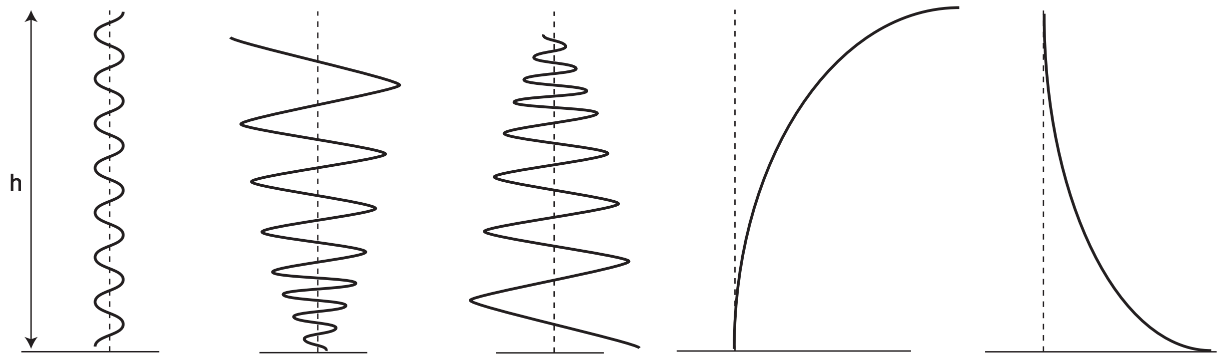
Mid-Term: Modern Physics

Attempt all questions. Write clearly and systematically, avoiding unnecessary details.

1. A heavy atom bounces up and down against the ground and is trapped within a height h above the ground. It possesses gravitational potential energy $V_0 = mgh$. Which of the following diagrams represents a plausible sketch of the atom's wavefunction (only real part is drawn)? The dashed line shows the zero of the wavefunction.



(a) (b) (c) (d) (e)

[3 Marks]

2. The levels inside an infinite well of length L are quantized according to,

$$E_n = n^2 \frac{\hbar^2 \pi^2}{2mL^2}.$$

What is the maximum wavelength of light that can be absorbed by the system?

(a) $\dfrac{4mL^2c}{3\pi\hbar}$

(b) The maximum wavelength is infinity.

