Assignment 5: Spin waves and magnons

1. In class we derived the dispersion relation for spin waves inside a ferromagnet. The equation of motion of the n'th spin is given by

$$\frac{d\,\boldsymbol{S}_n}{dt} = -2J\ \boldsymbol{S}_s \times \big(\boldsymbol{S}_{n-1} + \boldsymbol{S}_{n+1}\big). \tag{1}$$

For an antiferromagnet, there are two kinds of lattices with the lowest energy spin vectors being $\boldsymbol{S}_n$ and $-\boldsymbol{S}_n$. Fluctuations to these spin vectors inside the two lattices allows one to define the spin vectors,

$$\boldsymbol{S}_n^{\uparrow} = \boldsymbol{S}_n + \boldsymbol{\sigma}_n^{\uparrow} \tag{2}$$

$$\boldsymbol{S}_n^{\downarrow} = -\boldsymbol{S}_n + \boldsymbol{\sigma}_n^{\downarrow}. \tag{3}$$

Assume that $\pm\boldsymbol{S}_n$ is along the z axis and the fluctuations $\boldsymbol{\sigma}$ are solely in the xy plane.

   (a) Write equations of motion for $\boldsymbol{S}_n^{\uparrow}$ and $\boldsymbol{S}_n^{\downarrow}$.

   (b) Suppose the fluctuations are small, i.e., the spin wave excitations are low energy. Write equations of motion for $\boldsymbol{\sigma}_n^{\uparrow}$ and $\boldsymbol{\sigma}_n^{\downarrow}$.

   (c) Now write these fluctuations in component form, i.e., x and y components.

   (d) Construct equations for the complex vector $\sigma_{n,+}^{\uparrow} = \sigma_{n,x}^{\uparrow} + i\sigma_{n,y}^{\uparrow}$ and similarly for the $\downarrow$ lattice.

   (e) Assume trial solutions for $\sigma_{n,+}^{\uparrow} = Ae^{i(kna-\omega t)}$ and $\sigma_{n,+}^{\downarrow} = Be^{i(kna-\omega t)}$ and derive the dispersion relation for the spin waves.

   (f) Find the density of states $g(\omega)$ for magnons in the antiferromagnet.

   (g) What is the low temperature heat capacity of the antiferromagnet associated with magnons?

2. In molecular field theory, the magnetizations of the two sublattices in a ferrimagnet can be described by (high temperature approximation):

$$M_+ = \frac{C_1}{T}\big(B + \alpha_1 M_+ - \lambda M_-\big) \tag{4}$$

$$M_- = \frac{C_2}{T}\big(B + \alpha_2 M_- - \lambda M_+\big). \tag{5}$$

Here $\alpha_1 \neq \alpha_2$ and $C_1 \neq C_2$. The α's represent coupling to the *same* lattice while λ represents coupling to the *other* lattice. The Curie constants C_1 and C_2 are different since $g_J\,J$ is different for the sublattices.

(a) Find the critical temperature at which spontaneous magnetization can appear at $B = 0$. Do you recover the Neel temperature if $\alpha_1 = \alpha_2 = 0$ and $C_1 = C_2 = C$?

(b) Now suppose $\alpha_1 = \alpha_2 = 0$ but $C_1 \neq C_2$. Show that the high temperature susceptibility is given by

$$\chi = \frac{\mu_o}{T^2 - \lambda^2 C_1 C_2}\big(T(C_1 + C_2) - 2\lambda C_1 C_2\big). \tag{6}$$