Surface Tension

Suppose that you suspend a bent piece of clean wire from a sensitive spiral spring (Figure 13.25), lower the wire into water, and then raise it. As you attempt to free the wire from the water surface, you see from the stretched spring that the water surface exerts an appreciable force on the wire. The water surface resists being stretched, for it has a tendency to contract. You can also see this when a fine-haired paintbrush is wet. When the brush is under water, the hairs are fluffed pretty much as they are when the brush is dry, but when the brush is lifted out, the surface film of water contracts and pulls the hairs together (Figure 13.26). This contractive tendency of the surface of liquids is called **surface tension.**

Surface tension accounts for the spherical shape of liquid drops. Raindrops, drops of oil, and falling drops of molten metal are all spherical because their surfaces tend to contract and force each drop into the shape having the least surface area. This is a sphere, the geometrical figure that has the least surface area for a given volume. For this reason, the mist and dewdrops on spider webs or on the downy leaves of plants are nearly spherical blobs. (The larger they are, the more that gravity flattens them.)

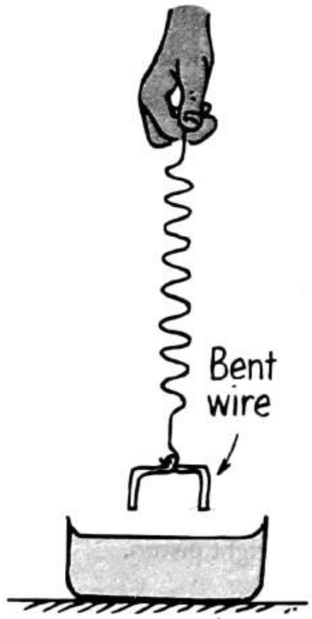


FIGURE 13.25
When the bent wire is lowered into the water and then raised, the spring will stretch because of surface tension.

CHECK YOUR ANSWERS

1. The car moves up a greater distance than the oil level drops, since the area of the piston is smaller than the surface area of the oil in the reservoir.
2. No, no, no! Although a hydraulic device, like a mechanical lever, can multiply *force*, it always does so at the expense of distance. Energy is the product of force and distance. If you increase one, you decrease the other. No device has ever been found that can multiply energy!

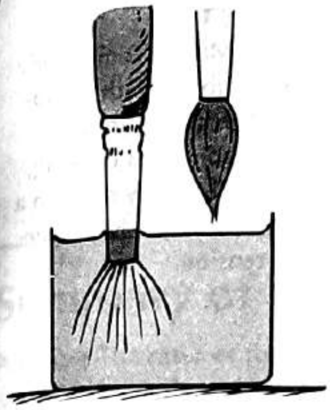


FIGURE 13.26
When the brush is taken out of the water, the hairs are held together by surface tension.

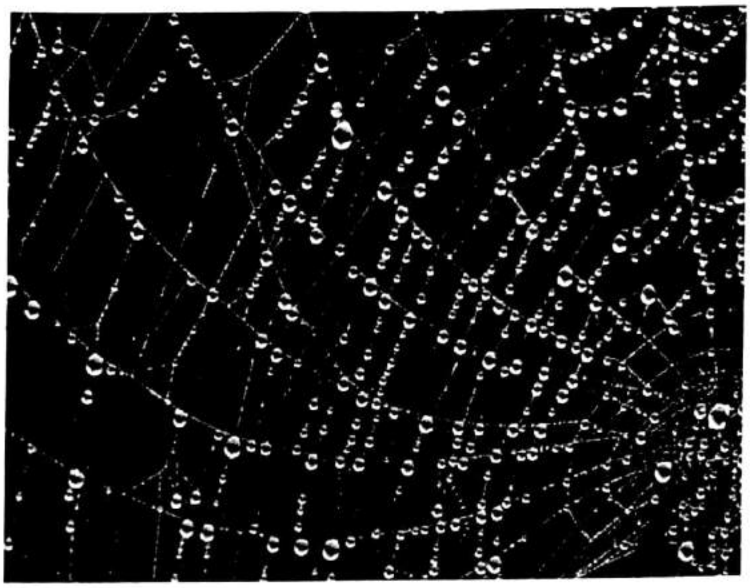


FIGURE 13.27
Small blobs of water are drawn by surface tension into spherelike shapes.

