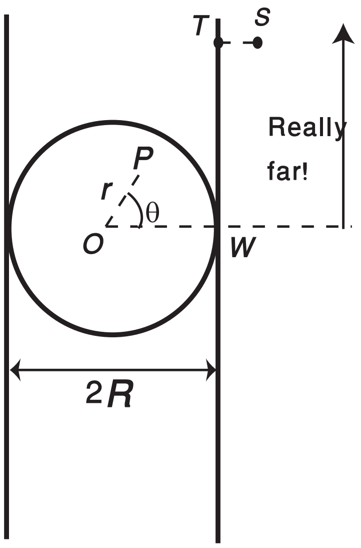


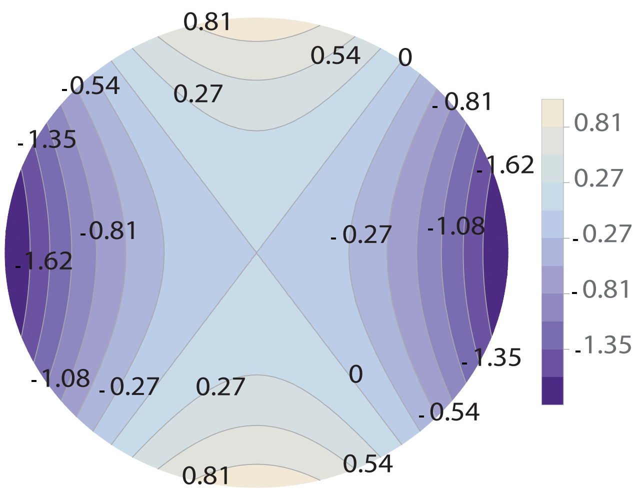
5. In class, it was pointed out the electrostatic energy U_E is semi-definite positive. Prove this fact. Start by writing

$$U_E = \frac{1}{2} \int d^3r \rho(\mathbf{r}) \phi(\mathbf{r}) \tag{1}$$

and subsequently use integration by parts.

6. Consider a slab of thickness $2R$ that extends to infinity along the other two dimensions. The slab carries a uniform charge density ρ with the exception of a circular cavity that is carved out from the slab. The cavity has a radius R. The accompanying diagram helps visualize this configuration.





(a) Find the electric field everywhere in the vertical plane shown in the diagram.

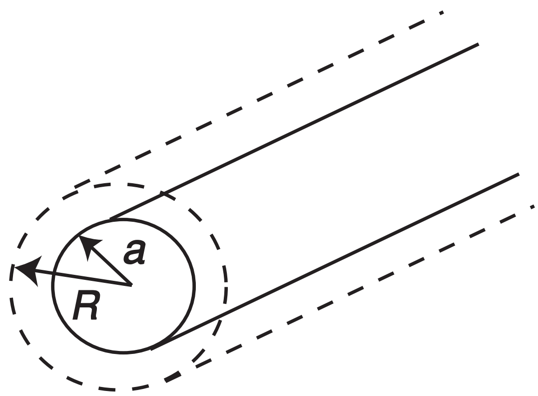
(b) Find the potential everywhere in the vertical plane shown in the diagram. Compute potential with respect to the centre of the cavity. In particular I would like to know the potential at points P and W. The latter point is at the intersection of the cavity with the surface of the slab.

(c) How does the potential change as we move along the surface, from W to a far away point T? I want you to determine the *change* in potential as we slide along the surface, upwards from W.

(d) Identify the location of the point S where the potential is the same as at W. Sketch an equipotential line that touches the surface at W and connects with S. A rough sketch is fine.

(e) The accompanying diagram also shows the equipotential lines *inside* the cavity. There are straight lines at which the potential (w.r.t. the centre) is zero. What are the slopes of these lines? Connect one of these equipotential lines to the point T.

7. We have a cylindrical rod of radius a that extends to infinity. Charge of density ρ is uniformly distributed throughout the volume of the rod.



(a) Find the potential with respect to a surface at radius $R > a$ form the axis of the cylinder. The surface at R is the reference.

(b) Determine the electrostatic potential energy per unit length with respect to the reference.

8. We have some charge distribution $\rho(\mathbf{r})$ that generates a potential

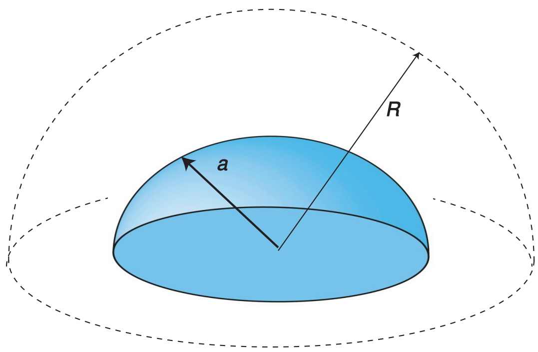
$$\phi(\mathbf{r}) = A\frac{e^{-\lambda r}}{r} \tag{2}$$

where A and λ are positive constants. Find mathematical expressions for the electric field, charge density and the total charge.

9. This question determines the force on a hemispherical section of a charge of uniform density ρ. The total charge on the sphere is Q. The radius of the charged ball is a. In class we used the Maxwell stress tensor $\mathbf{T}$ to derive the total force acting on the northern hemisphere

$$F = \frac{3}{16}\frac{Q^2}{4\pi\varepsilon_o a^2}. \tag{3}$$

The stress tensor was computed over the outer surface of the hemisphere. Well and good! However, the result is independent of the surface and will work with *any* surface that enclose *all* of the charge in question.



Consider our surface to be all of the $z > 0$ plane which is modeled as a hemisphere of infinite radius R. Compute the stress tensor on this surface and verify that the force computed using this semi-infinite surface is identical to what is given above.

10. Two equal charges of magnitude q each are separated by a distance $2a$. Draw an infinite plane equidistant between the charges. Find the stress tensor on this plane and use this tensor to compute the force one charge exerts on the other. Note that this can be done much easily using Coulomb's law. We are taking the longer route here.