Assignment 9: Quantum Field Theory
Due Date: 19 April. 10 am

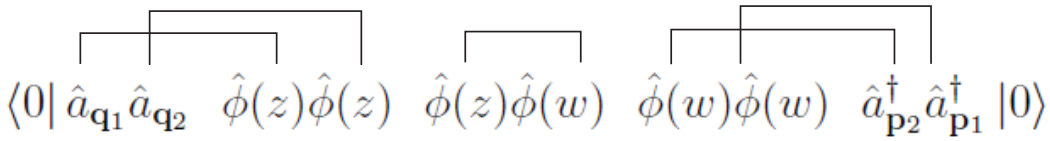
1. In the ϕ^3 scalar field theory, the Lagrangian density is given by,

$$\mathscr{L} = \frac{1}{2}(\partial^\mu \hat{\phi}(x))^2 - \frac{1}{2}m^2(\hat{\phi}(x))^2 - \frac{\eta}{3!}(\hat{\phi}(x))^3.$$

   (a) Motivate the first and second order single-particle scattering amplitudes, $\langle q|\,\hat{S}\,|p\rangle$, where $|p\rangle$ and $|q\rangle$ are input and output states. For each term in the amplitude you are required to (i) show the pairwise contraction, (ii) draw the associated Feynman diagram, (iii) identify the number of possible permutations corresponding to the same diagram and hence compute the symmetry factor$D$. [15 **marks**]

   (b) Draw at least six terms in the fourth order expansion and write down the corresponding pairwise contractions. Only fully connected Feynman diagrams are allowed. [10 **marks**]

   (c) Consider the two particle scattering $\langle q_1 q_2|\,\hat{S}\,|p_1 p_2\rangle$. Write down the integral expansion (in momentum space) and draw the Feynman diagram corresponding to the contraction pattern.



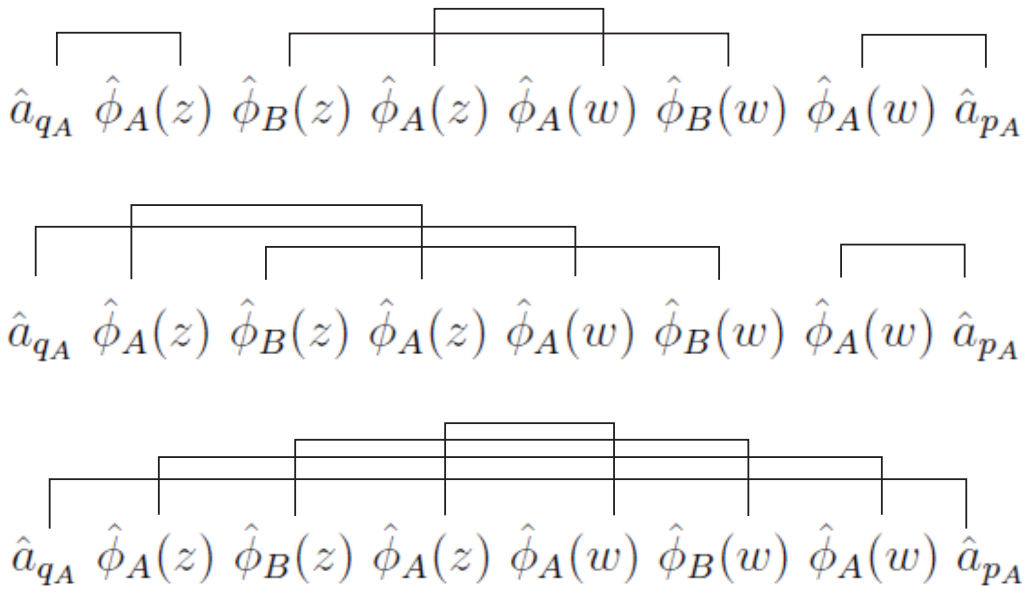
   [10 **marks**]

   (d) Draw eight possible Feynman diagrams and the contraction patterns in the fourth order, two-particle transition amplitude. You are allowed to show both fully connected and unconnected diagrams. [10 **marks**]

2. The ABA theory involves interaction between two types of scalar fields, labeled A and B. The Lagrangian density is,

$$\mathscr{L} = \frac{1}{2}(\partial^\mu \hat{\phi}_A(x))^2 - \frac{1}{2}m_A^2(\hat{\phi}_A(x))^2 + \frac{1}{2}(\partial^\mu \hat{\phi}_B(x))^2 - \frac{1}{2}m_B^2(\hat{\phi}_B(x))^2 - \frac{g}{2}\hat{\phi}_A(x)\hat{\phi}_B(x)\hat{\phi}_A(x).$$

   (a) Draw Feynman diagram for the following second order contractions. [5 **marks**]



(b) Write down the momentum-space integrals for the propagation corresponding to these Feynman diagrams. [5 **marks**]