HW5

**Q1**

$$\hat{S}_+ = \sqrt{2S}\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2} \hat{a}$$

$$\hat{S}_- = \sqrt{2S}\,\hat{a}^\dagger\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)$$

$$[\hat{S}_+, \hat{S}_-] = 2S\left[\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2} \hat{a}\,,\ \hat{a}^\dagger\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2}\right]$$

Now $\left[\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2} \hat{a}\,,\ \hat{a}^\dagger\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2}\right]$

$$= \left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2} \hat{a}\,\hat{a}^\dagger \left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2}$$

$$- \hat{a}^\dagger\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2}\left(1 - \frac{\hat{a}^\dagger \hat{a}}{2S}\right)^{1/2} \hat{a}$$

Let $\hat{a}^\dagger \hat{a} = \hat{n}$ (number operator) $\hat{n}^\dagger = (\hat{a}^\dagger \hat{a})^\dagger = \hat{a}^\dagger \hat{a} = \hat{n}$

The commutator is,

$$\left(1 - \frac{\hat{n}}{2S}\right)^{1/2} \hat{a}\,\hat{a}^\dagger \left(1 - \frac{\hat{n}}{2S}\right)^{1/2} - \hat{a}^\dagger (1 - \hat{n})\, \hat{a}$$

$$\hat{S}_+ |S_z\rangle = \sqrt{2S}\sqrt{1 - \frac{\hat{a}^\dagger \hat{a}}{2S}}\ \underbrace{\hat{a}|\hat{S}_z\rangle}$$

$$\hat{S}_z = -\hat{a}^\dagger \hat{a}$$

$$\hat{a}\underbrace{|S_z\rangle} = |S_z - 1\rangle$$

$$\hat{S}_+ |n\rangle = \sqrt{2S}\sqrt{1 - \frac{\hat{n}}{2S}}\ \underbrace{\hat{a}|n\rangle}$$

$$= \sqrt{2S}\sqrt{n+1}\sqrt{1 - \frac{\hat{n}}{2S}}\ |n+1\rangle$$

$$= \sqrt{2S}\sqrt{n+1}\sqrt{1 - \frac{(n+1)}{2S}}\ |n+1\rangle$$

$$S_z |n\rangle = (S-n)|n\rangle$$

$$S_z |n+2\rangle = (S-n+1)\ |n+1\rangle$$

$$\hat{S}_z S_+ |n\rangle = \sqrt{2S}\sqrt{n+1}\sqrt{1 - \frac{n+1}{2S}}\ (S-n+1)\ |n+1\rangle$$

$$\hat{S}_+ \hat{S}_z |n\rangle = \hat{S}_+ (S-n)\ |n\rangle$$

$$= (S-n)\ \hat{S}_+\ |n\rangle$$

$$= (S-n)\sqrt{2S}\sqrt{n+1}\sqrt{1 - \left(\frac{n+1}{2S}\right)}\ |n+1\rangle$$

$$[\hat{S}_z, \hat{S}_+] = \sqrt{2S}\sqrt{n+1}\sqrt{1 - \frac{n+1}{2S}}\left(\cancel{S} - \cancel{n} + 1 - \cancel{S} + \cancel{n}\right) = \sqrt{(n+1)(2S-n-1)}$$

HW5

$$\frac{d\vec{S}_n}{dt} = -2J\,\vec{S}_n \times (\vec{S}_{n-1} + \vec{S}_{n+1})$$

$$\vec{S}_n^{\uparrow} = \vec{S}_n + \vec{\sigma}_n^{\uparrow}$$ $\vec{S}_n$ is the ground state