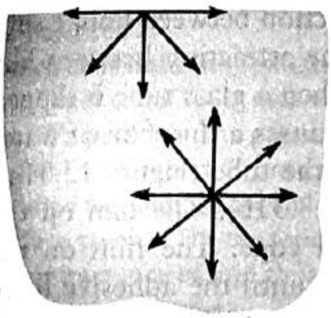


FIGURE 13.28
A molecule at the surface is pulled only sideways and downward by neighboring molecules. A molecule beneath the surface is pulled equally in all directions.

Surface tension is caused by molecular attractions. Beneath the surface, each molecule is attracted in every direction by neighboring molecules, resulting in no tendency to be pulled in any specific direction. A molecule on the surface of a liquid, however, is pulled only by neighbors on each side and downward from below; there is no pull upward (Figure 13.28). These molecular attractions thus tend to pull the molecule from the surface into the liquid, and this tendency minimizes the surface area. The surface behaves as if it were tightened into an elastic film. This is evident when dry steel needles or razor blades seem to float on water. They don't float in the usual sense, but they are supported by the surface molecules opposing an increase in surface area. The water surface sags like a piece of plastic wrap, which allows certain insects, such as water striders, to run across the surface of a pond.

The surface tension of water is greater than that of other common liquids, and pure water has a stronger surface tension than soapy water. We can see this when a little soap film on the surface of water is effectively pulled out over the entire surface. This minimizes the surface area of the water. The same thing happens for oil or grease floating on water. Oil has less surface tension than cold water, and it is drawn out into a film covering the whole surface. But hot water has less surface tension than cold water because the faster-moving molecules are not bonded as tightly. This allows the grease or oil in hot soups to float in little bubbles on the surface of the soup. When the soup cools and the surface tension of the water increases, the grease or oil is dragged out over the surface of the soup. The soup becomes "greasy." Hot soup tastes different from cold soup primarily because the surface tension of water in the soup changes with temperature.

Capillarity

When the end of a thoroughly clean glass tube with a small inside diameter is dipped into water, the water wets the inside of the tube and rises in it. In a tube with a bore of about $\frac{1}{2}$ millimeter in diameter, for example, the water rises

FIGURE 13.29
Bubble Master Tom Noddy blows bubbles within bubbles. The large bubble is elongated due to blowing, but it will quickly settle to a spherical shape due to surface tension.

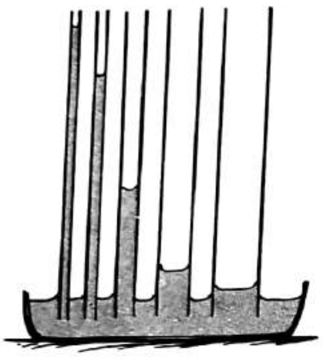


FIGURE 13.30
Capillary tubes.

slightly higher than 5 centimeters. With a still smaller bore, the water rises much higher (Figure 13.30). This rise of a liquid in a fine, hollow tube or in a narrow space is **capillarity.**

When thinking of capillarity, think of molecules as sticky balls. Water molecules stick to glass more than to each other. The attraction between unlike substances such as water and glass is called *adhesion*. The attraction between like substances, molecular stickiness, is called *cohesion*. When a glass tube is dipped into water, the adhesion between the glass and water causes a thin film of water to be drawn up over the inner and outer surfaces of the tube (Figure 13.31a). Surface tension causes this film to contract (Figure 13.31b). The film on the outer surface contracts enough to make a rounded edge. The film on the inner surface contracts more and raises water with it until the adhesive force is balanced by the weight of the water lifted (Figure 13.31c). In a narrower tube, the weight of the water in the tube is small and the water is lifted higher than it would be if the tube were wider.

If a paintbrush is dipped part way into water, the water will rise up into the narrow spaces between the bristles by capillary action. If your hair is long, let it hang into the sink or bathtub, and water will seep up to your scalp in the same way. This is how oil soaks upward in a lamp wick and water soaks into a bath towel when one end hangs in water. Dip one end of a lump of sugar in coffee, and the entire lump is quickly wet. Capillary action is essential for plant growth. It brings water to the roots of plants and carries sap and nourishment to high branches of trees. Just about everywhere we look, we can see capillary action at work. That's nice.

