Solution Quiz 4: Modern Physics

Date: 5 **May** 2018

Useful Formulae

$$h = 6.63 \times 10^{-34} \text{ Js} \qquad m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$k_B = 1.38 \times 10^{-23} \text{ JK}^{-1} \qquad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$c = 3 \times 10^{8} \text{ m/s} \qquad a_0 = 5.29 \times 10^{-11} \text{ m.}$$

1. In the state $n = 1$, $\ell = 0$, at what distance from the nucleus, is their maximum probability of locating the electron in the atom? The radial wave function is

$$R_{10}(r) = 2\sqrt{\frac{1}{a_0^3}}\, e^{-r/a_0}$$ **(5 marks)**

Answer 1

The radial probability density

$$p_r(r) = 4\pi r^2 |R_{10}|^2 = 4\pi r^2 \left(\frac{4}{a_0^3}\right) e^{-2r/a_0}$$

Now to find the maximum, we need to take the derivative.

$$\frac{dp_r(r)}{dr} = 4\pi r^2 \left(\frac{4}{a_0^3}\right) e^{-2r/a_0} \left(\frac{-2}{a_0}\right) + \left(\frac{4}{a_0^3}\right) e^{-2r/a_0} 4\pi(2r)$$

When $p_r(r)$ is maximum, $\frac{dp_r(r)}{dr} = 0$

$$4\pi r^2 \left(\frac{4}{a_0^3}\right) e^{-2r/a_0} \left(\frac{-2}{a_0}\right) + \left(\frac{4}{a_0^3}\right) e^{-2r/a_0} 4\pi(2r) = 0$$

$$\Rightarrow \qquad r = a_0.$$

2. Free electrons in a metal behave like a quantum gas. At what minimum temperature, would these electrons behave like a classical gas? The density of electrons is 1 per atom and atoms are roughly 0.3 nm apart. (6 **marks)**

Answer 2

In order to behave like classical gas $a > \lambda$, where a is the separation between atoms. Since

$$\frac{p^2}{2m} = \frac{3}{2}k_BT$$
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mk_BT}}$$

Therefore

$$T > \frac{h^2}{3mk_Ba^2} = \frac{(6.63\times10^{-34})^2}{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times(0.3\times10^{-9})^2}$$
$$T > 1.3\times10^5 \text{ K}.$$

3. In a liquid sample certain nuclei acts like spin$-\frac{1}{2}$ particles. They are modeled by two-state quantum levels (qubits). The energy separation between these levels is 1 μeV at a field of 1 Tesla. At what temperature, would 75% of the lower state and 25% of the upper state will be occupied? **(5 marks)**

   Answer 3

$$\frac{N_1}{N_2} = \exp\left(\frac{\Delta E}{k_BT}\right) = \frac{0.75}{0.25} = 3$$
$$\ln\left(\frac{N_1}{N_2}\right) = \left(\frac{\Delta E}{k_BT}\right)$$
$$T = \left(\frac{\Delta E}{k_B\ln\left(\frac{N_1}{N_2}\right)}\right) = \frac{1\ \mu\text{eV}}{1.38\times10^{-23}\times\ln(3)}$$
$$= \frac{10^{-6}\times1.6\times10^{-19}}{1.38\times10^{-23}\times\ln(3)} \approx 0.01 \text{ K} \approx 10 \text{ mK}.$$

   We need a really low temperature to allow 75% of the atoms go in the ground state.

4. ZnTe laser has a band gap of about 2.2 eV. What wavelength is this laser expected to produce? **(3 marks)**

   Answer 4

$$E_g = hf = \frac{hc}{\lambda}$$
$$\Rightarrow \quad \lambda = \frac{hc}{E_g} = \frac{6.63\times10^{-34}\times3\times10^8}{2.2\times1.6\times10^{-19}} = 565 \text{ nm} \quad \text{(green light)}.$$

5. Diamond is barely transparent at a UV wavelength of 250 nm. What is its approximate band gap?How can we predict transparency or opaqueness based on band gap? (4 **marks)**

Answer 5

$$E_g = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{250 \times 10^{-9}}$$
$$= 7.956 \times 10^{-19} \text{ J} = 5 \text{ eV}.$$

If the wavelength corresponds to an energy greater than the band gap $\left(\frac{hc}{\lambda} > E_g\right)$, the photon will be absorbed exciting electrons from valance to conduction band. Hence the material will be opaque.

On the other hand for long wavelengths $\left(\text{i.e.} \frac{hc}{\lambda} < E_g\right)$, the energy input is not sufficient to cross the band gap. The material will be transparent to such wavelengths.