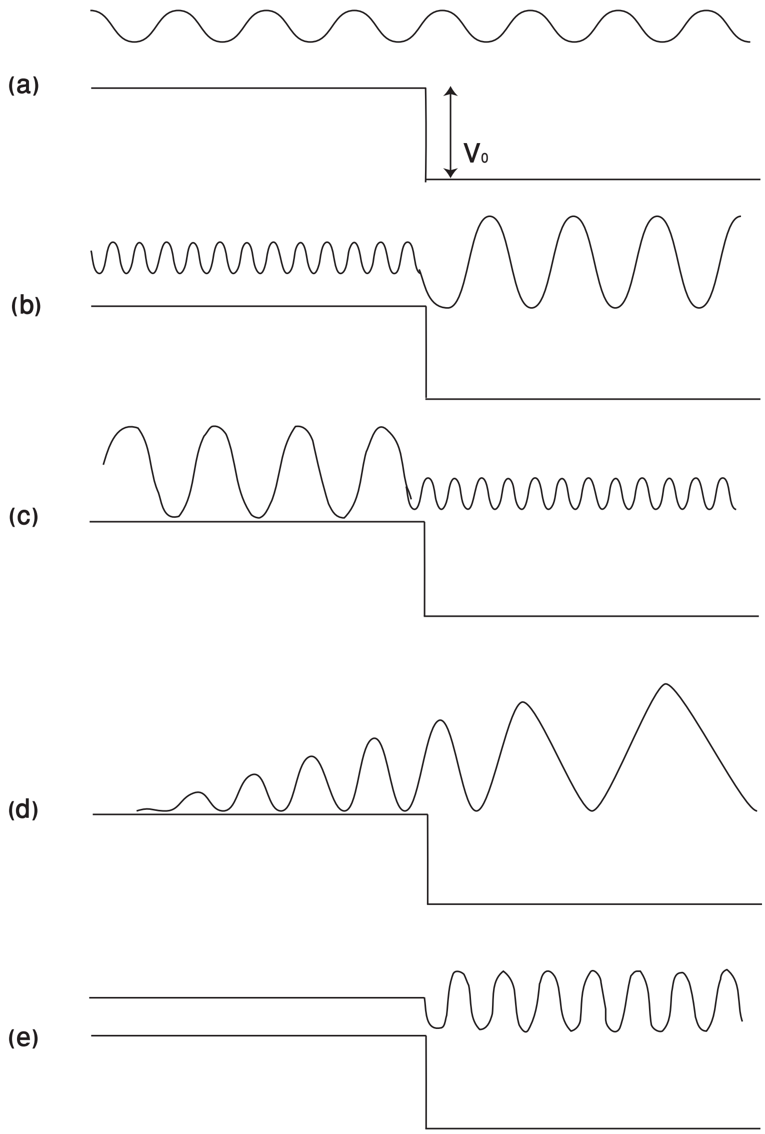
(c) $\dfrac{16\pi mL^2c}{\hbar}$

(d) none of these answers is correct.

(e) $\dfrac{\hbar^2\pi^2}{2mL^2}$

[3 Marks]

3. Which of the following is a reasonable sketch of the real part of the wavefunction where a particle encounters a potential drop.?



[3 Marks]

4. An electron in an infinite well has energy 1eV, while the ground state energy is 1 μeV. ($1\mu\text{eV} = 10^{-6}$ eV). What is the approximate probability of locating the particle in the middle half of the well?

   (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$.

   (c) $\frac{1}{4}$ (d) 1.

   (e) This is a meaningless question. **[3 Marks]**

5. A free electron of energy E has a de Broglie wavelength $\lambda = h/p = h/\sqrt{2mE}$ and speed v. In the presence of an electric field, it acquires a potential energy $V(x)$. Hence the total energy changes, and the speed of the electron changes to v'. What is the value of refractive index $n = v/v'$? (Assume $E > V(x)$.)

   (a) 1 (one)

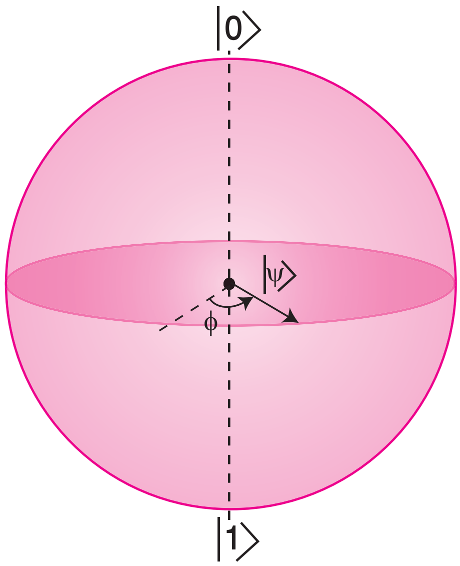
   (b) $\sqrt{\frac{E}{V(x)}}$

   (c) $\sqrt{\frac{E}{E - V(x)}}$

   (d) $\sqrt{\frac{E - V(x)}{E}}$

   (e) $\sqrt{\frac{E - V(x)}{V(x)}}$ **[3 Marks]**

6. A qubit is in the state $|\psi\rangle = \frac{1}{\sqrt{\sqrt{2}}}\left(\left(\frac{1+i}{\sqrt{2}}\right)|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$. On the Bloch sphere, this state is represented by a vector whose tip lies on the equatorial plane. What is its azimuthal angle ϕ?



