[redacted]

Q.1.

$$4s^2 3d^8$$

Maximize S $\qquad$ $\boxed{\uparrow\downarrow \,|\, \uparrow\downarrow \,|\, \uparrow\downarrow \,|\, \uparrow \,|\, \uparrow}$

$$S = 2 \times 1/2 = 1.$$

$$\Sigma m_l = 1 + 2 = 3 = L$$

$$J = L + S \qquad \text{More than half filled}$$
$$= 3 + 1 = 4$$

Multiplicity $\quad 2S + 1 = 3$

The allowed values of mag. moment along The field axis are given by $g m_j \mu_B$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$= 1 + \frac{20 + 2 - 12}{40}$$

$$= 1.25$$

since $J = 4$, $m_j = -4, -3, -2, -1, 0, 1, 2, 3, 4$

Mag. moment along the field direction $= 1.25 \mu_B m_j$

Q.2

As $N^+ = \frac{1}{2}N(1+\eta)$

$N^- = \frac{1}{2}N(1-\eta)$

Total no. of modes/states is given by

$$N = 2\left(L/2\pi\right)^3 \, 4/3\, \pi K^3 \qquad \text{where } K \text{ is radius of sphere.}$$

$$K = \left(2mE/\hbar^2\right)^{1/2}$$

$$N = 2\frac{V}{8\pi^3} \, 4/3\, \pi \left(2mE/\hbar^2\right)^{3/2}$$

$$N/V = 1/3\pi^2 \left(2mE/\hbar^2\right)^{3/2}$$

$$E_{K.E} = \hbar^2/2m \left(3\pi^2 N/V\right)^{2/3} = E_F$$

Energy of the spin up band electrons is

$$E^+ = 3/5\, N^+ E_F \qquad U = 3/5\, N E_F$$

plugging in the value of N^+ and E_F, we get

$$\boxed{E^+_{K.E} = 3/10\, E_F \,(1+\eta)^{5/3}}$$ Kinetic energy term for spin up electrons.

Now energy in a mag. field for spin up electron

$$E^+_m = -N^+ \mu_B B$$

$$E^+_m = -\tfrac{1}{2} N(1+\eta)\mu_0 B$$

total energy of spin up electron

$$E^+_0 = E^+_{K.E} + E^+_m$$

$$\boxed{E^+ = E_0(1+\eta)^{5/3} - \tfrac{1}{2} N\mu_0 B(1+\eta)}$$

Similarly for E^-

$$\boxed{E^- = E_0(1-\eta)^{5/3} + \tfrac{1}{2} N\mu_0 B(1-\eta)}$$

b)

$$E_{tot} = E^+ + E^-$$

$$= E_0\left[(1+\eta)^{5/3} + (1-\eta)^{5/3}\right] - N\eta\mu_0 B$$

$$\frac{\partial E_{tot}}{\partial \eta} = E_0 \, 5/3 \left[(1+\eta)^{2/3} \mp (1-\eta)^{2/3}\right] - N\mu_0 B = 0$$

$$(1+\eta)^{2/3} \mp (1-\eta)^{2/3} = 3/5 \, \frac{N\mu_0 B}{E_0}$$

For $\eta << 1$

$$\left(1 + 2/3\,\eta + \text{---}\right) - \left(1 - 2/3\,\eta + \text{---}\right) = 3/5 \, \frac{N\mu_0 B}{E_0}$$

$$\Rightarrow \boxed{\eta = \frac{9}{20}\frac{N\mu_0 B}{E_0}}$$

Q.3

Dispersion relation is

$$E = \alpha k^{3/2}$$

Density of states in 2-D can be simplifield as

$$D(\epsilon)d\epsilon = \frac{Ak}{2\pi}\left(\frac{dk}{d\epsilon}\right)d\epsilon.$$

A = area = L^2

Now $K = \left(\frac{E}{\alpha}\right)^{2/3}$

$2/3 - 1$

$2/3 - 1/3$

$$D(\epsilon) = \frac{A}{2\pi}\left(E/\alpha\right)^{2/3} \frac{d}{d\epsilon}\left(E/\alpha\right)^{2/3}$$

$$= \frac{A}{2\pi\,\alpha^{4/3}} E^{2/3} \cdot \frac{2}{3} E^{-1/3}$$

$$\boxed{D(\epsilon) = \frac{A}{3\pi\,\alpha^{4/3}} E^{1/3}}$$

Q.4

since we are in ground state and $T=0$

$$F = U - TS$$

$$F = U$$

magnetization can be found

$$M = -\partial U/\partial B = -\partial F/\partial B$$

lowest landau level is split by mag. field into two sublevels with energies

$$E^- = \tfrac{1}{2}\hbar\omega_c - \mu_B B \quad , \quad E_+ = \tfrac{1}{2}\hbar\omega_c + \mu_s B$$

Diamagnetic Contribution comes from the diamagnetic part of energy $N\hbar\omega_c/2$

$$M_D = -\frac{\partial}{\partial B}\left(N\hbar\frac{eB}{2m}\right)$$

$$= -\frac{Ne\hbar}{2m}$$

$$\boxed{M_D = -N\mu_B}$$