# Solution Final Examination: Modern Physics
# Spring 2018

1. Which of the following two qubit states is antisymmetric (odd) under exchange?

   (a) $|0\rangle\,|0\rangle$

   (b) $\dfrac{|0\rangle\,|1\rangle + |1\rangle\,|0\rangle}{\sqrt{2}}$

   (c) $\dfrac{|0\rangle\,|1\rangle - |1\rangle\,|0\rangle}{\sqrt{2}}$

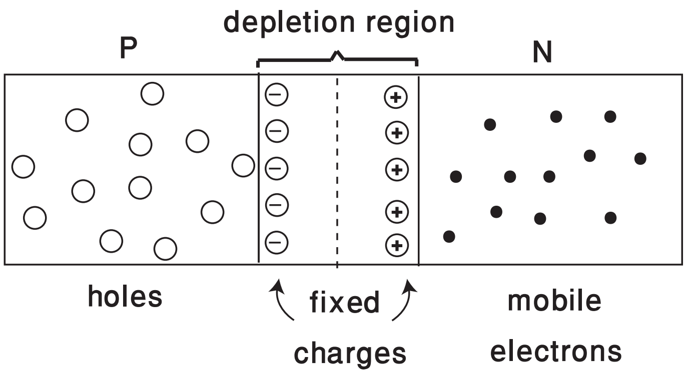
   (d) $|0\rangle\,|1\rangle$

   (e) $\dfrac{|0\rangle\,|1\rangle + |1\rangle\,|1\rangle}{\sqrt{2}}$ **(3 marks)**

   **Answer 1**

   Option (c) is the correct answer.

2. In a PN junction, fixed charges are formed in the two sides of the depletion region as shown below.




   If the diode is reverse-biased, what happens to the fixed charges?

   (a) The charges increase (more negative on the P side and more positive on the N side).

   (b) The charges decrease (less negative on the P side and less positive on the N side).

   (c) They remain unchanged.

   (d) They disappear altogether.

(e) It depends whether the crystal is made of silicon or germanium. **(3 marks)**

## Answer 2

Option (a) is the correct answer.

3. There are as much as $10^{40}$ oxygen molecules in this auditorium. They are evenly spaced. If all of these molecules were to spontaneously congregate in one small corner, all of us would probably suffocate. When is this dangerous configuration of molecules more likely? **(3 marks)**


(a) When there is a higher number of $O_2$ molecules.

(b) When there are fewer $O_2$ molecules.

(c) The likelihood of this congregation of molecules is independent of the number of molecules.

(d) By increasing the pressure of $O_2$ inside the auditorium.

(e) None of the above.

## Answer 3

Option (b) is the correct answer.

4. An electron is known to be in the $n = 3$ state in the hydrogen atom. An experiment is devised to measure the quantum number $m_\ell$ of the electron. What is the probability that the outcome of the measurement is $m_\ell = 0$?

(a) $\frac{1}{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{2}$

(d) 0

(e) $\frac{1}{5}$ **(3 marks)**

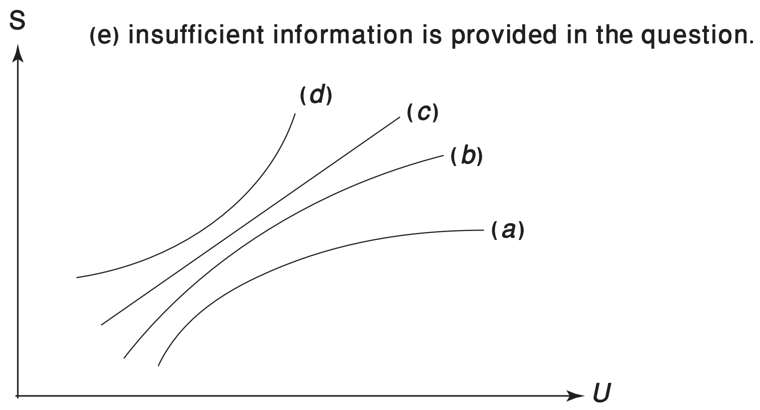
**Answer 4**

Option (a) is the correct answer.

$$n = 3 \quad \text{has} \quad \begin{cases} \ell = 2 & (m_\ell = 2, 1, 0, -1, -2) \\ \ell = 1 & (m_\ell = 1, 0, -1) \\ \ell = 0 & (m_\ell = 0) \end{cases}$$

So desired probability is $\frac{3}{9} = \frac{1}{3}$.