$$\vec{S}_{n+1}^{\downarrow} = -\vec{S}_n + \vec{\sigma}_n^{\downarrow}$$

$\vec{S}_n \times \vec{S}_n = 0$

$\vec{S}_n \times \vec{S}_{n+1} = 0$

$\vec{S}_n \times \vec{S}_{n-1} = 0$

For the $\uparrow$ sublattice

$$\frac{d\vec{S}_n^{\uparrow}}{dt} = \overbrace{\frac{d\vec{S}_n}{dt}}^{0} + \frac{d\vec{\sigma}_n^{\uparrow}}{dt} = -2J\,(\vec{S}_n + \vec{\sigma}_n^{\uparrow}) \times \left(-\vec{S}_n + \vec{\sigma}_{n-1}^{\downarrow} + (-\vec{S}_n + \vec{\sigma}_{n+1}^{\downarrow}\right)$$

$$= -2J\left[\overbrace{-\vec{S}_n \times \vec{S}_n}^{0} + \vec{S}_n \times \vec{\sigma}_{n-1}^{\downarrow} + \vec{S}_n \times \vec{\sigma}_{n+1}^{\downarrow} + \vec{\sigma}_n^{\uparrow} \times (-\vec{S}_n) + \vec{\sigma}_n^{\uparrow} \times \vec{\sigma}_{n-1}^{\downarrow} + \vec{\sigma}_n^{\uparrow} \times (-\vec{S}_n) + \vec{\sigma}_n^{\uparrow} \times \vec{\sigma}_{n+1}^{\downarrow}\right]$$

$$\frac{d\vec{S}_n^{\uparrow}}{dt} = \frac{d\vec{\sigma}_n^{\uparrow}}{dt} = -2J\left[\vec{S}_n \times \vec{\sigma}_{n-1}^{\downarrow} + \vec{S}_n \times \vec{\sigma}_{n+1}^{\downarrow} - \vec{S}_n \times \vec{\sigma}_n^{\uparrow} + \vec{\sigma}_n^{\uparrow} \times \vec{\sigma}_{n-1}^{\downarrow} - \vec{S}_n \times \vec{\sigma}_n^{\uparrow} + \vec{\sigma}_n^{\uparrow} \times \vec{\sigma}_{n+1}\right]$$

$$\boxed{\frac{d\vec{\sigma}_n^{\uparrow}}{dt} = -2J\Big[\vec{S}_n \times \vec{\sigma}_{n-1}^{\downarrow} + \vec{S}_n \times \vec{\sigma}_{n+1}^{\downarrow} - 2\vec{S}_n \times \vec{\sigma}_n^{\uparrow} + \vec{\sigma}_n^{\uparrow} \times \vec{\sigma}_{n-1}^{\downarrow} + \vec{\sigma}_n^{\uparrow} \times \vec{\sigma}_{n+1}\Big]}$$

If $|\vec{\sigma}_n^{\uparrow}|, |\vec{\sigma}_n^{\downarrow}| \ll |\vec{S}_n|$ (fluctuations are small)

$$\boxed{\frac{d\vec{\sigma}_n^{\uparrow}}{dt} = -2J\Big[\vec{S}_n \times (\vec{\sigma}_{n-1}^{\downarrow} + \vec{\sigma}_{n+1}^{\downarrow} - 2\vec{\sigma}_n^{\uparrow})\Big]} \quad -(1)$$

Similarly for the $\downarrow$ sublattice

$$\boxed{\frac{d\vec{\sigma}_n^{\downarrow}}{dt} = -2J\Big[-\vec{S}_n \times (\vec{\sigma}_{n-1}^{\uparrow} + \vec{\sigma}_{n-1}^{\uparrow} - 2\vec{\sigma}_n^{\downarrow})\Big]} \quad -(2)$$

