Vertical Pendulum in Phase Space

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The bicycle wheel can serve as a very good example of a physical pendulum. In this experiment, we study the motion of the bicycle wheel using the smartphone's built-in accelerometer and gyroscope sensors. In the end, we explore the phase space trajectories of the motion.

Essential pre-lab reading:

"*Physics for Engineers and Scientists : Extended 3rd Edition*" by Ohanian, H.C. and Markert, J.T. (W.W. Norton and Company, 2007); (Sections 12.1-12.5 and 13.2-13.3)

M. Monteiro, C. Cabeza, A.C. Marti, "*Exploring Phase Space Using Smartphone Acceleration and Rotation Sensors Simultaneously*" European Journal of Physics **35**, 045013 (2014)

1 Test your understanding

1. What is the equation of motion of a pendulum
   (a) rotating in the vertical plane; and
   (b) oscillating about its mean position in the vertical plane?
2. Consider a wheel with moment of inertia I and a mass m attached to the wheel at its periphery at a distance R from the centre of mass. What is the total moment of inertia?
3. In the case of no translation, what is the total energy of the wheel rotating in a vertical plane?

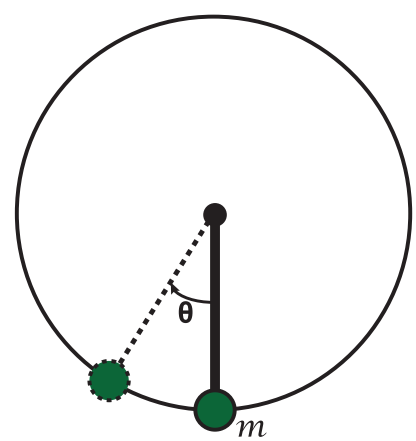
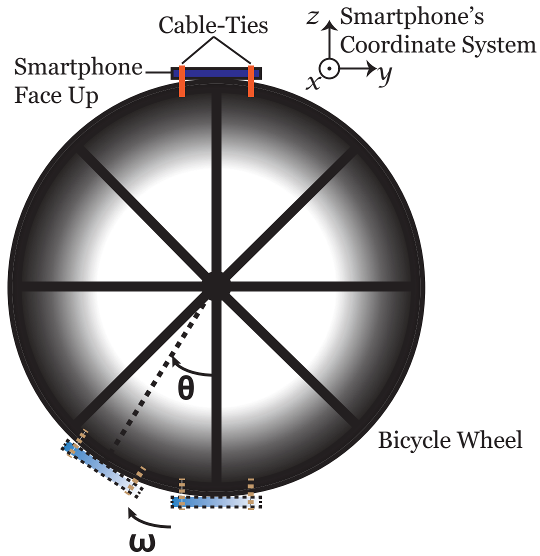


Figure 1: A physical pendulum in a vertical plane.



(a) Schematic of the experiment.

(b)

(c)

Figure 2: Experimental Setup

2 The Experiment

We will attach the mobile phone the outer rim of the bicycle wheel and use the AndroSensor (v1.9.6.3) to collect the data simultaneously from the smartphone's accelerometer and gyroscope sensors. After data analysis, we will explore the phase space trajectories of the different states of motion of the bicycle wheel. So now let's get on with the experiment.

Start by attaching the smartphone to the periphery of the bicycle wheel. You can use the cable-ties to securely attach the smartphone to the outer rim (use figure (2) as reference). Ensure that the smartphone is oriented correctly and not askew. Even a slight tilt will record a component in two fixed coordinate axes which is undesirable. When ready tap the "**Record**" button.

Now, hold the edge of the wheel and give it a push so that the wheel starts rotating in a manner that the smartphone is moving in the $+y$ direction with respect to its native fixed coordinate

system. Closely observe the motion as it goes from full rotations to large oscillations. The large oscillations will also die down eventually to small harmonic oscillations. Allow the motion to almost cease before stopping data recording. Save this file and transfer it to the file to your workstation using the data cable. You can also email the file to yourself. App will generate a .csv file containing data from every sensor on the smartphone.

3 Data Analysis

The data of our concern; one is the raw acceleration data along the three fixed coordinate axes and the second is the gyroscope data about the three fixed coordinate axes. Simply open the .csv file in Excel, copy the data of concern and paste it to new variables in the workspace of Matlab.

3.1 Dynamical Variables

Q 1. Along which axes do you expect accelerometer data corresponding to

(a) centripetal acceleration; and

(b) tangential acceleration?

Q 2. About which axes do you expect to find ω? (Hint: This is the gyroscope data about one of the three axes.)

Q 3. Using the plot for ω, try to find the point at which the system stops rotating and starts oscillating. What is the timestamp at this point?

We will now integrate ω to obtain θ. To integrate, we use the MATLAB's built-in function for cumulative trapezoidal numerical integration **cumtrapz**. The syntax for using this function is "z = ***cumtrapz***(x,y);" where x is the time vector and y is the angular velocity data. The output z is the numerical integral which is the angular displacement.

Q 4. Draw the following plots and describe your graphs. For example, the maxima and minima of the physical quantities and if the observed behaviour can be described coherently.

(a) ω vs t and a_r vs t (plot both together).

(b) θ vs t and a_r vs t.

It is also interesting to study the temporal evolution of kinetic and potential energies of the system. The kinetic energy here is purely rotational as the system is not translating but rotating about a fixed axis.

Q 5. If we take the bottom of the wheel to be the zero of potential energy, what is the potential energy at

(a) the top of the wheel;

(b) an angle θ from its lowest point?

The total energy at the bottom is therefore only the rotational kinetic energy whereas at the top, the total energy is the sum of the rotational kinetic energy and the gravitational potential energy.

Q 6. Using the gyroscope data, calculate the rotational kinetic energy at each point and plot it versus time. Plot the total energy as a function of time.

3.2 Phase Space Portraits

Now, we will plot the trajectories in phase space for this motion. The state of a mechanical system can be represented by a unique point in this geometrical representation which consists of all possible values of the generalized coordinates and conjugate variables (generalized momenta). For systems with only one degree of freedom such as our physical pendulum, the dimension of the phase space is equal to two. If we take the generalized coordinate to be θ, then the conjugate variable is the corresponding component of angular momentum $I\omega$.

As the system evolves, it traces a path or trajectory in the phase space. We will represent the phase space as a function of $\dot{\theta} = \omega$ (obtained from the gyroscope) and the angular acceleration $\ddot{\theta} = a_t/R$ (obtained from the accelerometer).

Q 7. Plot the trajectories in phase space ($\ddot{\theta}$ vs $\dot{\theta}$).

At the first stage of the experiment, the wheel is fully rotating performing full revolutions in the same direction. As the energy is dissipated and when the angular velocity changes sign for the first time, the wheel starts oscillating about the stable equilibrium position.

Q 8. Plot these two trajectories in different colours to differentiate between the rotations and oscillations in phase space.

With each oscillation, further energy is dissipated and the amplitude of oscillations decreases. Finally, in the case of simple harmonic oscillations, the trajectories can be approximated by ellipses in the phase space. You should be able to fully describe the phase space trajectories.