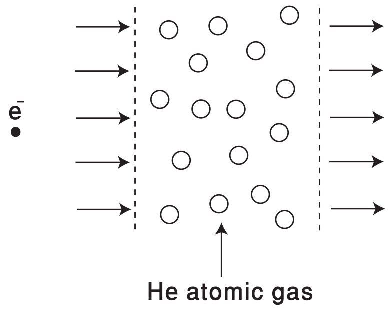
5. The entropy-internal energy curves for five objects are shown. Which of these objects has the smallest "heat-capacity" which is defined as the energy required to raise the temperature by 1 K?**(3 marks)**



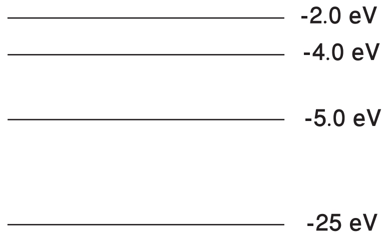

**Answer 5**

Option (a) is the correct answer. In (a), the slope becomes flatter more rapidly as we go along the energy axis. So temperature goes up in (a) more rapidly.

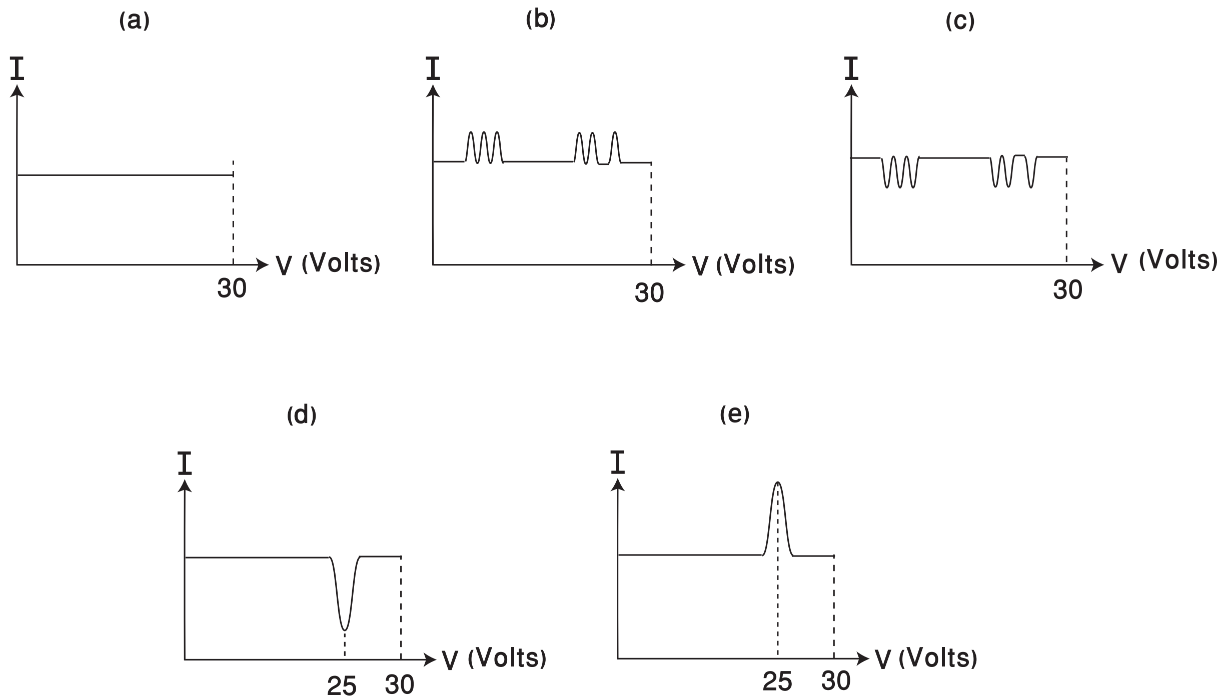
6.