$\cancel{\sigma^{\uparrow}}$

$$\vec{S}_n = S\,\hat{e}_z$$

$$\vec{\sigma}^{\uparrow\downarrow}_{n,n\pm1} = \begin{pmatrix} \sigma^{\uparrow\downarrow}_{n,n\pm1,x} \\ \sigma^{\uparrow\downarrow}_{n,n\pm1,y} \\ \sigma^{\uparrow\downarrow}_{n,n\pm1,z} \end{pmatrix} \simeq \begin{pmatrix} \sigma^{\uparrow\downarrow}_{n,n\pm1,x} \\ \sigma^{\uparrow\downarrow}_{n,n\pm1,y} \\ 0 \end{pmatrix}$$
, fluctuation is $\perp$ to the $z$ direction

$$\vec{S}_n \times \vec{\sigma}_{n-1}^{\downarrow} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & S \\ \sigma^{\downarrow}_{n-1,x} & \sigma^{\downarrow}_{n-1,y} & \sigma^{\downarrow}_{n-1,z} \end{vmatrix} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & S \\ \sigma^{\downarrow}_{n-1,x} & \sigma^{\downarrow}_{n-1,y} & 0 \end{vmatrix}$$

$$= -\hat{e}_x S \sigma^{\downarrow}_{n-1,y} + \hat{e}_y S \sigma^{\downarrow}_{n-1,x} = S\left(-\sigma^{\downarrow}_{n-1,y}\hat{e}_x + \sigma^{\downarrow}_{n-1,x}\hat{e}_y\right)$$

Using this general scheme, (1) becomes:

$$\frac{d\sigma^{\uparrow}_{n,x}}{dt} = -2JS\left[-\sigma^{\downarrow}_{n-1,y} - \sigma^{\downarrow}_{n+1,y} + 2\sigma^{\uparrow}_{n,y}\right] \quad -(3)$$

Similarly

$$\frac{d\sigma^{\uparrow}_{n,y}}{dt} = -2JS\left[\sigma^{\downarrow}_{n-1,x} + \sigma^{\downarrow}_{n+1,x} - 2\sigma^{\uparrow}_{n,x}\right] \quad -(4)$$

We can also find equations of motion for the components of the other sublattice:

$$\frac{d\sigma^{\downarrow}_{n,x}}{dt} = -2JS\left[\sigma^{\uparrow}_{n-1,y} + \sigma^{\uparrow}_{n+1,y} - 2\sigma^{\downarrow}_{n,y}\right] \quad -(5)$$

$$\frac{d\sigma^{\downarrow}_{n,y}}{dt} = -2JS\left[-\sigma^{\uparrow}_{n-1,x} - \sigma^{\uparrow}_{n+1,x} + 2\sigma^{\downarrow}_{n,x}\right] \quad -(6)$$

Define

$$\sigma^{\uparrow}_{n,+} = \sigma^{\uparrow}_{n,x} + i\,\sigma^{\uparrow}_{n,y} \quad \text{(a ladder operator)}$$

$$\sigma^{\downarrow}_{n,+} = \sigma^{\downarrow}_{n,x} + i\,\sigma^{\downarrow}_{n,y}$$

Form combinations of (3) and (4)

$i \quad i$

$$\frac{d\sigma^{\uparrow}_{n,+}}{dt} = -2JS\Big[-\sigma^{\downarrow}_{n-1,y} + i\,\sigma^{\downarrow}_{n-1,x}$$
$$-\sigma^{\downarrow}_{n+1,y} + i\,\sigma^{\downarrow}_{n+1,x}$$
$$+2\sigma^{\uparrow}_{n,y} - 2i\,\sigma^{\uparrow}_{n,x}\Big]$$

$-i^2$

$-2i(\sigma^{\uparrow}_{n,x} + i\sigma_{n,y})$

$$= -2JS\,[i]\big[\sigma^{\downarrow}_{n-1,+} + \sigma^{\downarrow}_{n+1,+} - 2\sigma^{\uparrow}_{n,+}\big]$$

$-\frac{1}{i}$

$$\cancel{= -2iJS\left(\sigma^{\downarrow}_{n-1,+} + \sigma^{\downarrow}_{n+1,+} + 2\sigma^{\uparrow}_{n,x}\right)}$$

$-i.i$

$i,i$

$$\boxed{\frac{d\sigma^{\uparrow}_{n,+}}{dt} = -2iJS\left(\sigma^{\downarrow}_{n-1,+} + \sigma^{\downarrow}_{n+1,+} - 2\sigma^{\uparrow}_{n,+}\right)} \quad \text{— (7)}$$

