Solution Quiz 1: Modern Physics

1. **Answer 1**

$$\psi(x) = Ae^{i(10^{10}x)}. \tag{1}$$

(a) As we know for a free particle,

$$\psi(x) = Ae^{ikx}. \tag{2}$$

From equation (1) and (2),

$$\begin{aligned} k &= 10^{10}\ \text{m}^{-1} \quad \Rightarrow \quad \frac{2\pi}{\lambda} = 10^{10} \\ \lambda &= \frac{2\pi}{10^{10}}\ \text{m} = 2\pi\ \mathring{A} \end{aligned}$$

(b)

$$p = \hbar k = 1.05 \times 10^{-34} \times 10^{10} = 1.05 \times 10^{-24}\ \text{kgms}^{-2}.$$

(c)

$$K = \frac{mv^2}{2} = \frac{p^2}{2m} = \frac{(1.05 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}} = 6.06 \times 10^{-19}\ \text{J} = 3.79\ \text{eV}.$$

(d) As we have found momentum by virtue of (b),

$$\Delta p = 0, \qquad \Rightarrow \qquad \Delta x = \infty.$$

2. **Answer 2**

$$\begin{aligned} E_g &= 5E_X \\ \Rightarrow \frac{hc}{\lambda_g} &= \frac{5hc}{\lambda_X} \\ \Rightarrow \lambda_g &= \frac{\lambda_X}{5}. \end{aligned}$$

3. **Answer 3**

$$\begin{aligned} K &= \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}, \qquad \text{since} \;\; p = \hbar k \\ &= \frac{\hbar^2}{2m} \cdot \left(\frac{2\pi}{\lambda}\right)^2 = \frac{h^2}{2m\lambda^2} \\ \text{As} \;\; K_m &= K_e \\ \frac{h^2}{2m_m\lambda_m^2} &= \frac{h^2}{2m_e\lambda_e^2} \\ \Rightarrow \;\; \lambda_e^2 &= \frac{m_m}{m_e}\lambda_m^2 \\ \lambda_e &= \sqrt{200}\lambda_m. \end{aligned}$$

4. **Answer 4**

$$\begin{aligned} \psi(x) &= Ae^{ikx} \\ \mathrm{Re}\{\psi(x)\} &= A\cos(kx), \end{aligned}$$

shorter the wave length, higher will be wave number.

$$p = \hbar k$$

As (a) has shortest wave length, its k will be highest hence the momentum.

5. **Answer 5**

$$\begin{aligned} \frac{p_0 a z}{\hbar L} &= 2\pi \\ \Rightarrow \qquad z &= \frac{2\pi\hbar L}{p_0 a} = \frac{hL}{p_0 a}, \qquad \text{since} \;\; \hbar = \frac{h}{2\pi}. \end{aligned}$$

6. **Answer 6**

$$\psi(x) = A\sin(kx).$$

The length $x \in [0, L]$ fits two wavelengths of the standing wave,

$$\begin{aligned} k &= \frac{2\pi}{L/2} = \frac{4\pi}{L} \\ \Rightarrow \quad \psi(x) &= A\sin\left(\frac{4\pi x}{L}\right). \end{aligned}$$

7. **Answer 7**

$$\begin{aligned}
P(I) &= \int_0^{L/4} |\psi(x)|^2 \, dx \\
&= |A|^2 \int_0^{L/4} \sin^2\left(\frac{4\pi x}{L}\right) dx \\
&= \frac{|A|^2}{2} \int_0^{L/4} \left[1 - \cos^2\left(\frac{8\pi x}{L}\right)\right] dx \\
&= \frac{|A|^2}{2} \left| \frac{L}{4} - \frac{\sin^2\left(\frac{8\pi x}{L}\right)}{\left(\frac{8\pi x}{L}\right)} \right|_0^{L/4} \\
&= \frac{|A|^2}{2} \left[\frac{L}{4} - 0\right] \\
&= \frac{|A|^2 L}{8}.
\end{aligned}$$

Normalization:

$$\begin{aligned}
\int_0^L \left(A \sin\left(\frac{4\pi x}{L}\right)\right)^2 dx &= 1 \\
\frac{|A|^2}{2} \int_0^L \left[1 - \cos^2\left(\frac{8\pi x}{L}\right)\right] dx &= 1 \\
\frac{|A|^2}{2} L &= 1 \\
|A|^2 &= \frac{2}{L} \\
\Rightarrow \quad P(I) &= \frac{(2/L)L}{8} = \frac{1}{4}.
\end{aligned}$$

Similarly we can see that,

$$\begin{aligned}
P(I) &= P(II) \quad \text{and} \\
P(I) + P(II) &= P(III).
\end{aligned}$$

8. **Answer 8**

$$\Delta z = \frac{\lambda L}{a}$$

As $p = h/\lambda$, and $K = \dfrac{p^2}{2m}$, $\Rightarrow \quad \lambda = \dfrac{h}{\sqrt{2mK}}$.

$$\begin{aligned}
\Delta z &= \frac{hL}{a\sqrt{2mK}} \\
\text{As} \qquad m &= 200\ m_e
\end{aligned}$$

Δz would decrease $\sqrt{200}$ times.

9. **Answer** 9

According to uncertainty principle:

$$
\begin{aligned}
\Delta E\ \Delta t &\geq \frac{\hbar}{2} \\
\Delta t &\geq \frac{\hbar}{2\Delta E} \\
\Delta E &= 0.1\%\ E = \frac{0.1}{100} \times \frac{1}{2} m v^2 \\
\Delta E &= 4.56 \times 10^{-32}\ \text{J} \\
\Rightarrow \quad \Delta t &\geq \frac{1.05 \times 10^{-34}}{2 \times 4.56 \times 10^{-32}} \\
&\geq 1\ \text{ms}.
\end{aligned}
$$