Electrons are accelerated from 0 eV to 30 eV as they pass through an evacuated tube containing He gas. The approximate energy level structure of the He atom is given below:




Which of the following graphs represents the most plausible (likely) variation of electron current ($I$) with accelerating voltage ($V$)? **(3 marks)**



## Answer 6

Option (c) is the correct answer.

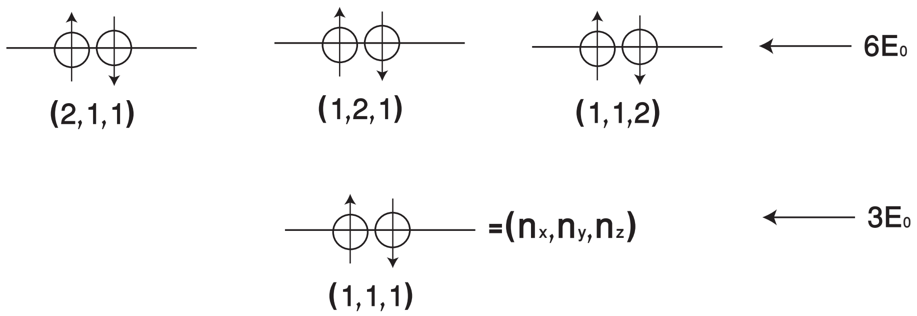
7. Eight indistinguishable electrons (fermions) are placed inside a $3D$ infinite well. The energy of the electron is quantized according to

$$E_{n_x,n_y,n_z} = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2)\cdot$$

The system is in its ground state. What is the *total* energy of the electrons? **(10 marks)**

## Answer 7

Let $E_0 = \frac{\hbar^2\pi^2}{2mL^2}$. Electrons are fermions. They obey the Pauli exclusion principle and hence each energy level can populate only two electrons and that too with opposite spins.




$$\begin{aligned}\text{Total energy} &= 2(3E_0) + 6(6E_0) \\ &= 6E_0 + 36E_0 \\ &= 42E_0.\end{aligned}$$

8. The energy gap for Si at 300 K is 1 eV.

   (a) Photons of energy less than 1 eV will be absorbed by silicon. (True or false? No description please.) **(2 marks)**

   (b) Which wavelengths will be absorbed by the silicon? I want you to mention a minimum or a maximum wavelength. **(3 marks)**

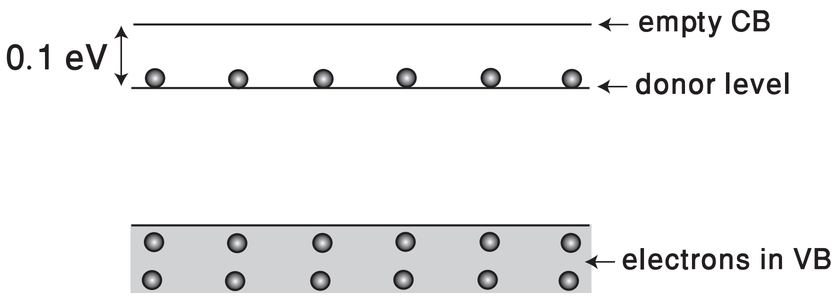
## Answer 8

(a) False. These photons cannot be absorbed as the energy gap is bigger.

