Solution Quiz $3b$: Modern Physics

Date: 12 **April** 2018

Useful Formulae

$$h = 6.63 \times 10^{-34} \text{ Js} \qquad m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$E_n = \frac{-13.6}{n^2} \text{ eV in a } H \text{ atom} \qquad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J.}$$

1. What is the maximum and minimum wavelength emitted from a Hydrogen atom.

(5 marks)

Answer 1

Minimum Wavelength

Transition is from $n = \infty$ to $n = 1$.

$$\begin{aligned}
\Delta E &= -\frac{E_0}{n^2} - \left(-\frac{E_0}{n_1^2}\right) \\
&= 0 + \frac{13.6}{1^2} = 13.6 \text{ eV} \\
\Delta E &= \frac{hc}{\lambda} \\
\Rightarrow \qquad \lambda &= \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ m/s}}{13.6 \times 1.6 \times 10^{-19} \text{ J}} = 91 \text{ nm.}
\end{aligned}$$

Maximum Wavelength

As n goes large, the levels are finely spaced. this means that transitions between these levels results in extremely long wavelengths, theoretically approaching infinity.

2. The energy of an electron in the ground state is

$$E_1 = \frac{e^2}{4\pi\varepsilon_0(2a_0)},$$

where a_0 = Bohr's radius. Show that in the ground state of the bound H atom, the maximum distance of the electron from the nucleus can be $2a_0$. (5 marks)

Answer 2

At the maximum possible distance, the kinetic energy is zero. so the overall energy equals the potential energy $-\frac{e^2}{4\pi\varepsilon_0 r} \cdot$ So,

$$-\frac{e^2}{4\pi\varepsilon_0 r} = E_1 = -\frac{e^2}{4\pi\varepsilon_0(2a_0)}$$

yields $2 = 2a_0$ which is the maximum distance an electron can be in the ground state of the bound H atom. Beyond $2 = 2a_0$, the K.E. will become negative which is not meaningful.

3. A particle in one dimension is described by the normalized wavefunction

$$\psi(x) = \begin{Bmatrix} 0, & x < 0 \\ Ce^{-x}(1 - e^{-x}), & x \geq 0. \end{Bmatrix}$$

(a) Find C.

(b) An experiment is performed with a microscope aimed at locating the particle somewhere. At what location is it most likely to be found? (5 marks)

Answer 3

(a) C can be found by normalization as below.

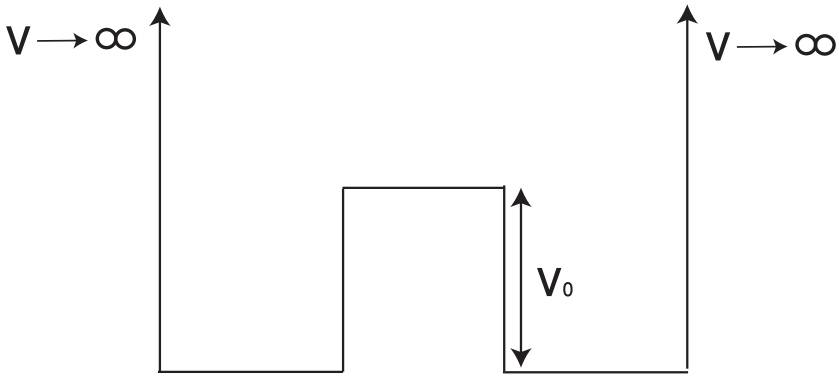
$$\begin{aligned}
\int_{-\infty}^{\infty} dx\ |\psi(x)|^2 &= \int_{-\infty}^{0} dx\ 0 + \int_{0}^{\infty} dx|C|^2\Big(e^{-x}(1 - e^{-x})\Big)^2 \\
&= |C|^2 \int_0^\infty dx\Big(e^{-x} - e^{-2x}\Big)^2 \\
&= |C|^2 \int_0^\infty dx\Big(e^{-2x} + e^{-4x} - 2e^{-3x}\Big) \\
&= |C|^2 \left(\frac{e^{-2x}}{-2} + \frac{e^{-4x}}{-4} + \frac{2}{3}e^{-3x}\right)\Bigg|_0^\infty \\
&= |C|^2\left(\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right) = \frac{|C|^2}{12} = 1 \\
\Rightarrow \qquad C &= \pm\sqrt{12}
\end{aligned}$$

(b)

$$\begin{aligned}
|\psi|^2 &= |C|^2(e^{-2x} + e^{-4x} - 2e^{-3x}) = p(x) \\
\frac{d}{dx}p(x) &= |C|^2(-2e^{-2x} - 4e^{-4x} + 6e^{-3x}) = 0 \\
\Rightarrow \qquad &- 2e^{-2x}(1 + 2e^{-2x} - 3e^{-x}) = 0 \\
\Rightarrow \qquad 1 &= -2e^{-2x} + 3e^{-x}
\end{aligned}$$

This equation will be graphically or numerically solved to determine x.

4. Consider the potential well shown below. Assume $E > V_0$. Sketch an approximate wavefunction for the eighth excited level. It has 8 nodes excluding the nodes at the walls. (5 marks)



Answer 4