Tutorial 10: Modern Physics

1. Consider a system of two Einstein solids, A and B, each containing 10 oscillators, sharing a total of 10 units of energy. Assume that the solids are weakly coupled, and that the total energy is fixed.

   (a) How many different macrostates are available to this system?

   (b) How many different microstates are available to this system?

   (c) Assuming that this system is in thermal equilibrium, what is the probability of finding all the energy in solid A?

   (d) What is the probability of finding exactly half of the energy in solid A?

   (e) Under what circumstances would this system exhibit irreversible behavior?

   For convenience, a table listing the macrostates and number of microstates in each macrostate is also given.

$\mathbf{q_A}$	0	1	2	3	4	5	6	7	8	9	10
$\mathbf{q_B}$	10	9	8	7	6	5	4	3	2	1	0
$\mathbf{\Omega_A}$	1	10	55	220	715	2002	5005	11440	24310	48620	92378
$\mathbf{\Omega_B}$	92378	48620	24310	11440	5005	2002	715	220	55	10	1
$\mathbf{\Omega_A \Omega_B}$	92378	486200	1337050	2516800	3578575	4008004	3578578	2516800	1337050	486200	92378

2. Suppose you flip four fair coins.

   (a) How many possible outcomes (microstates) are there?

   (b) What is the probability of getting the sequence HTHHTTTHTHHHTHHHHTHT (in exactly that order)?

   (c) What is the probability of getting 12 heads and 8 tails (in any order)?

3. (a) A paramagnet has all its dipoles aligned parallel to a magnet field. What is its entropy?

   (b) What is the energy of the system in this state? N is the total number of dipoles.

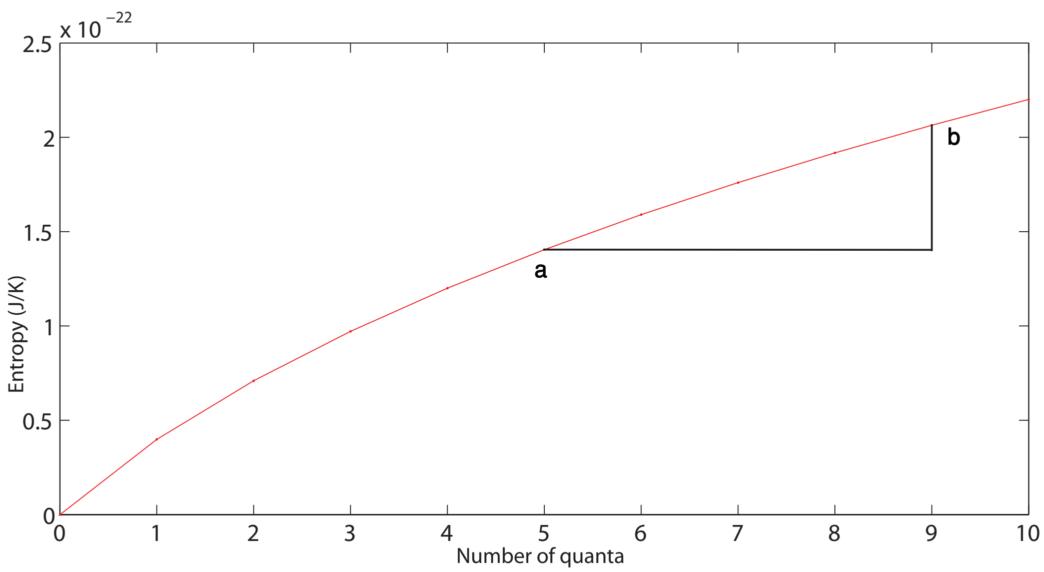
   (c) If $N = 10^{23}$, how many microstates are accessible to the system?

   (d) If $N = 10^{23}$, and with the huge number of microstates accessible, can the energy still be zero?

   (e) Suppose that the microstates of the system changes a billion times per second.

How many microstates will it explore in ten billion years (the age of the universe)? Suppose 1yr $\sim 10^7$ s.

4. A nanoparticle containing 6 atoms can be modeled approximately as an Einstein solid of 18 independent oscillators. The evenly spaced energy levels of each oscillator are 4×10^{-21} J apart.
(a) When the nanoparticle's energy is in the range $5 \times 4 \times 10^{-21}$ J to $6 \times 4 \times 10^{-21}$ J, what is the approximate temperature? (In order to keep precision for calculating the heat capacity, give the result to the nearest tenth of a Kelvin.
(b) When the nanoparticle's energy is in the range $8 \times 4 \times 10^{-21}$ J to $9 \times 4 \times 10^{-21}$ J, what is the approximate temperature? (In order to keep precision for calculating the heat capacity, give the result to the nearest tenth of a degree.)
(c) When the nanoparticle's energy is in the range $5 \times 4 \times 10^{-21}$ J to $9 \times 4 \times 10^{-21}$ J, what is the approximate heat capacity per atom? For your convenience, the entropy-energy graph is also shown.



5. For a certain metal the stiffness of the interatomic bond and the mass of one atom are such that the spacing of the quantum oscillator energy levels is 1.5×10^{-23} J. A nanoparticle of this metal consisting of 10 atoms has a total thermal energy of 18×10^{-23} J.
Assume all the internal energy is of the disordered kind. (a) What is the entropy of

this nanoparticle?

(b) The temperature of the nanoparticle is 87 K. Next we add 18×10^{-23} J to the nanoparticle. By how much does the entropy increase?

6. A 50 gram block of copper (one mole has a mass of 63.5 grams) at a temperature of 35° C is put in contact with a 100 gram block of aluminum (molar mass 27 grams) at a temperature of 20° C. The blocks are inside an insulated enclosure, with little contact with the walls. At these temperatures, the high temperature limit is valid for the specific heat capacity, $C_v = 3k_B$. Calculate the final temperature of the two blocks. Do NOT look up the specific heat capacities of aluminum and copper; these have been provided to you.