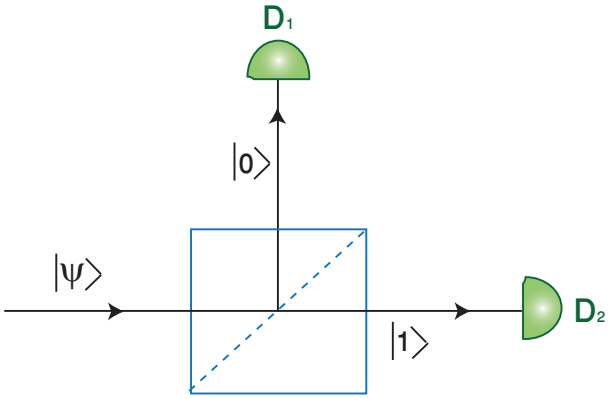
Tutorial 2: Modern Physics

1. (a) A quantum state of a qubit is,

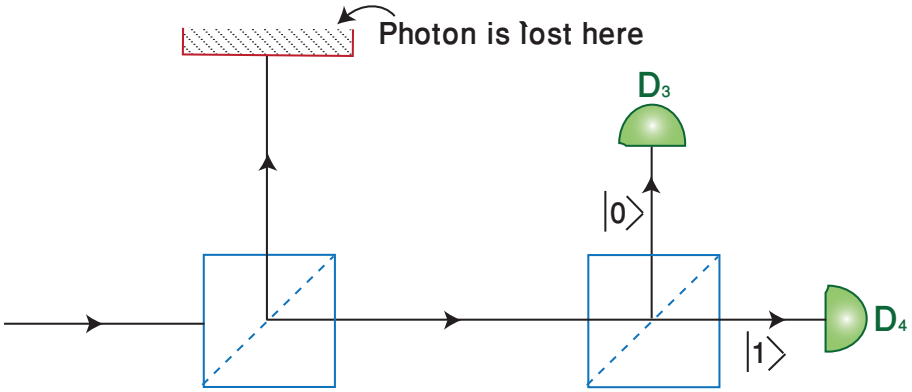
$$|\psi\rangle \frac{1}{\sqrt{2}}\Big(|0\rangle + e^{i\phi}|1\rangle\Big),$$

where ϕ is equal to $\pi/2$ $(= 90°)$. Plot the state on a Bloch sphere.

(b) This qubit is physically implemented by the path of a photon and hits a $50:50$ beamsplitter producing two paths $|0\rangle$ and $|1\rangle$. Detectors are placed on these paths. See the figure. What is the probability that D_1 clicks? D_2 clicks? Both click at the same time.



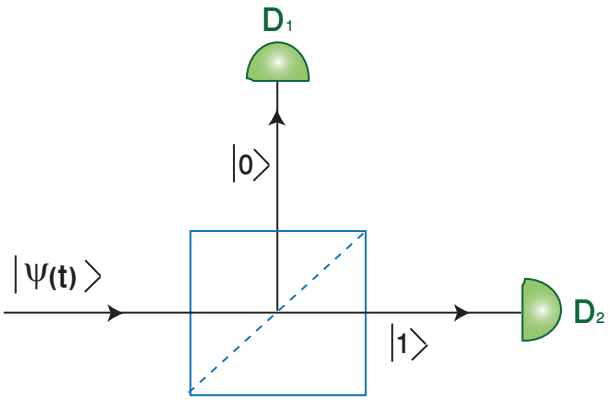
(c) D_1 is now replaced by an opaque screen. What is the probability that D_3 clicks? D_4 clicks? Both D_3 and D_4 click?



2. A quantum state changes with time and is given by

$$|\psi(t)\rangle = \frac{1}{\sqrt{3}}\,|0\rangle + \sqrt{\frac{2}{3}}\,e^{i\omega t}|1\rangle,$$

