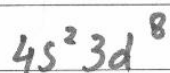
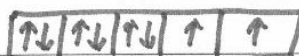


~~Q.1.~~

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Maximize S



$$S = 2 \times \frac{1}{2} = 1$$

$$\sum m_l = 1 + 2 = 3 = L$$

$$J = L + S \quad \text{More than half filled}$$
$$= 3 + 1 = 4$$

$$\text{Multiplicity} \quad 2S + 1 = 3$$

The allowed values of mag. moment along the field axis are given by $g m_j \mu_B$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$= 1 + \frac{20 + 2 - 12}{40}$$

$$= 1.25$$

Since $J = 4$, $m_j = -4, -3, -2, -1, 0, 1, 2, 3, 4$

Mag. moment along the field direction = $1.25 \mu_B m_j$

Q.2

As $N^+ = \frac{1}{2} N(1+\eta)$

$N^- = \frac{1}{2} N(1-\eta)$

Total no. of modes/states is given by

$N = 2 \left(\frac{L}{2\pi}\right)^3 \frac{4}{3} \pi k^3$ where k is radius of sphere.

$k = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$

$N = 2 \frac{V}{8\pi^3} \frac{4}{3} \pi \left(\frac{2mE}{\hbar^2}\right)^{3/2}$

$N/V = \frac{1}{3\pi^2} \left(\frac{2mE}{\hbar^2}\right)^{3/2}$

$E_{k.E} = \frac{\hbar^2}{2m} \left(3\pi^2 N/V\right)^{2/3} = E_F$

Energy of the spin up band electrons is

$E^+ = \frac{3}{5} N^+ E_F$

$U = \frac{3}{5} N E_F$

plugging in the value of N^+ and E_F , we get

$E_{k.E}^+ = \frac{3}{10} E_F (1+\eta)^{5/3}$

Kinetic energy term for spin up electrons.

Now energy in a mag. field for spin up electron

$E_m^+ = -N^+ \mu_B B$

$$E_m^+ = -\frac{1}{2} N(1+\eta) \mu_B$$

total energy of spin up electron

$$E^+ = E_{k,E}^+ + E_m^+$$

$$E^+ = E_0 (1+\eta)^{5/3} - \frac{1}{2} N \mu_B (1+\eta)$$

Similarly for E^-

$$E^- = E_0 (1-\eta)^{5/3} + \frac{1}{2} N \mu_B (1-\eta)$$

b)

$$E_{tot} = E^+ + E^-$$

$$= E_0 \left[(1+\eta)^{5/3} + (1-\eta)^{5/3} \right] - N \eta \mu_B$$

$$\frac{\partial E_{tot}}{\partial \eta} = E_0 \frac{5}{3} \left[(1+\eta)^{2/3} - (1-\eta)^{2/3} \right] - N \mu_B = 0$$

$$(1+\eta)^{2/3} - (1-\eta)^{2/3} = \frac{3}{5} \frac{N \mu_B}{E_0}$$

For $\eta \ll 1$

$$\left(1 + \frac{2}{3} \eta + \dots \right) - \left(1 - \frac{2}{3} \eta + \dots \right) = \frac{3}{5} \frac{N \mu_B}{E_0}$$

$$\Rightarrow \boxed{\eta = \frac{q}{20} \frac{N \mu_B}{E_0}}$$

Q.3

Dispersion relation is

$$E = \alpha k^{3/2}$$

Density of states in 2-D can be simplified as

$$D(E) dE = \frac{AK}{2\pi} \left(\frac{dk}{dE} \right) dE.$$

$$A = \text{area} = L^2$$

$$\text{Now } k = \left(\frac{E}{\alpha} \right)^{2/3}$$

$$\begin{aligned} \frac{2}{3} - 1 \\ \frac{2}{3} - \frac{1}{3} \end{aligned}$$

$$\begin{aligned} D(E) &= \frac{A}{2\pi} \left(\frac{E}{\alpha} \right)^{2/3} \frac{d}{dE} \left(\frac{E}{\alpha} \right)^{2/3} \\ &= \frac{A}{2\pi \alpha^{4/3}} E^{2/3} \cdot \frac{2}{3} E^{-1/3} \end{aligned}$$

$$\boxed{D(E) = \frac{A}{3\pi \alpha^{4/3}} E^{1/3}}$$

Q.4

Since we are in ground state and $T=0$

$$F = U - TS$$

$$F = U$$

Magnetization can be found

$$M = - \frac{\partial U}{\partial B} = - \frac{\partial F}{\partial B}$$

lowest Landau level is split by mag. field into two sublevels with energies

$$E^- = \frac{1}{2} \hbar \omega_c - \mu_B B, \quad E_+ = \frac{1}{2} \hbar \omega_c + \mu_B B$$

Diamagnetic Contribution comes from the diamagnetic part of energy $N \hbar \omega_c / 2$

$$M_D = - \frac{\partial}{\partial B} \left(N \hbar \frac{eB}{2m} \right)$$

$$= - \frac{Ne \hbar}{2m}$$

$$\boxed{M_D = - N \mu_B}$$