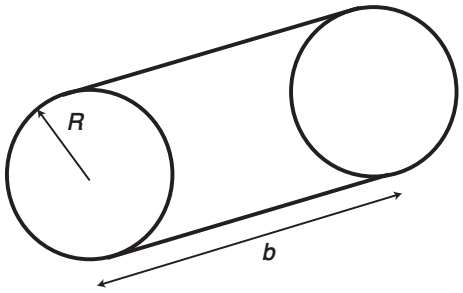
Mid Term Exam

Draw neat diagrams where necessary. The nomenclature is evident from the text. Allowed time is two hours.

1. Consider a circular cylinder, hollow from the inside and with both ends open. It has radius R and a length b. Find the expression for computing the electric potential at any point on the axis of the cylinder. There is no need to perform the integration. Writing down the integral is enough. **[5 marks]**



2. The electric potential due to a charge distribution $\rho(x)$ is given by $\phi(x) = A(l^2 - x^2)$ for $|x| \leq l$ and 0 otherwise. The constant $A > 0$. **[10 marks]**

   (a) Plot $\phi(x)$ versus x.

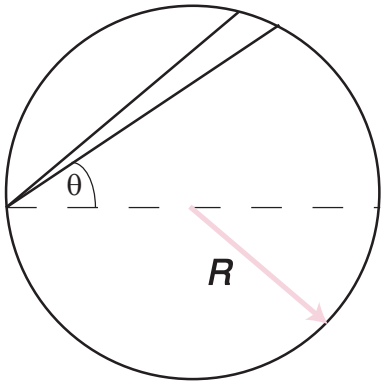
   (b) Find and plot the electric field as function of x.

   (c) Now find and plot $\rho(x)$. Express $\rho(x)$ in closed form describing it in terms of $\delta(x \pm l)$ and the Heaviside function $\Theta(x \pm l)$.

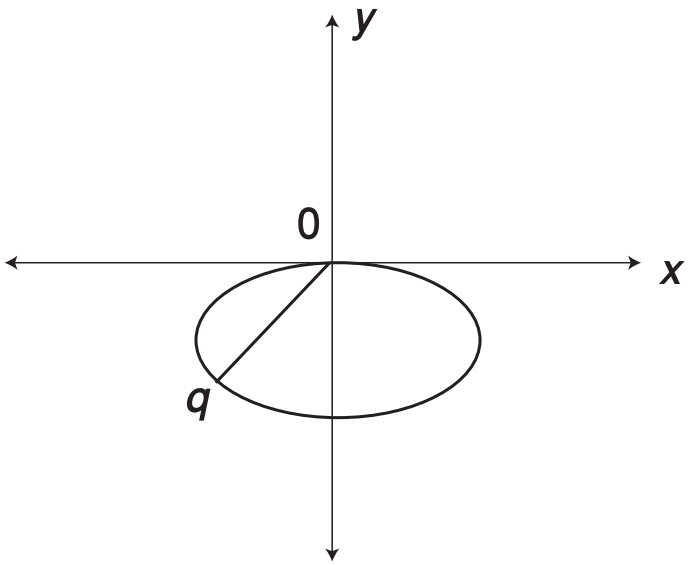
3. Two infinitely large plates are oppositely charged and placed parallel to one another. They carry uniform charge densities $\pm\sigma$ and the electric field between them is uniform with magnitude E_o. Use Maxwell's stress tensor formulation to compute the force per unit area on either of the plates. Note that the elements of the stress tensor are given by $T_{ij} = \varepsilon_o\big(E_iE_j - \delta_{ij}E^2/2\big)$ and the force on an area $d\mathbf{a}$ is given by $(\mathbf{T}\cdot d\mathbf{a})_l = T_{lm}da_m$. **[10 marks]**

4. Consider a thin disk of radius R. It is uniformly charged with a density σ. With is the electric potential at any point on the periphery (outer boundary)? It will be a good idea to decompose the disk into wedges as shown in the accompanying diagram. **[8 marks]**

5. Now use the previous result to find the electrostatic potential energy U_E of a think disk of radius a that carries a uniform charge density σ. **[7 marks]**



6. A charge q moves on a contour such that the y component of the electric field at the origin remains constant. What is the shape of the curve (find its equation)? **[5 marks]**



7. Find the electric dipole moment of a charged shell (no thickness) whose surface charge density varies as $\sigma_o \cos\theta$. **[5 marks]**