$-2\sigma_{n,x}$

$+\frac{1}{i}$

$-2\sigma_{n,x} - 2i\,\sigma_{n,y}$

$-i(2\sigma_{n,x} + 2i\sigma_{n,y})$

Now form an appropriate superposition of (5) and (6):

$$\frac{d\sigma^{\downarrow}_{n,+}}{dt} = -2JS\left(\sigma^{\uparrow}_{n-1,y} - i\,\sigma^{\uparrow}_{n-1,x} + \sigma^{\uparrow}_{n+1,y} - i\,\sigma^{\uparrow}_{n+1,x} - 2\sigma^{\downarrow}_{n,y} + 2i\,\sigma^{\downarrow}_{n,x}\right)$$

$$= -2JS\,(-i)\left[\sigma^{\uparrow}_{n-1,t} + \sigma^{\uparrow}_{n+1,t} - 2\sigma^{\downarrow}_{n,t}\right]$$

$$\boxed{\frac{d\sigma^{\downarrow}_{n,t}}{dt} = 2iJS\left[\sigma^{\uparrow}_{n-1,t} + \sigma^{\uparrow}_{n+1,t} - 2\sigma^{\downarrow}_{n,t}\right]} \quad — (8)$$

(7) and (8) represent two coupled equations. If "a" is the lattice spacing, we propose the ansatz

$$\sigma^{\uparrow}_{n,t} = A\, e^{i(kna - \omega t)}$$

$$\sigma^{\downarrow}_{n,t} = B\, e^{i(kna - \omega t)}$$

$$\frac{d\sigma^{\uparrow}_{n,t}}{dt} = -i\omega A\, e^{i(kna-\omega t)} \quad \otimes$$

$$\frac{d\sigma^{\downarrow}_{n,t}}{dt} = -i\omega B\, e^{i(kna-\omega t)} \quad \otimes$$

Inserting these values into (7) and (8) yields:

$$-i\omega A\, e^{ikna} = -2iJS\left(B e^{ik(n-1)a} + B e^{ik(n+1)a} - 2A\, e^{ikna}\right)$$

$$\boxed{-i\omega A = -2iJS\left(B e^{-ika} + B e^{ika} - 2A\right)} \quad — (9)$$

Similarly,

$$-i\omega B e^{ikna} = 2iJS\left(A\, \cancel{e^{ikna}}\, e^{ik(n-1)a} + Ae^{ik(n+1)a} - 2Be^{ikna}\right)$$

$$\boxed{-i\omega B = 2iJS\left(Ae^{-ika} + Ae^{ika} - 2B\right)} \quad - \; (10)$$

Expressing (9) and (10) in matrix form allows us to come up with the following constraint:

$$\begin{vmatrix} 2iJS\left(e^{-ika} + e^{ika}\right) & -2(2iJS) + i\omega \\ -i\omega - 4iJS & 2iJS\left(e^{-ika} + e^{ika}\right) \end{vmatrix} = 0$$

$$\begin{vmatrix} 4iJS\cos(ka) & -4iJS + i\omega \\ -4iJS - i\omega & 4iJS\cos(ka) \end{vmatrix} = 0$$

$$-\left(4JS\cos(ka)\right)^2 - \left(-4iJS + i\omega\right)\left(-4iJS - i\omega\right) = 0 \qquad -i^2$$

$$-\left(4JS\cos(ka)\right)^2 - \left[\left(-4iJS\right)^2 - (i\omega)^2\right] = 0$$

$$-\left(4JS\cos(ka)\right)^2 - \left[-16J^2S^2 + \omega^2\right] = 0$$

$$\left(4JS\cos(ka)\right)^2 + \left(\omega^2 - 16J^2S^2\right) = 0 \Rightarrow \omega^2 = 16J^2S^2 - 16J^2S^2\cos^2(ka)$$

