Assignment 2: Quantum Field Theory
Due Date: 15 Feb. 2018, 4 pm

1. Show that $\partial_v F^{\mu\nu} = 0$ reproduces the source-free (homogenous) Maxwell equations:

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = 0 \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{2}$$

2. A charge is at rest in an unprimed frame. What are the electric and magnetic fields as measured in the primed frame that moves with a speed v parallel to the x axis towards the right? The electrognetic field tensor is given by,

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}. \tag{3}$$

3. Prove that the Lorentz transformation preserves the antisymmetry property of a rank-2 tensor.

4. Consider a Lagrangian given by

$$\mathscr{L} = -m\sqrt{1 - \dot{x}^2} - mAx \tag{4}$$

where A is a constant. Find the relativistic trajectory of the particle $x(t)$.