HW 6: Dielectrics and Polarization

Draw neat sketches where applicable.

1. An LIH dielectric sphere of radius R has a uniform polarization $\mathbf{P}$. What are the polarization charge densities on the surface and in the volume? Would the sphere have a dipole moment?

2. In the previous question, we computed a charge distribution from a given polarization. Let's now do the converse. Suppose we confine polarization charges to be uniformly distributed within a sphere of radius R centred at the origin. What kind of $\mathbf{P}$ does this produce? Inside and outside the dielectric? (I expect mathematical expressions.) Hint: Use the mathematical structure of Gauss'a law. Plot the magnitude of the polarization as r, the distance from the origin.

3. A dielectric sphere of radius R, relative permittivity κ, is placed inside a uniform electric field $\mathbf{E}_o$. The sphere is electrically neutral. The sphere is uniformly polarized and an internal field is developed given by,

$$\mathbf{E}_{\text{in}} = -\frac{\mathbf{P}}{3\varepsilon_o}. \tag{1}$$

   We derived this expression in class. Find the (a) electric field *only* due to the polarized charges, (b) total electric field, and (c) polarization vector expressing each one of them in terms of *only* $\mathbf{E}_o$.

4. We return to a uniformly polarized dielectric sphere of polarization $\mathbf{P}$. There is no electric field due to free charges.

   (a) Since there are no free charges, $D_\perp$ is continuous across the surface of the sphere, where $D_\perp$ is the component normal to the surface. Prove this assertion by explicitly calculating this component on both sides of the surface. The electric field inside is given by Eq. (1) while the electric field outside is:

$$\mathbf{E}_{\text{out}} = \frac{V}{4\pi\varepsilon_o}\left(\frac{3(\mathbf{P}\cdot\hat{\mathbf{r}}) - \mathbf{P}}{r^3}\right). \tag{2}$$

   (b) Is the normal component of $\mathbf{E}$ continuous across the surface? What is the discontinuity in terms of $\mathbf{P}$?

(c) Using the polarization charge on the uniformly polarized sphere, calculate, through *explicit integration* the electric field at the centre of the sphere. This should be identical to Eq. (1).

5. We have an LIH dielectric sphere of permittivity κ and radius R. Free charge is uniformly imbedded into the sphere. It has a density ρ_f.

   (a) Compute the polarization field $\mathbf{P}$ inside the dielectric.

   (b) Compute the volume and surface charge densities of the polarization charge (if any).

   (c) What is the total polarization charge on the surface?