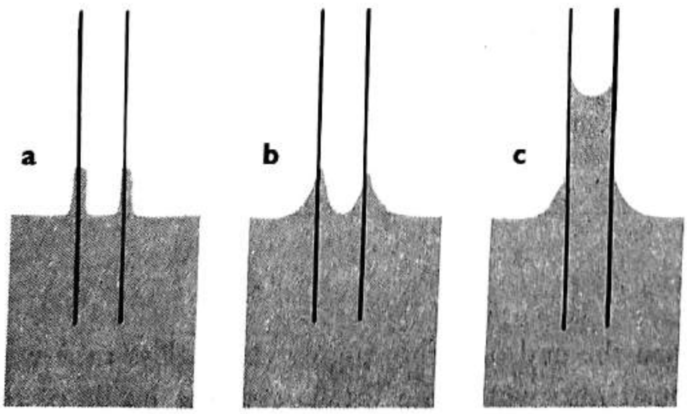


FIGURE 13.31
Hypothetical stages of capillary action, as seen in a cross-sectional view of a capillary tube.

But from the point of view of an insect, capillarity is not so nice. Recall from the previous chapter that, because of an insect's relatively large surface area, it falls slowly in air. Gravity poses almost no risk at all—but not so with capillarity. Being in the grip of water may be fatal to an insect—unless it is equipped for water like a water strider.

Summary of Terms

Pressure The ratio of force to the area over which that force is distributed:

$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{Liquid pressure} = \text{weight density} \times \text{depth}$$

Buoyant force The net upward force that a fluid exerts on an immersed object.

Archimedes' principle An immersed body is buoyed up by a force equal to the weight of the fluid it displaces.

Principle of flotation A floating object displaces a weight of fluid equal to its own weight.

Pascal's principle The pressure applied to a motionless fluid confined in a container is transmitted undiminished throughout the fluid.

Surface tension The tendency of the surface of a liquid to contract in area and thus to behave like a stretched elastic membrane.

Capillarity The rise of a liquid in a fine, hollow tube or in a narrow space.

Suggested Reading

Rogers, E. *Physics for the Inquiring Mind.* Princeton, N.J.: Princeton University Press, 1960. Chapter 6 of this textbook, an oldie but goodie (and a wonderful influence on this author's writing), treats surface tension in fascinating detail.

Review Questions

1. Give two examples of a fluid.

Pressure

2. Distinguish between *force* and *pressure*.

Pressure in a Liquid

3. What is the relationship between liquid pressure and the depth of a liquid? Between liquid pressure and density?
4. If you swim beneath the surface in salt water, will the pressure be greater than in freshwater at the same depth? Why or why not?
5. How does water pressure one meter below the surface of a small pond compare with water pressure one meter below the surface of a huge lake?
6. If you punch a hole in a container filled with water, in what direction does the water initially flow outward from the container?

Buoyancy

7. Why does buoyant force act upward on an object submerged in water?
8. Why is there no horizontal buoyant force on a submerged object?
9. How does the volume of a completely submerged object compare with the volume of water displaced?

Archimedes' Principle

10. How does the buoyant force on a submerged object compare with the weight of water displaced?
11. Distinguish between a *submerged* body and an *immersed* body.
12. What is the mass of 1 L of water? What is its weight in newtons?
13. If a 1-L container is immersed halfway into water, what is the volume of water displaced? What is the buoyant force on the container?

What Makes an Object Sink or Float?

14. Is the buoyant force on a submerged object equal to the weight of the object itself or equal to the weight of the fluid displaced by the object?
15. There is a condition in which the buoyant force on an object does equal the weight of the object. What is this condition?
16. Does the buoyant force on a submerged object depend on the volume of the object or the weight of the object?
17. Fill in the blanks: An object denser than water will ______ in water. An object less dense than water will ______ in water. An object with the same density of water will ____________ in water.
18. How is the density of a fish controlled? How is the density of a submarine controlled?

8.7 Surface Tension

In discussing the liquid phase, the effect of the boundary surface on the properties of the liquid has been neglected. In the absence of a gravitational field, a liquid droplet will assume a spherical shape, because in this geometry the maximum number of molecules is surrounded by neighboring molecules. Because the interaction between molecules in a liquid is attractive, minimizing the surface-to-volume ratio minimizes the energy. How does the energy of the droplet depend on its surface area? Starting with the equilibrium spherical shape, assume that the droplet is distorted to create more area while keeping the volume constant. The work associated with the creation of additional surface area at constant V and T is

$$dA = \gamma d\sigma \tag{8.23}$$

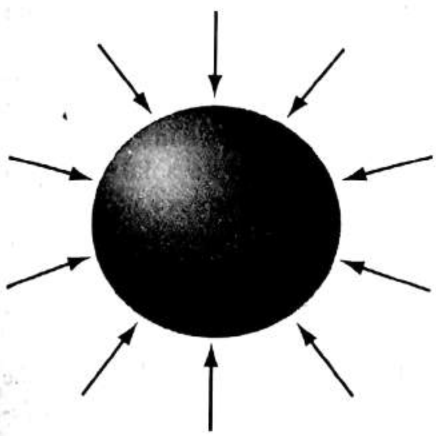


FIGURE 8.15
The forces acting on a spherical droplet that arise from surface tension.