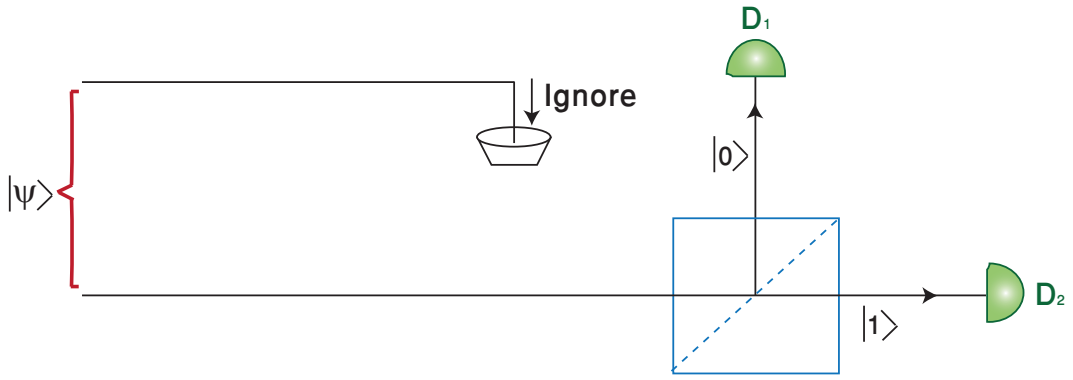
where $\omega > 0$ is a frequency. See the accompanying figure.



What is the probability P that D_1 clicks, D_2 clicks at various times t? Plot P_1, P_2 as a function of t.

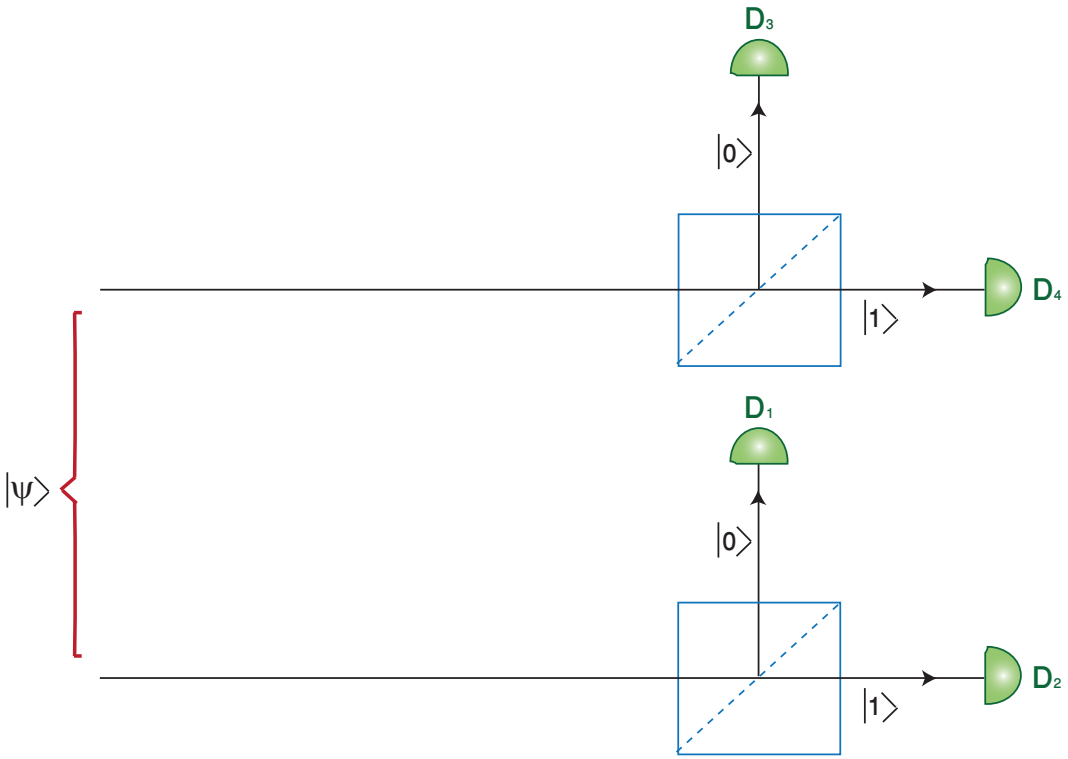
3. Consider an entangled pair of photons with the state description.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\,0\rangle|\,1\rangle - |\,1\rangle|\,0\rangle\Big).$$



(a) The first photon is not observed, while the second photon passes through a beam splitter. What is the probability that D_1 clicks? D_2 clicks?

(b)



Now both photons are observed. Compute the following probabilities.

i. $P(D_1$ clicks)

ii. $P(D_2$ clicks)

iii. $P(D_1$ clicks and D_3 clicks)

iv. $P(D_1$ clicks and D_4 clicks)

v. $P(D_1$ clicks and D_2 clicks).

What is "strange" about these outcomes?