(b) Photons with energy greater than 1 eV will be absorbed. Since $E = \frac{hv}{\lambda}$, the maximum wavelength that will be absorbed will be

$$\begin{aligned}\lambda_{\max} = \frac{hc}{\Delta E} &= \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^{8} \text{ m/s}}{1 \times 1.6 \times 10^{-19} \text{ J}} \\ &= 1.24 \times 10^{-6} \text{ m} \\ &= 1.24\ \mu\text{m}.\end{aligned}$$

9. The energy band diagram for an N type semiconductor crystal at 0 K is shown below.

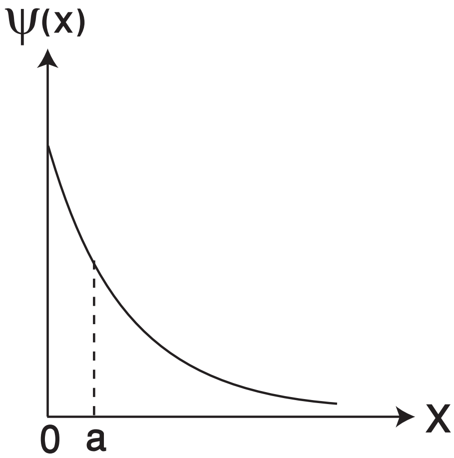



At what minimum temperature would the crystal start to conduct? Box your final answer. I will only consider the final answer. **(3 marks)**

## Answer 9

$$\begin{aligned}k_B T_{\min} &\sim 0.1 \times 1.6 \times 10^{-19} \text{ J} \\ T_{\min} &\sim \frac{0.1 \times 1.6 \times 10^{-19} \text{ J}}{1.38 \times 10^{-23} \text{ JK}^{-1}} \\ &= \boxed{1159 \text{ K.}}\end{aligned}$$

10. A wave function decays in a region of space as $\psi(x) = Ae^{-x/a}$ where $A$ and $a$ are real constants.



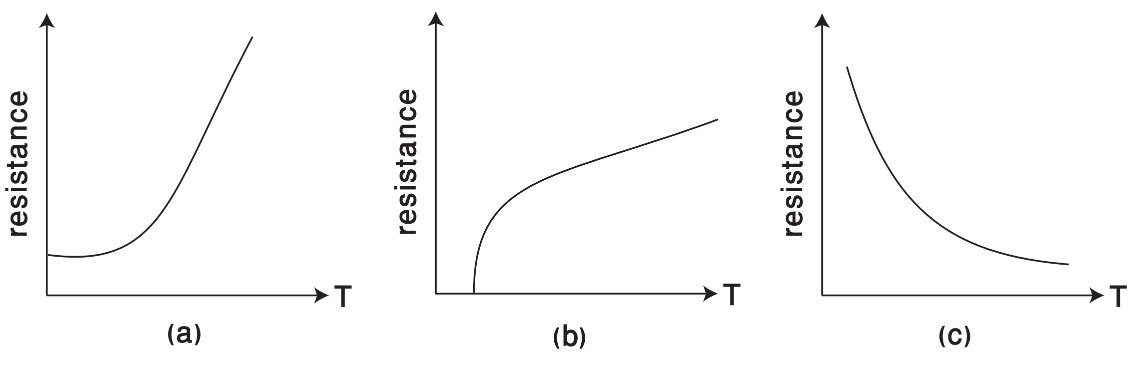

At what length ($x = x_0$) will the probability density decay to one-half $\left(\frac{1}{2}\right)$ the probability density at $x = 0$? **(3 marks)**