where A is the Helmholtz energy, γ is the surface tension, and σ is the unit element of area. The **surface tension** has the units of energy/area or J m^{-2}, which is equivalent to N m^{-1} (Newtons per meter). Because $dA < 0$ for a spontaneous process at constant V and T, Equation (8.23) predicts that a liquid, or a bubble, or a liquid film suspended in a wire frame will tend to minimize its surface area.

Consider the spherical droplet depicted in Figure 8.15. There must be a force acting on the droplet in the radially inward direction for the liquid to assume a spherical shape. An expression for the force can be generated as follows. If the radius of the droplet is increased from r to $r + dr$, the area increases by

$$d\sigma = 4\pi(r+dr)^2 - 4\pi r^2 = 4\pi(r^2 + 2rdr + (dr)^2) - 4\pi r^2 \approx 8\pi r dr \qquad (8.24)$$

From Equation (8.23), the work done in the expansion of the droplet is $8\pi\gamma r dr$. The force, which is normal to the surface of the droplet, is the work divided by the distance or

$$F = 8\pi\gamma r \qquad (8.25)$$

The net effect of this force is to generate a pressure differential across the droplet surface. At equilibrium, there is a balance between the inward and outward acting forces. The inward acting force is the sum of the force exerted by the external pressure and the force arising from the surface tension, whereas the outward acting force arises solely from the pressure in the liquid:

$$4\pi r^2 P_{outer} + 8\pi\gamma r = 4\pi r^2 P_{inner} \quad \text{or} \qquad (8.26)$$
$$P_{inner} = P_{outer} + \frac{2\gamma}{r}$$

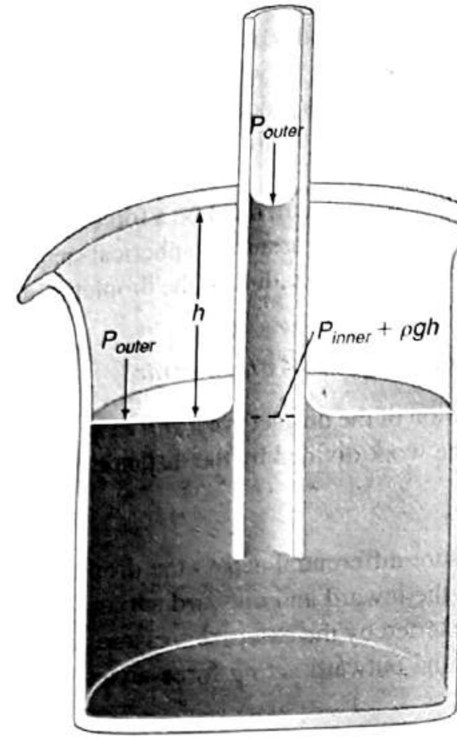
Note that $P_{inner} - P_{outer} \rightarrow 0$ as $r \rightarrow \infty$. Therefore, the pressure differential exists only for a curved surface. From the geometry in Figure 8.15, it is apparent that the higher pressure is always on the concave side of the interface. Values for the surface tension for a number of liquids are listed in Table 8.5.

One effect of this pressure differential is that the vapor pressure of a droplet depends on its radius. By substituting numbers in Equation (8.26) and using Equation (8.22) to calculate the vapor pressure, we find that the vapor pressure of a 10^{-7} m water droplet is increased by 1%, that of a 10^{-8} m droplet is increased by 11%, and that of a 10^{-9} m droplet is increased by 270%. [At such a small diameter, the application of Equation (8.26) is questionable because the size of an individual water molecule is comparable to the droplet diameter. Therefore, a microscopic theory is needed to describe the forces within the droplet.] This effect plays a role in the formation of liquid droplets in a condensing gas such as fog. Small droplets evaporate more rapidly than large droplets, and the vapor condenses on the larger droplets, allowing them to grow at the expense of small droplets.