$$\omega^2 = 16 J^2 S^2 \left( \sin^2(ka) \right)$$

$$\boxed{\omega = 4JS \sin(ka)}$$

$\boxed{AF}$

SEE THIS COMPARISON

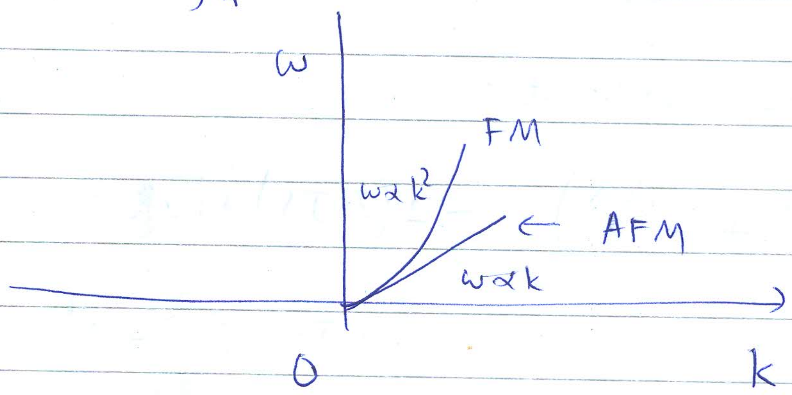
$$\boxed{\omega = 8JS \sin^2\left(\frac{ka}{2}\right)}$$

FM

$\omega \propto k$

long wavelength

$\omega \propto k^2$ (long wavelength




<u>Finding the D.O.S. of magnons in an antiferromagnet in the long wavelength regime</u>

$$\omega \approx 4JS\,ka \quad \Rightarrow \quad \boxed{k = \frac{\omega}{4JSa}} \quad \text{and} \quad \boxed{dk = \frac{d\omega}{4JSa}}$$

$$g(k)\,dk = \frac{4\pi k^2}{(2\pi)^3}\,dk$$

$$= \frac{k^2}{8\pi^2}\,dk \quad \text{— (11)}$$

Inserting $k$ and $dk$ into (11) yields:

$$g(k)\,dk = \frac{1}{8\pi^2}\,\frac{\omega^2}{(4JSa)^2}\,\frac{d\omega}{(4JSa)}$$

$$= \frac{\omega^2}{8\pi^2(4JSa)^3}\,d\omega$$

$$\Rightarrow \boxed{g(\omega) = \frac{\omega^2}{8\pi^2(4JSa)^3}}$$

<u>Heat capacity of an antiferromagnet at low temperatures</u>

$$E = \overset{\text{zero point}}{E_0} + \int_0^\infty d\omega\,\hbar\omega\,g(\omega)\,n_{BE}(\omega,T)$$

$$= E_0 + \hbar\int_0^\infty d\omega\,\frac{\omega^3}{8\pi^2(4JSa)^3}\cdot\frac{1}{e^{\hbar\omega/k_BT}-1}$$

Let $\dfrac{\hbar\omega}{k_BT} = x \Rightarrow d\omega = \dfrac{k_BT}{\hbar}\,dx$ ; $\omega = \dfrac{k_BT}{\hbar}\,x$

$$E = E_0 + \hbar\left(\frac{k_BT}{\hbar}\right)^4 \int_0^\infty dx\,\frac{x^3}{8\pi^2(4JSa)^3}\cdot\frac{1}{e^x-1}$$

$$C_v = \frac{\partial E}{\partial T} = \frac{4k_B^4}{\hbar^3}\,T^3\cdot\frac{1}{8\pi^2(4JSa)^3}\int_0^\infty\frac{x^3\,dx}{e^x-1} \propto T^3.$$

