

## Assignment 11: Quantum Field Theory

**Due Date: 7 May. 4 pm**

1. Starting with

$$G^+(x, y) = \Theta(x - y) \langle x | e^{-i\hat{H}(t_x - t_y)} | y \rangle$$

compute the Green function for a free particle with end points  $\vec{x} = \vec{y} = 0$ . Show that this equals:

$$G^+(x = 0, y = 0) = \Theta(x - y) \left( \frac{-im}{2\pi T} \right)^{1/2}.$$

Note that  $\hat{H} = \hat{p}^2/2m$  and  $T = t_x - t_y$ .

[5 Marks]

2. Using

$$G(x, y) = T \langle 0 | \psi(\hat{x}) \hat{\psi}(y) | 0 \rangle$$

and

$$\hat{\psi}(x) = \int \frac{d^3p}{(2\pi)^{3/2}(2E_{\mathbf{p}})^{1/2}} \sum_s \left( u^s(p) \hat{a}_{s\mathbf{p}} e^{-ip \cdot x} + v^s(p) \hat{b}_{s\mathbf{p}}^\dagger e^{+ip \cdot x} \right)$$

show a complete, neat and systematic derivation of the fermionic free propagator

$$G_0(x, y) = \int \frac{d^4p}{(2\pi)^4} i \frac{\not{p} + m}{p^2 - m^2 + i0^+} e^{-ip \cdot (x-y)}$$

[15 Marks]

3. In the minimal coupling technique, ordinary derivatives are replaced by covariant derivatives. Apply this to a complex massive scalar field and a non-interacting electromagnetic field. The resulting theory is not only gauge invariant but leads to an interaction between the complex scalar field and the electromagnetic field. What is the interaction term?

[5 Marks]

4. (a) Show that the momentum space representation of the fermionic free propagation

$$\tilde{G}(p) = i \frac{\not{p} + m}{p^2 - m^2 + i0^+}$$

can be written as

$$\tilde{G}(p) = \frac{i}{\not{p} - m + i0^+} .$$

[5 Marks]

(b) The free propagator

$$G = \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} - m + i0^+} e^{-ip \cdot (x-y)}$$

is a Green's function of which operator? Substantiate your answer by a calculation. [5 Marks]

5. (a) Find the conserved Noether current for the massless Dirac equation. [5 Marks]

(b) Consider the Global chiral symmetry

$$\psi \mapsto e^{-i\alpha\gamma^5} \psi$$

where  $\gamma^5$  is the chirality operator. Show that the massless Dirac Lagrangian is invariant under the chiral transformation. [5 Marks]

(c) What is the corresponding Noether current? [5 Marks]