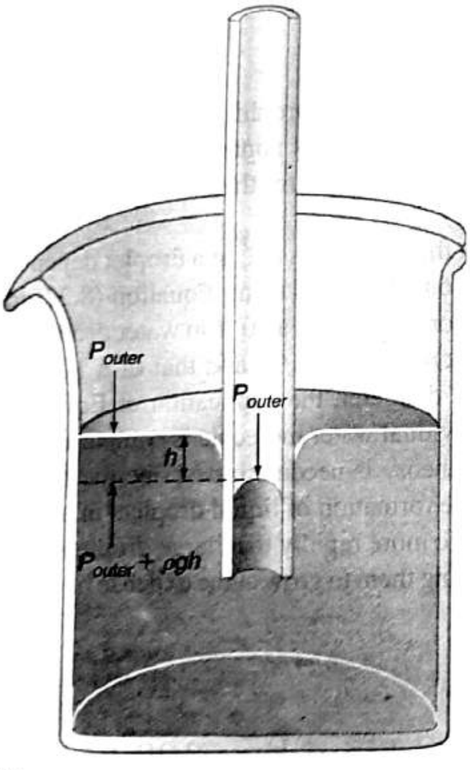
TABLE 8.5
Surface Tension of Selected Liquids at 298 K

Formula	Name	γ (mN m^{-1})	Formula	Name	(mN m^{-1})
Br_2	Bromine	40.95	CS_2	Carbon disulfide	31.58
H_2O	Water	71.99	C_2H_5OH	Ethanol	21.97
Hg	Mercury	485.5	C_6H_5N	Pyridine	36.56
CCl_4	Carbon tetrachloride	26.43	C_6H_6	Benzene	28.22
CH_3OH	Methanol	22.07	C_8H_{18}	Octane	21.14

Source: Data from Lide, D. R., Ed., *Handbook of Chemistry and Physics,* 83rd ed. CRC Press, Boca Raton, FL, 2002.



(a)



(b)

FIGURE 8.16
(a) If the liquid wets the interior wall of the capillary, a capillary rise is observed. The combination Pyrex–water exhibits this behavior. **(b)** If the liquid does not wet the capillary surface, a capillary depression is observed. The combination Pyrex–mercury exhibits this behavior.

Capillary rise and **capillary depression** are other consequences of the pressure differential across a curved surface. Assume that a capillary of radius r is partially immersed in a liquid. When the liquid comes in contact with a solid surface, there is a natural tendency to minimize the energy of the system. If the surface tension of the liquid is lower than that of the solid, the liquid will wet the surface, as shown in Figure 8.16a. However, if the surface tension of the liquid is higher than that of the solid, the liquid will avoid the surface, as shown in Figure 8.16b. In either case, there is a pressure differential in the capillary across the gas–liquid interface, because the interface is curved. If we assume that the liquid–gas interface is tangent to the interior wall of the capillary at the solid–liquid interface, the radius of curvature of the interface is equal to the capillary radius.

The difference in the pressure across the curved interface, $2\gamma/r$, is balanced by the weight of the column in the gravitational field, ρgh. Therefore, the capillary rise or depression is given by

$$h = \frac{2\gamma}{\rho g r} \tag{8.27}$$

In the preceding discussion, it was assumed that either (1) the liquid completely wets the interior surface of the capillary, in which case the liquid coats the capillary walls, but does not fill the core, or (2) the liquid is completely nonwetting, in which case the liquid does not coat the capillary walls, but fills the core. In a more realistic model, the interaction is intermediate between these two extremes. In this case, the liquid–surface is characterized by the **contact angle** θ, as shown in Figure 8.17.

Complete **wetting** corresponds to $\theta = 0°$ and complete **nonwetting** corresponds to $\theta = 180°$. For intermediate cases,

$$P_{inner} = P_{outer} + \frac{2\gamma}{r\cos\theta} \text{ and } h = \frac{2\gamma}{\rho g r \cos\theta} \tag{8.28}$$

The measurement of the contact angle is one of the main experimental methods used to measure the difference in surface tension at the solid–liquid interface.

EXAMPLE PROBLEM 8.3

The six-legged water strider supports itself on the surface of a pond on four of its legs. Each of these legs causes a depression to be formed in the pond surface. Assume that each depression can be approximated as a hemisphere of radius 1.2×10^{-4} m and that θ (as in Figure 8.17) is 0°. Calculate the force that one of the insect's legs exerts on the pond.

