13/12/2023 3.7:

9:00am Polarization Peanuts with Fourier Analysis

In many optical experiments, it is extremely important to accurately determine the polarization state of light. The complete polarization state of light can be described in terms of the Stokes parameters, which are related to the polarization ellipse.

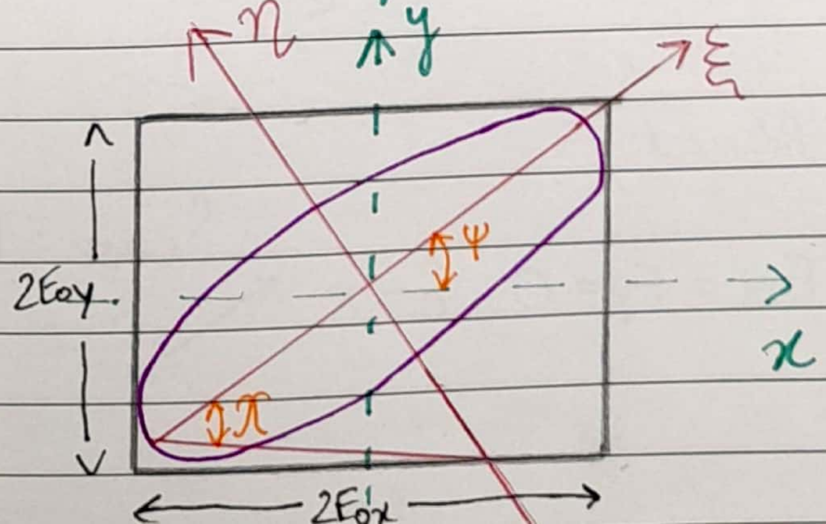
(1) Theoretical Background:

According to Fresnel's theory, $E_x(z,t)$ and $E_y(z,t)$ describe sinusoidal oscillations in the x-z and y-z planes. This gives rise to the equation of the polarization ellipse:

$$\frac{E_x(z,t)^2}{E_{0x}^2} + \frac{E_y(z,t)^2}{E_{0y}^2} - \frac{2E_x(z,t)E_y(z,t)}{E_{0x}E_{0y}}\cos\delta = \sin^2\delta$$

where $\delta = \delta_x - \delta_y$.

Lets look at a sketch of the ellipse



Because the amplitudes E_{0x} and E_{0y}, and the phase δ are constant, the polarization ellipse remains fixed as the polarized beam propagates.

There are a few combinations of parameters that have special features. These are called the degenerate polarization states.

i) Linearly Horizontally Polarized:

LHP: $E_{0y} = 0$ ↔ $S_{LHP} = I_0 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

ii) Linearly Vertically Polarized:

LVP: $E_{0x} = 0$ ↕ $S_{LVP} = I_0 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

iii) Linear + 45 Polarized:

L+45P: $E_{0x} = E_y = 0$, $\delta = 0$ ⤢ $S_{L+45P} = I_0 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

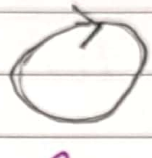
iv) Linear − 45 Polarized:

L−45P: $E_{0x} = E_y = 0$, $\delta = \pi$ ⤡ $S_{L-45P} = I_0 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

v) Right Circularly Polarized:

$$S_{RCP} = I_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

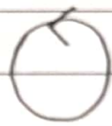
RCP: $E_{0x} = E_{0y} = E_0$, $\delta = \frac{\pi}{2}$



vi) Left Circularly Polarized:

$$S_{LCP} = I_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

LCP: $E_{0x} = E_{0y} = E_0$, $\delta = \frac{-\pi}{2}$



For all other cases, general elliptically polarized light will be observed.

All linear polarizations lie on the equator of the Poincaré Sphere. The RCP lies on the north pole and the LCP lies on the south pole. All other elliptically polarized states are represented elsewhere on the surface of the sphere.