# Ferrimagnetism

↑ ↑ ↑ ↑

↓ ↓ ↓ ↓

staggered type

$\cancel{M_A \neq M_B}$

$M_+ \neq -M_-$

$$M_+ = M_{s+} B_J \left( \frac{g_J \mu_B J}{k_B T} \left( B + \alpha_1 M_+ + \beta M_- \right) \right)$$

unequal          equal

$$M_- = M_{s-} B_J \left( \frac{g_J \mu_B J}{k_B T} \left( B + \alpha_2 M_- + \beta M_+ \right) \right)$$

$$\cancel{M_+ + M_- = M_{s+} \frac{g_J \mu_B J}{k_B T}}$$

$$M_+ = M_{s+} \frac{g_J \mu_B J}{k_B T} \left( B + \alpha_1 M_+ - \beta M_- \right) \left( \frac{J+1}{3J} \right)$$

Q2 [redacted]

$$M_+ = \frac{C_1}{T} \left( B + \alpha_1 M_+ - \beta M_- \right)$$

$$M_- = \frac{C_2}{T} \left( B + \alpha_2 M_- - \beta M_+ \right)$$

$$M_+ \left( 1 - \frac{\alpha_1 C_1}{T} \right) + M_- \beta \frac{C_1}{T} = \frac{C_1 B}{T}$$

$$M_+ \left( + \beta \frac{C_2}{T} \right) + M_- \left( 1 - \frac{\alpha_2 C_2}{T} \right) = \frac{C_2 B}{T}$$

→ Spontaneous magnetization will build up at $B = 0$

$$\therefore \begin{vmatrix} 1 - \dfrac{\alpha_2 C_1}{T_c} & +\dfrac{\beta C_1}{T_c} \\[2ex] +\dfrac{\beta C_2}{T_c} & 1 - \dfrac{\alpha_2 C_2}{T_c} \end{vmatrix} = 0$$

$$\left(1 - \frac{\alpha_1 C_1}{T_c}\right)\left(1 - \frac{\alpha_2 C_2}{T_c}\right) - \frac{\beta^2 C_1 C_2}{T_c^2} = 0$$

$$\frac{(T_c - \alpha_1 C_1)(T_c - \alpha_2 C_2)}{T_c^2} - \frac{\beta^2 C_1 C_2}{T_c^2} = 0.$$

$$T_c^2 \cancel{- (\alpha_1 + \alpha_2) C_1 C_2} - (\alpha_2 C_2 + \alpha_1 C_1) T_c + \left(\alpha_1 \alpha_2 C_1 C_2 - \beta^2 C_1 C_2\right) = 0.$$

$$T_c = \frac{1}{2}\left[\alpha_2 C_2 + \alpha_1 C_1 \pm \sqrt{(\alpha_1 C_1 + \alpha_2 C_2)^2 - 4(\alpha_1 \alpha_2 C_1 C_2 - \beta^2 C_1 C_2)}\right]$$

$$\alpha_1^2 C_1^2 + \alpha_2^2 C_2^2 + 2\alpha_1 C_1 \alpha_2 C_2 - 4\alpha_1 \alpha_2 C_1 C_2 + 4\beta^2 C_1 C_2$$

$$(\alpha_1 C_1 - \alpha_2 C_2)^2 + 4\beta^2 C_1 C_2$$

critical point

$$\therefore \boxed{T_c = \frac{1}{2}\left[\alpha_2 C_2 + \alpha_1 C_1 \pm \sqrt{(\alpha_1 C_1 - \alpha_2 C_2)^2 + 4\beta^2 C_1 C_2}\right]}$$

If $\alpha_1 = \alpha_2 = 0$, $\quad C_1 = C_2 = C$

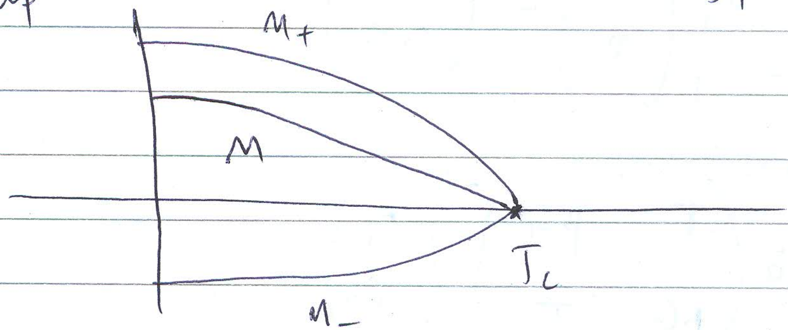
$$T_c = +\frac{1}{2}\sqrt{4\beta^2 \underset{\sim}{C_1}\underset{\sim}{C_2}} = \frac{1}{\cancel{2}}\cancel{2}\beta C$$