**Answer** 10

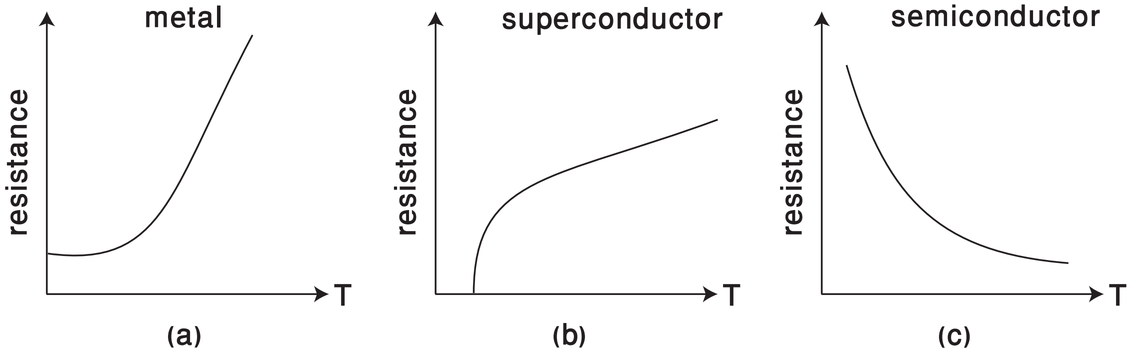
Probability density is

$$
\begin{aligned}
|\psi(x)|^2 &= A^2 e^{-2x/a} \\
\text{At } x = 0, \quad p(0) &= A^2 \\
\text{At } x = x_0, \quad p(x_0) &= A^2 e^{-2x_0/a} = \frac{1}{2}A^2. \\
\text{So} \quad e^{-2x_0/a} &= \frac{1}{2} \\
-\frac{2x_0}{a} &= -\ln 2 \\
\frac{2x_0}{a} &= \ln 2 \\
x_0 &= \frac{a \ln 2}{2} .
\end{aligned}
$$

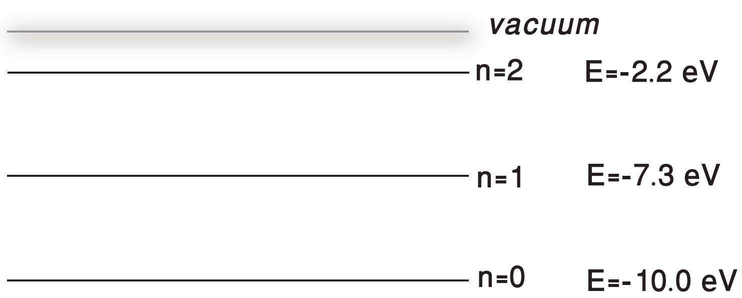
11. Write metal, semiconductor, superconductor against these curves. **(6 marks)**



**Answer** 11

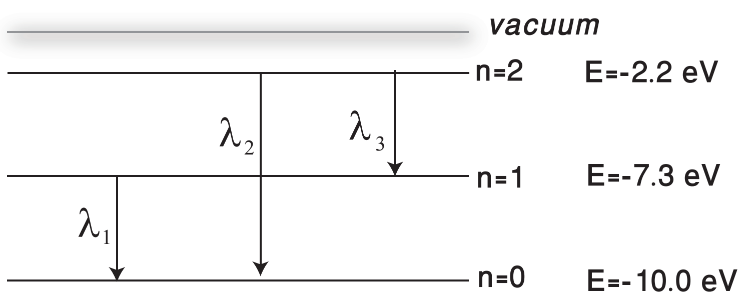



12. The energy level diagram of an artificial atom is shown here.




Sketch the emission spectrum expected from a gas comprising of these artificial atoms. Identify the wavelengths. **(5 marks)**

**Answer** 12



$$
\begin{aligned}
\lambda &= \frac{hc}{\Delta E} \\
\lambda_1 &= \frac{6.63 \times 10^{-34}\ \text{Js} \times 3 \times 10^{8}\ \text{m/s}}{[-7.3 - (-10)] \times 1.6 \times 10^{-19}\ \text{J}} = 460\ \text{nm} \\
\lambda_2 &= \frac{6.63 \times 10^{-34}\ \text{Js} \times 3 \times 10^{8}\ \text{m/s}}{[-2.2 - (-10)] \times 1.6 \times 10^{-19}\ \text{J}} = 159\ \text{nm} \\
\lambda_3 &= \frac{6.63 \times 10^{-34}\ \text{Js} \times 3 \times 10^{8}\ \text{m/s}}{[-2.2 - (-7.3)] \times 1.6 \times 10^{-19}\ \text{J}} = 243\ \text{nm}
\end{aligned}
$$