Applying time averaging to the equation of the polarization ellipse yields:

$$S_0^2 = S_1^2 + S_2^2 + S_3^2$$

where $S_0 = E_{0x}^2 + E_{0y}^2$

$$S_1 = E_{0x}^2 - E_{0y}^2$$

$$S_2 = 2E_{0x}E_{0y}\cos\delta$$

$$S_3 = 2E_{0x}E_{y}\sin\delta$$

S_0, S_1, S_2 and S_3 are the Stokes Polarization Parameters. These form the Stokes vector.

The Stokes parameters can be expressed in terms of the orientation ψ and ellipticity χ angles as:

$$S_0 = 1 \quad \text{(normalized to unity)}$$

$$S_1 = S_0\cos(2\chi)\cos(2\psi)$$

$$S_2 = S_0\cos(2\chi)\sin(2\psi)$$

$$S_3 = S_0\sin(2\chi)$$

where $\psi = \frac{1}{2}\tan^{-1}\left(\frac{S_2}{S_1}\right) \qquad 0 \le \psi \le \pi$

$$\chi = \frac{1}{2}\sin^{-1}\left(\frac{S_3}{S_0}\right) \qquad \frac{-\pi}{4} \le \chi \le \frac{\pi}{4}$$

Refer to experiment 3.4 in this notebook for schemes for generating different polarizations at arbitrary angles.

② The Fourier Series for the Intensity profile goes as:

$$I = \frac{1}{2} + \frac{1}{2}\cos(2\theta)\cos(2\theta-2\beta)\cos(2\alpha) + \frac{1}{2}\sin(2\theta)\cos(2\theta-2\beta)(\sin(2\alpha))$$

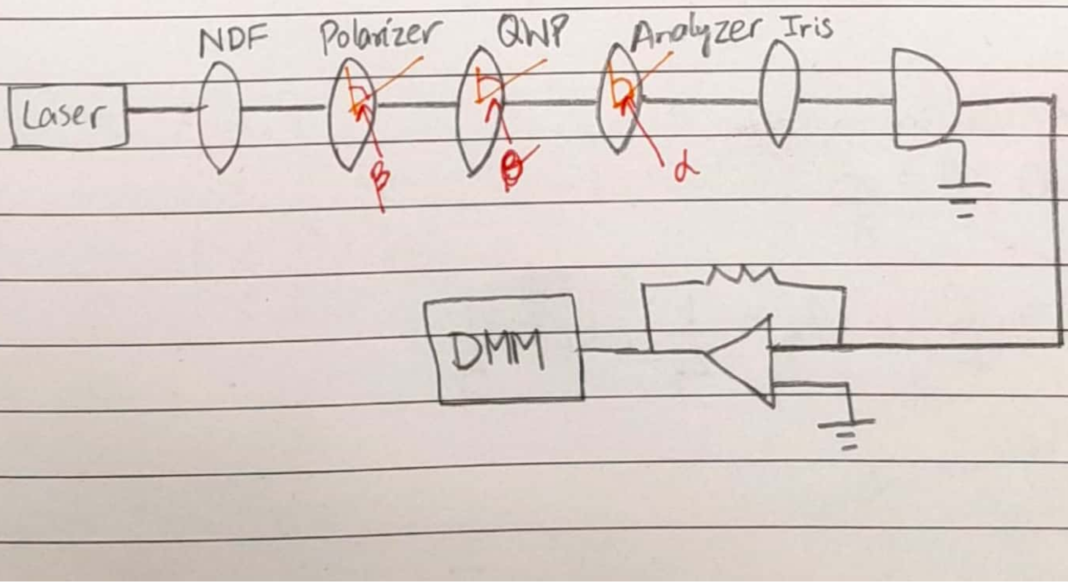
reading off the Fourier coefficients:

$$a_0 = \frac{1}{2}$$

$$a_2 = \frac{1}{2}\cos 2\theta \cos(2\theta-2\beta)$$

$$b_2 = \frac{1}{2}\sin 2\theta \cos(2\theta-2\beta)$$

$\beta \rightarrow$ polarizer angle
$\theta \rightarrow$ QWP angle



(3) Calculating Fourier Coefficients for linear, circ, ellip polarizations

① Horizontal:

$\beta = 0, \theta = 0$

$a_0 = \frac{1}{2}, \quad a_2 = \frac{1}{2}, \quad b_2 = 0$

$S = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

② Vertical:

$\beta = \pi, \theta = 0$

$a_0 = \frac{1}{2}, \quad a_2 = \frac{-1}{2}, \quad b_2 = 0$

$S = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

③ Circular:

$\beta = 0, \theta = \frac{\pi}{4}$

$a_0 = \frac{1}{2}, \quad a_2 = 0, \quad b_2 = 0$

$S = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{bmatrix}$

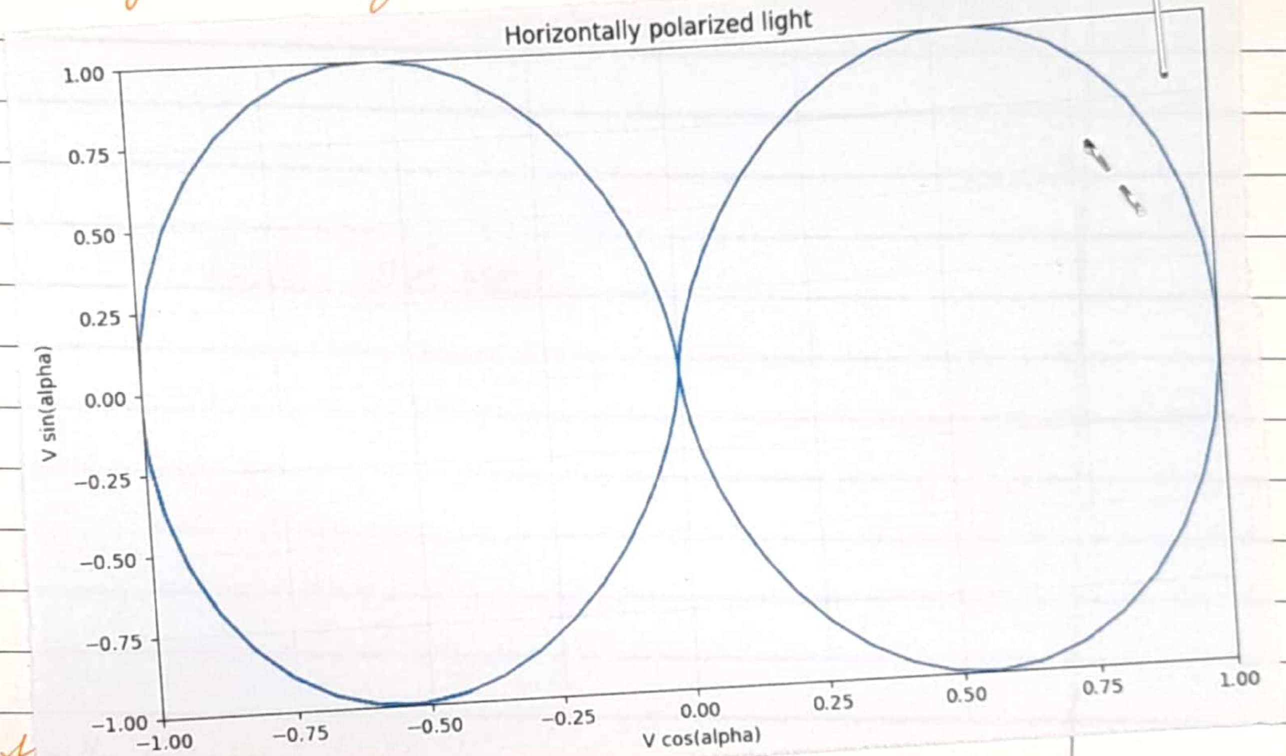
④ Elliptical:

$\beta = 0, \theta = \frac{\pi}{8}$

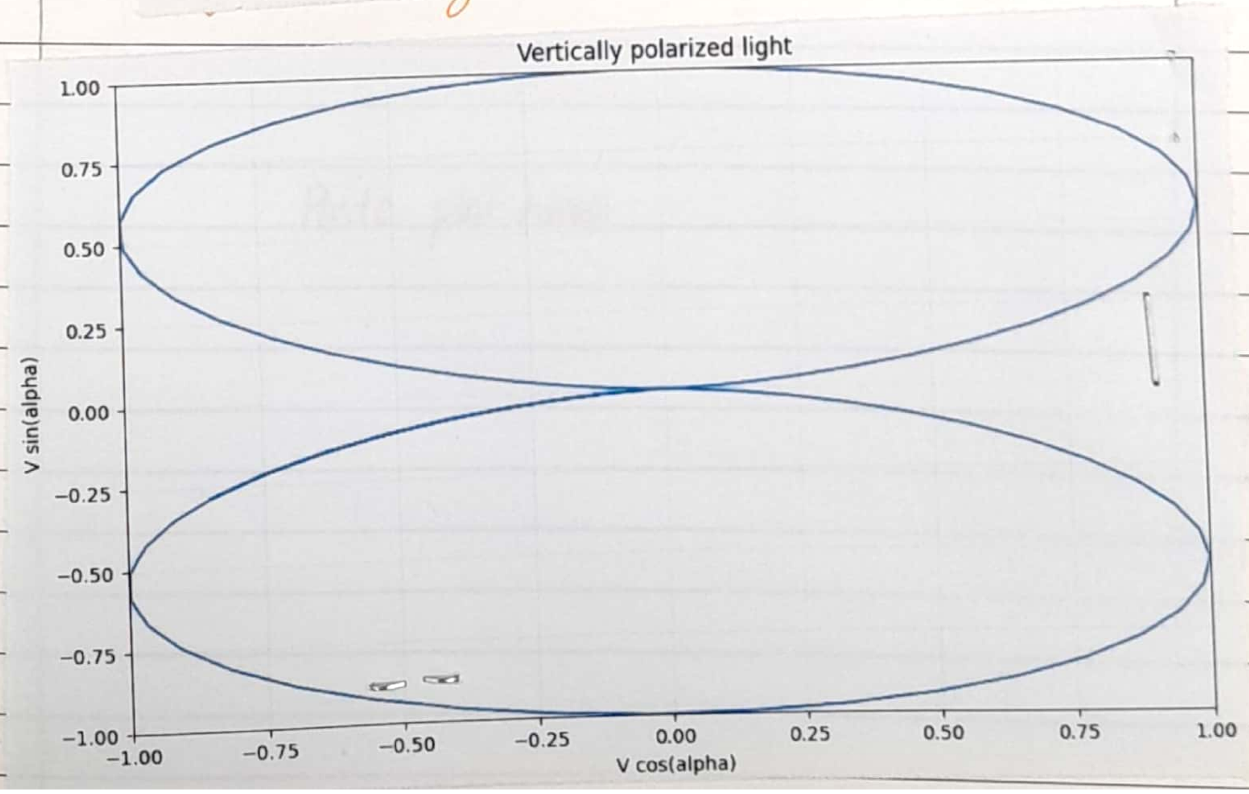
$a_0 = \frac{1}{2}, \quad a_2 = \frac{1}{4}, \quad b_2 = \frac{1}{4}$