Solution

$$\Delta P = \frac{2\gamma}{r\cos\theta} = \frac{2 \times 71.99 \times 10^{-3}\ \mathrm{N\ m^{-1}}}{1.2 \times 10^{-4}\ \mathrm{m} \times 1} = 1.20 \times 10^{3}\ \mathrm{Pa}$$

$$F = PA = P \times \pi r^2 = 1.20 \times 10^{3}\ \mathrm{Pa} \times \pi (1.2 \times 10^{-4}\ \mathrm{m})^2 = 5.4 \times 10^{-5}\ \mathrm{N}$$

EXAMPLE PROBLEM 8.4

Water is transported upward in trees through channels in the trunk called xylem. Although the diameter of the xylem channels varies from species to species, a typical value is 2.0×10^{-5} m. Is capillary rise sufficient to transport water to the top of a redwood tree that is 100 m high? Assume complete wetting of the xylem channels.

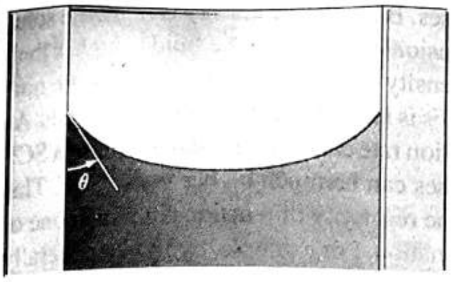


FIGURE 8.17
For cases intermediate between wetting and nonwetting, the contact angle θ lies in the range $0^\circ < \theta < 180^\circ$.

Solution
From Equation (8.28),

$$h = \frac{2\gamma}{\rho g r \cos\theta} = \frac{2 \times 71.99 \times 10^{-3}\ \mathrm{N\,m^{-1}}}{997\ \mathrm{kg\,m^{-3}} \times 9.81\ \mathrm{m\,s^{-2}} \times 2.0 \times 10^{-5}\ \mathrm{m} \times 1} = 0.74\ \mathrm{m}$$

No, capillary rise is not sufficient to account for water supply to the top of a redwood tree.

As Example Problem 8.4 shows, capillary rise is insufficient to account for water transport to the leaves in all but the smallest plants. The property of water that accounts for water supply to the top of a redwood is its high **tensile strength.** Imagine pulling on a piston and cylinder containing only liquid water to create a negative pressure. How hard can you pull on the water without "breaking" the water column? The answer to this question depends on whether bubbles are nucleated in the liquid. This phenomenon is called cavitation. If cavitation occurs, the bubbles will grow rapidly as the piston is pulled outward, and the bubble pressure is given by Equation (8.28), where $P_{external}$ is the vapor pressure of water. The height of the water column in this case is limited to about 9.7 m. However, bubble nucleation is a kinetic phenomenon initiated at certain sites at the wall surrounding the water, and under the conditions present in xylem tubes, it is largely suppressed. In the absence of bubble nucleation, theoretical calculations predict that the tensile strength of water is sufficient that negative pressure in excess of 1000 atm can be generated. The pressure is negative because the water is under tension rather than compression. Experiments on water inclusions in very small cracks in natural rocks have verified these estimates. However, bubble nucleation occurs at much lower negative pressures in capillaries similar in diameter to xylem tubes. Even in these capillaries, negative pressures of more than 50 atm have been observed.

How does the high tensile strength of water explain the transport of water to the top of a redwood? If one cuts into a tree near its base, the sap oozes rather than spurts out, showing that the pressure in the xylem tubes is ~1 atm at the base of a tree. Imagine the redwood in its infancy as a seedling. Capillary rise is sufficient to fill the xylem tubes to the top of the plant. As the tree grows, the water can be pulled upward because of its high tensile strength. As the height of the tree increases, the pressure at the top becomes increasingly negative. As long as cavitation does not occur, the water column remains intact. As water evaporates from the leaves, it is resupplied from the roots through the pressure gradient in the xylem tubes that arises from the weight of the column. If the tree (and each xylem tube) grows to a height of ~100 m, and $P = 1$ atm at the base, the pressure at the top must be ~ -9 atm, from $\Delta P = \rho g h$. We again encounter a negative pressure because the water is under tension. If water did not have a sufficiently high tensile strength, gas bubbles would form in the xylem tubes. This would disrupt the flow of sap, and tall trees could not exist.