Assignment 5: Modern Physics
Due Date: 5 April 2018, 4 pm

This is a collaborative Home Work. Mention names of all collaborators in the group. Work alone or in groups upto four. Answer the questions on the same sheet and return. You must describe the reasoning in $Q1$ to 5. This assignment also requires you to make computer generated plots.

1). **Name:** **Roll No.:**

2). **Name:** **Roll No.:**

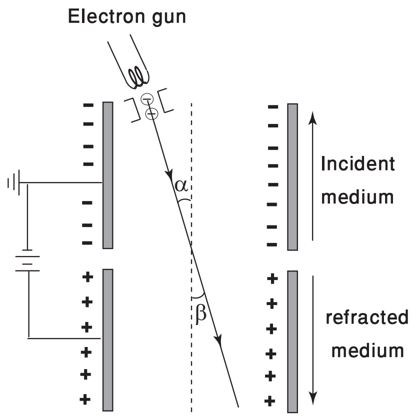
3). **Name:** **Roll No.:**

4). **Name:** **Roll No.:**

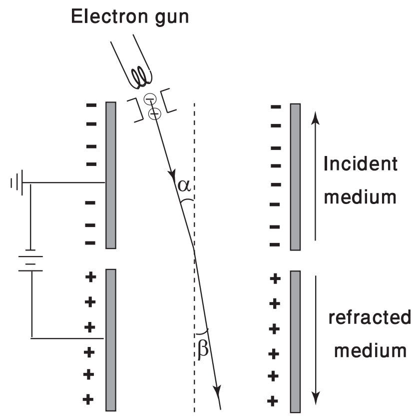
1. Snell's law of refraction determines the bending of light across an interface. For sure, electrons are also waves and can be refracted. The corresponding law for electrons is called Bethe's law and is given by,

$$\frac{\sin\alpha}{\sin\beta} = \frac{v_2}{v_1},$$

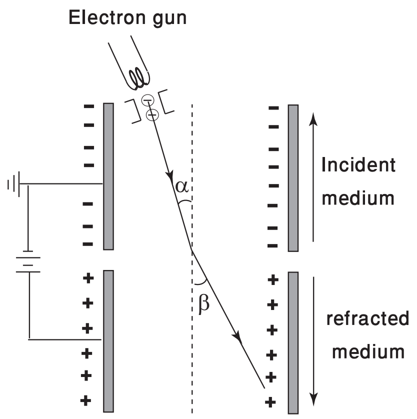
where α is the angle of incidence measured from the normal to the interface, β is the angle of refraction also measured from the normal, v_1 is the speed of electron in the incident medium and v_2 is the speed in the refracted medium. Now a beam of electrons is made to pass through two hollow cylinders with an applied voltage difference. Which of the following diagrams show the correct trajectory of electrons? Describe your reasoning.



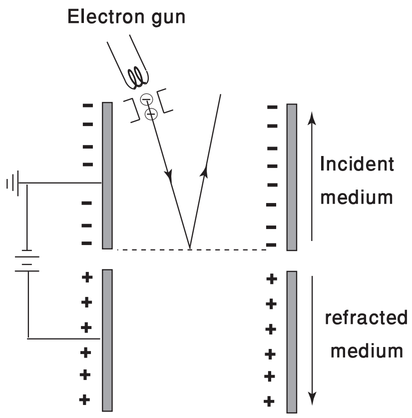
(a) No bending.



(b) Bending towards normal.



(c) Bending away from normal.



(d) No refraction takes place.

(e) None of these.

Answer 1 **and Reason:**

2. A particle moving in a region of zero force encounters a precipice—a sudden drop in the potential energy to an extremely large negative value. What is the probability that it will "go over the edge", i.e., it will enter the negative potential energy region?

(a) Almost zero. (b) Almost one.

(c) $\approx 1/2$. (d) $> 1/2$.

(e) None of the above.

Answer 2 **and Reason:**

3. An electron of energy 1 eV is trapped inside an infinite well of length 30 cm. What is the distance between two consecutive nodes of the electron's wavefunction? (A node is a point where the wavefunction goes to zero.)

   (a) There are no nodes in the electron's wavefunction.

   (b) The distance between consecutive nodes is zero.

