HW 7: Miscellaneous topics in electrostatics and steady currents

Draw neat sketches where applicable.

1. Using $\nabla \times \mathbf{E}(\mathbf{r}) = 0$, derive a matching condition foor current densities $\mathbf{j}_1$ and $\mathbf{j}_2$ on two sides of an interface with conductivities σ_1 and σ_2 and permittivities ε_1 and ε_2.

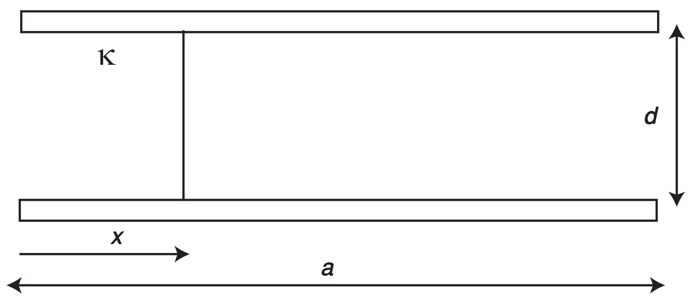
2. Consider a free ion of charge q and radius a. A spherical neutral atom is placed at a distance r from the ion. The atom has a polarizability α and $r \gg a$. Note that the electric field due to a dipole is given by (in spherical coordinates)

$$\mathbf{E}(r,\theta) = \frac{p}{4\pi\varepsilon_o r^3}\big(2\cos\theta\ \mathbf{e}_r + \sin\theta\ \mathbf{e}_\theta\big). \tag{1}$$

   (a) Find the electric field produced by the atom on the location of the ion. Hence find the force acting on the ion.

   (b) What is the potential energy of the system compared to when $d \to \infty$?

3. A dielectric of permittivity κ is introduced through a length x into the region between two metal plates. These plates are electrically isolated and hold a charge $\pm Q$. To the right of the dielectric, there is just vacuum. The separation between the plates is d, the length of the plates is a and the depth into the paper (not shown) is b.

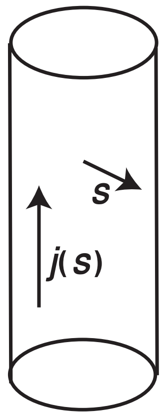


   (a) Find the charge densities σ_{f1} and σ_{f2} on a plate, in the region making contact with the dielectric and with vacuum, respectively. Are these densities the same or different?

   (b) Find the electrostatic energy of the system and the force on the dielectric (if any).

4. A steady current $\mathbf{j}(s)$ carries positive charge in the upwards ($\mathbf{e}_z$) direction. The current density is non-uniform and depends only on the distance s form the axis of the

cylinder. We would like to put a constraint on the system that none of the charges are accelerating. This means that the force on any single charge moving with velocity $\mathbf{v}$ is zero. (Note that $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$).



(a) What is the mgnetic field vcB inside the conductor (magnitude and direction)?

(b) What should be the electric field (magnitude and direction) to ensure that the force acting on any charge is zero?

(c) Where is this electric field coming from? Of course there are the mobile charges with density ρ_2 and fixed charges with density ρ_1. Use Gauss's law in integral form to derive a relationship between $\rho_1(s)$ and $\rho_2(s)$. How is quasi-neutrality inside the wire maintained?

5. A spherical conductor of radius R is imbedded inside ground and is therefore, at a potential zero. What should be the charge density on its surface?

6. Attempt Q. 9.24 from Zangwill.