Tutorial 8: Modern Physics

1. How much more likely is a $1s$ electron in hydrogen to be at a distance a_0 (Bohr's radius) from the nucleus than at a distance $a_0/2$? The $1s$ wavefunction is,

$$\psi_{n=1,\,\ell=0,\,m_l=0}(r,\theta,\phi) = \frac{1}{\sqrt{\pi}\,a_0^{3/2}}\,e^{-r/a_0}.$$

2. A spherical infinite well has potential energy,

$$U(r) = \begin{cases} 0 & r < a \\ \infty & r > a. \end{cases}$$

This is a central potential and the radial part of the Schrodinger equation in the region $r < a$ is given by,

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)R(r) + \frac{\hbar^2\,l(l+1)}{2mr^2}R(r) = ER(r),$$

where all symbols have the usual meanings.

(a) Show that

$$R(r) = A(\sin br)/r$$

is a solution for $0 < r < a$. What should be the angular momentum if this is a solution?

(b) Apply the appropriate boundary conditions and in doing so, find the quantized energies.

3. Draw polar diagrams for the following orbitals.

(a) $Y_{2,0}(\theta,\phi) = K(3\cos^2\theta - 1)$

(b) $\dfrac{Y_{2,1}(\theta,\phi) + Y_{2,-1}(\theta,\phi)}{\sqrt{2}} = K\sin\theta\cos\theta\cos\phi$

(c) $\dfrac{Y_{2,1}(\theta,\phi) - Y_{2,-1}(\theta,\phi)}{\sqrt{2}} = K\sin\theta\cos\theta\sin\phi$