$\uparrow$

sublattice

Neel temp.

$$= \lambda \frac{g_J \mu_B \cancel{J}}{k_B T}\left(\frac{J+1}{3\cancel{J}}\right) = \frac{g_J \mu_B (J+1)\lambda}{3 k_B}.$$




$\Rightarrow$ Susceptibility of a ferrimagnet

Let $\alpha_1 = \alpha_2 = 0$.

However $C_1 \neq C_2$

$$\cancel{M_+ = M_s + \frac{g_J \mu_B J}{k_B T}}$$

$$M_+ = \frac{C_1}{T}(B - \beta M_-) \quad \Rightarrow \quad M_+ + \frac{\beta C_1}{T} M_- = \frac{C_1 B}{T}$$

$$M_- = \frac{C_2}{T}(B - \beta M_+) \qquad \beta\frac{C_2}{T} M_+ + M_- = \frac{C_2 B}{T}$$

$$\cancel{M_+ + M_- = \frac{C_1}{T}B + \frac{C_2}{T}B - \beta\frac{C_1}{T}M_-} - \beta\frac{C_2}{T}M_+$$

$$\begin{pmatrix} 1 & \frac{\beta C_1}{T} \\ \frac{\beta C_2}{T} & 1 \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} \frac{C_1 B}{T} \\ \frac{C_2 B}{T} \end{pmatrix}$$

$$\begin{pmatrix} T & \beta C_1 \\ \beta C_2 & T \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} C_1 B \\ C_2 B \end{pmatrix}$$

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = B \underbrace{\begin{pmatrix} T & \beta C_1 \\ \beta C_2 & T \end{pmatrix}^{-1}}_{\text{find its inverse.}} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}.$$

$$\frac{1}{T^2 - \beta^2 C_1 C_2} \begin{pmatrix} T & -\beta C_1 \\ -\beta C_2 & T \end{pmatrix}$$ , $\beta$ is in fact $\lambda$.

$$M_1 = \frac{B}{T^2 - \lambda^2 C_1 C_2} \left( T C_1 - \lambda C_1 C_2 \right)$$

$$M_2 = \frac{B}{T^2 - \lambda^2 C_1 C_2} \left( -\lambda C_1 C_2 + T C_2 \right)$$

$$M_1 + M_2 = \frac{B}{T^2 - \lambda^2 C_1 C_2} \left( T(C_1 + C_2) - 2\lambda C_1 C_2 \right)$$

$$C_1 = C_2$$

$$\chi = \frac{\mu_0 T (C_1 + C_2)}{T^2 - \lambda^2 C_1 C_2} - \frac{2\mu_0 \lambda C_1 C_2}{T^2 - \lambda^2 C_1 C_2}$$

$$\frac{\mu_0 T 2C}{T^2 - \lambda^2 C^2} - \frac{\mu_0 \lambda C^2}{T^2 - \lambda^2 C^2}$$

$$= \frac{\mu_0}{T^2 - \lambda^2 C_1 C_2} \left( T (C_1 + C_2) - 2\lambda C_1 C_2 \right)$$

$$\mu_0 \frac{2TC - 2\lambda C^2}{T^2 - \lambda^2 C^2}$$

$$= 2\mu_0 C \frac{(T - \cancel{\lambda C})}{(T + \lambda C)(\cancel{T - \lambda C})}$$

an antiferromagnet.

$$= \frac{2\mu_0 C}{T + \lambda C}$$