   (c) 1.25×10^{-18} m.

   (d) 6×10^{-10} m.

   (e) None of the above.

   Answer 3 **and Reason:**

4. A free particle has a wavefunction $A(e^{ikx} + e^{-ikx})$ and energy E. A is a normalization constant. Mark **True** of **False** against these statements with the proper reason.

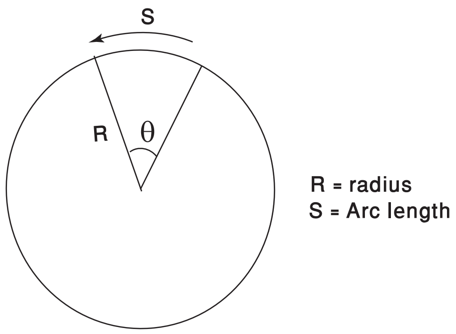
   (i) The probability density does not change with time.

   (ii) The probability density is constant in space x.

   (iii) The de Broglie wave associated with the particle is in fact a standing wave.

   Answer 4 **and Reason:**

5. The uncertainty relationship for a particle moving in a straight line is $\Delta p \Delta x \geq \hbar/2$.



If the particle is moving in a circle with angular momentum L, the uncertainty relationship becomes:

(HINT: Distance becomes the arc length!)

(a) $\Delta L \Delta \theta \geq \frac{\hbar}{2}$.

(b) $\Delta L \Delta S \geq \frac{\hbar}{2}$.

(c) $\Delta L \Delta R \geq \frac{\hbar}{2}$.

(d) $\Delta L \Delta \theta \leq \frac{\hbar}{2}$.

(e) None of the above.

Answer 5 **and Reason:**

6. The radioactive decay of certain heavy nuclei by emission of an alpha particle is a result of quantum tunneling. Imagine an alpha particle moving around inside a nucleus, such as thorium (mass number= 232). When the alpha particles bounces against the surface of the nucleus, it meets a barrier caused by the attractive nuclear force. The dimensions of barrier vary a lot from one nucleus to another, but as representative numbers you can assume that the barrier's width is $L \approx 35$ fm (1 fm $= 10^{-15}$ m) and the average barrier height is such that $V_0 - E \approx 5$ MeV. Find the probability that an alpha hitting the nucleus surface will escape. Given that the alpha hits the nuclear surface about 5×10^{21} times per second, what is the probability that it will escape in a day? The tunneling probability is $T = e^{-2\alpha L}$ where $\alpha = \sqrt{2m(V_0 - E)}/\hbar$ and L is the barrier length. (1 MeV= 10^6 eV). Plot T as a function of L. (Use the computer program.)

   Answer 6:

7. In an infinite well of length L, we have an electron in the superposition state

$$\psi = \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right) + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right).$$

   (a) Is the above state normalized?

   (b) How does the state evolve with time? Plot the real part of the state.

   (c) How does the probability density evolve with time? I am expecting the mathematical form.

   (d) Use a computer program to plot the probability density profile at various times. Is the probability density periodically varying in time? Explain your result.

   Answer 7:

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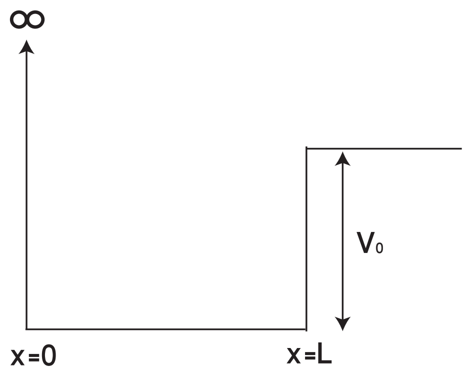
8. Consider the potential landscape shown in the diagram.



Where,

$$x < 0, \quad V \to \infty$$
$$L > x > 0, \quad V = 0$$
$$x > L, \quad V = V_0.$$

(a) Write the candidate wavefunction in the three distinct regions.

(b) Show that the energy of an electron inside this system is quantized according to

$$\sqrt{E}\ \cot\left(\sqrt{\frac{2mE}{h}}L\right) = -\sqrt{V_0 - E}.$$

Answer 8:

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