   (a) $\phi = \frac{\pi}{4}$

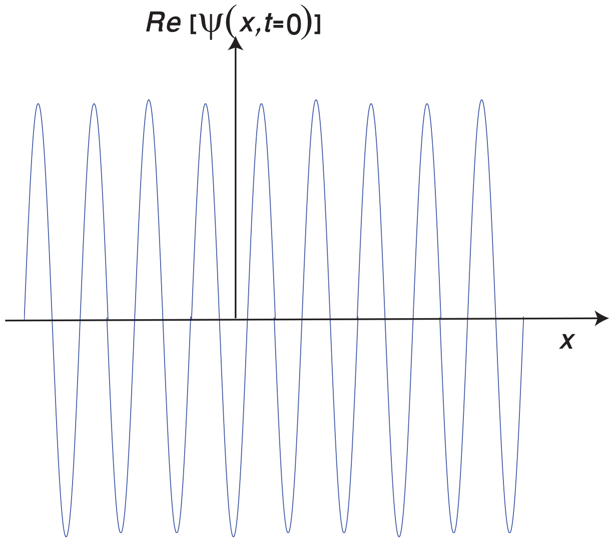
   (b) $\phi = -\frac{\pi}{4}$

   (c) $\phi = 0$

   (d) $\phi = \pi$

(e) It could be $\phi = \frac{\pi}{4}$ or $\phi = -\frac{\pi}{4}$. **[3 Marks]**

7. A particle is described by the wavefunction $\Psi(x,t) = e^{i(kx-\omega t)}$ and can be thought of a plane wave traveling along the x axis. The real part at $t = 0$ is shown in the accompanying diagram. (The wavefunction extends from $-\infty$ to ∞ which of course we cannot show on paper.) Which of the following statements *most* accurately describes the probability of finding the particle.



(a) It is equally likely to find the particle anywhere along the x axis.

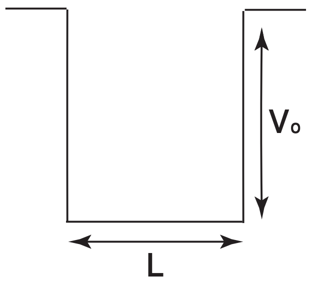
(b) It is most likely to be found in the peaks of the wave.

(c) It is most likely to be found in the peaks or the troughs the wave.

(d) The position of the particle depends on *when* I make a measurement.

(e) I have no idea how to answer this question. **[3 Marks]**

8. An electron is trapped in a quantum dot of diameter L. The electron is in a potential well of depth V_0.



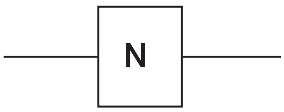
The energy values are approximately the same as the infinite well,

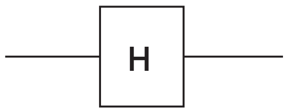
$$E = n^2 \frac{\pi^2 \hbar^2}{2mL^2}. \tag{1}$$

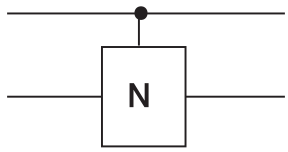
A laser photon of energy E_{photon} shines on the quantum dot in the ground state. What should be the minimum diameter if the electron is to always remain confined in the quantum dot? The dot absorbs the energy E_{photon}. **[5 Marks]**

9. Can you draw a quantum circuit (explaining how it works) that generates the three qubit entangled state $\frac{1}{\sqrt{2}}\left(|0\rangle\,|0\rangle\,|0\rangle + |1\rangle\,|1\rangle\,|1\rangle \right)$?

   You are allowed to use one or more of the following quantum gates, whose truth tables are provided below.







Input	Output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

Input	Output
$\|0\rangle$	$\frac{1}{\sqrt{2}}\left(\|1\rangle + \|0\rangle\right)$
$\|1\rangle$	$\frac{1}{\sqrt{2}}\left(\|1\rangle - \|0\rangle\right)$

Input	Output
$\|0\rangle\,\|0\rangle$	$\|0\rangle\,\|0\rangle$
$\|0\rangle\,\|1\rangle$	$\|0\rangle\,\|1\rangle$
$\|1\rangle\,\|0\rangle$	$\|1\rangle\,\|1\rangle$
$\|1\rangle\,\|1\rangle$	$\|1\rangle\,\|0\rangle$

   Note that your circuit will use three lines representing three qubits. [**10 Marks**]

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