~~$S = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$~~

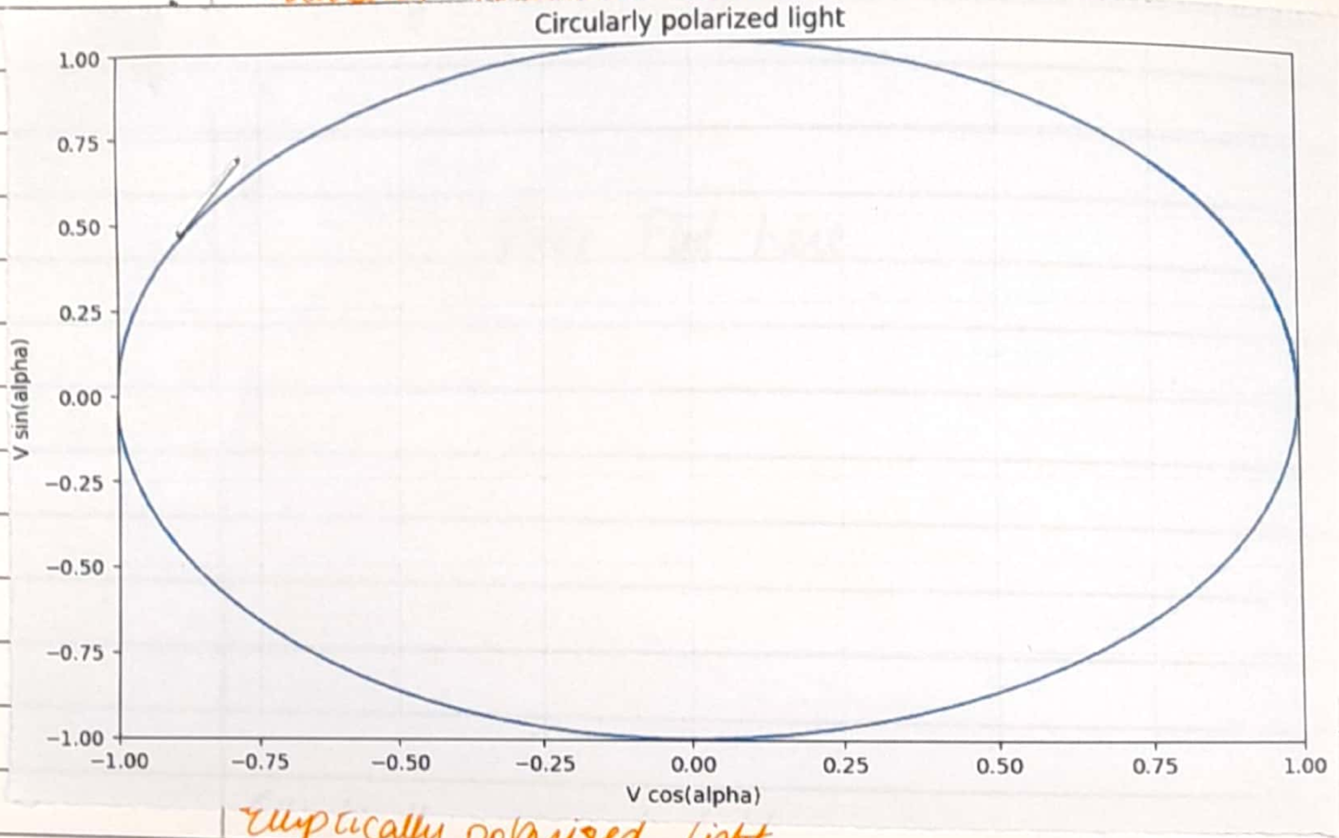
Horizontally Polarized Light



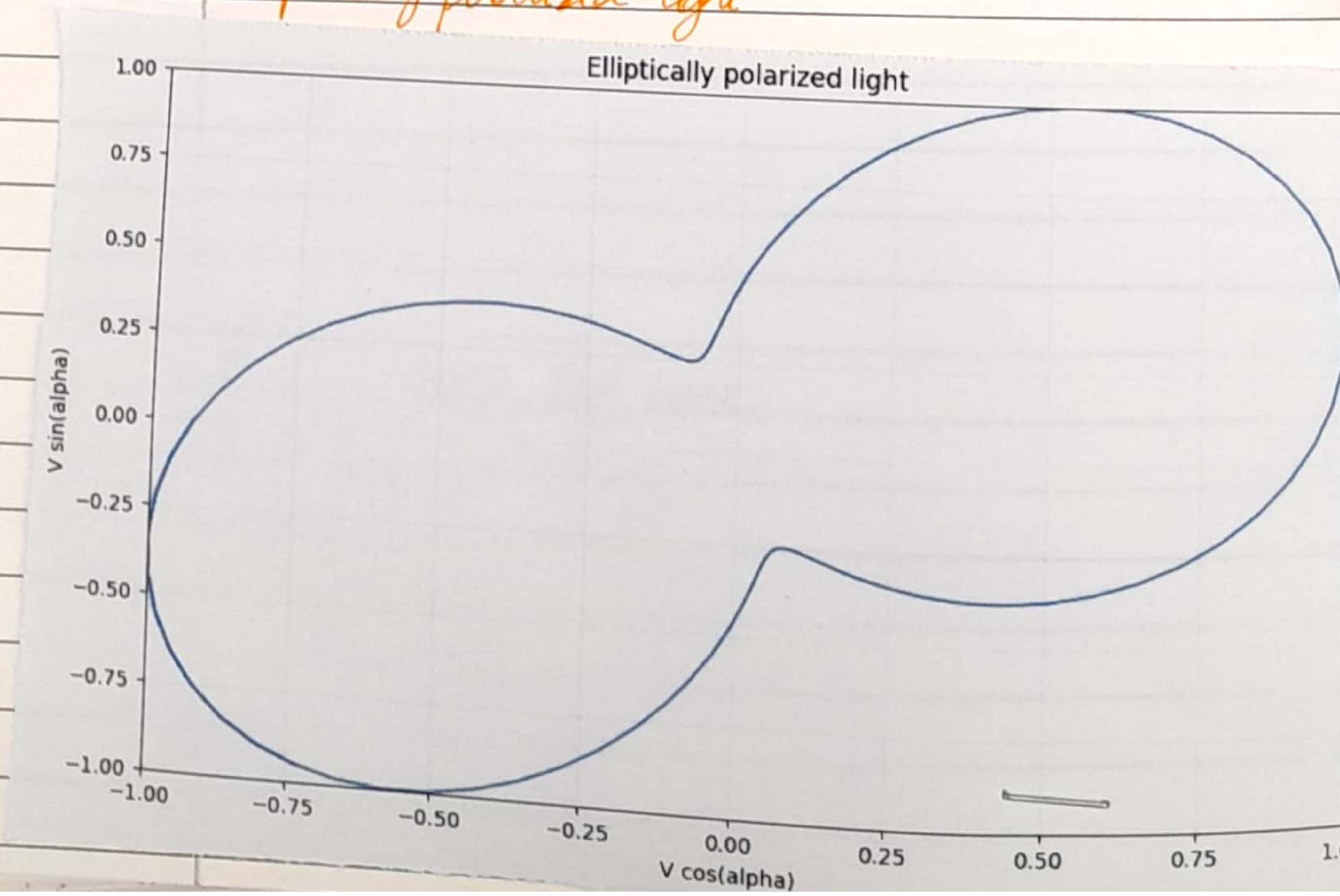
Vert



Circularly Polarized Light



Elliptically polarized light



(4) There is a significant positive correlation between the shape of the plots and the input state polarization falling on the analyzer.

Linear polarization states produce 2 buds with the orientation dependent on hor/vert polarization

Circular polarization states produce a perfectly shaped circular plot.

Elliptical polarization states produce peanut like plots with the angles χ and ψ dictating the handedness/skew of the plot and polarization state.

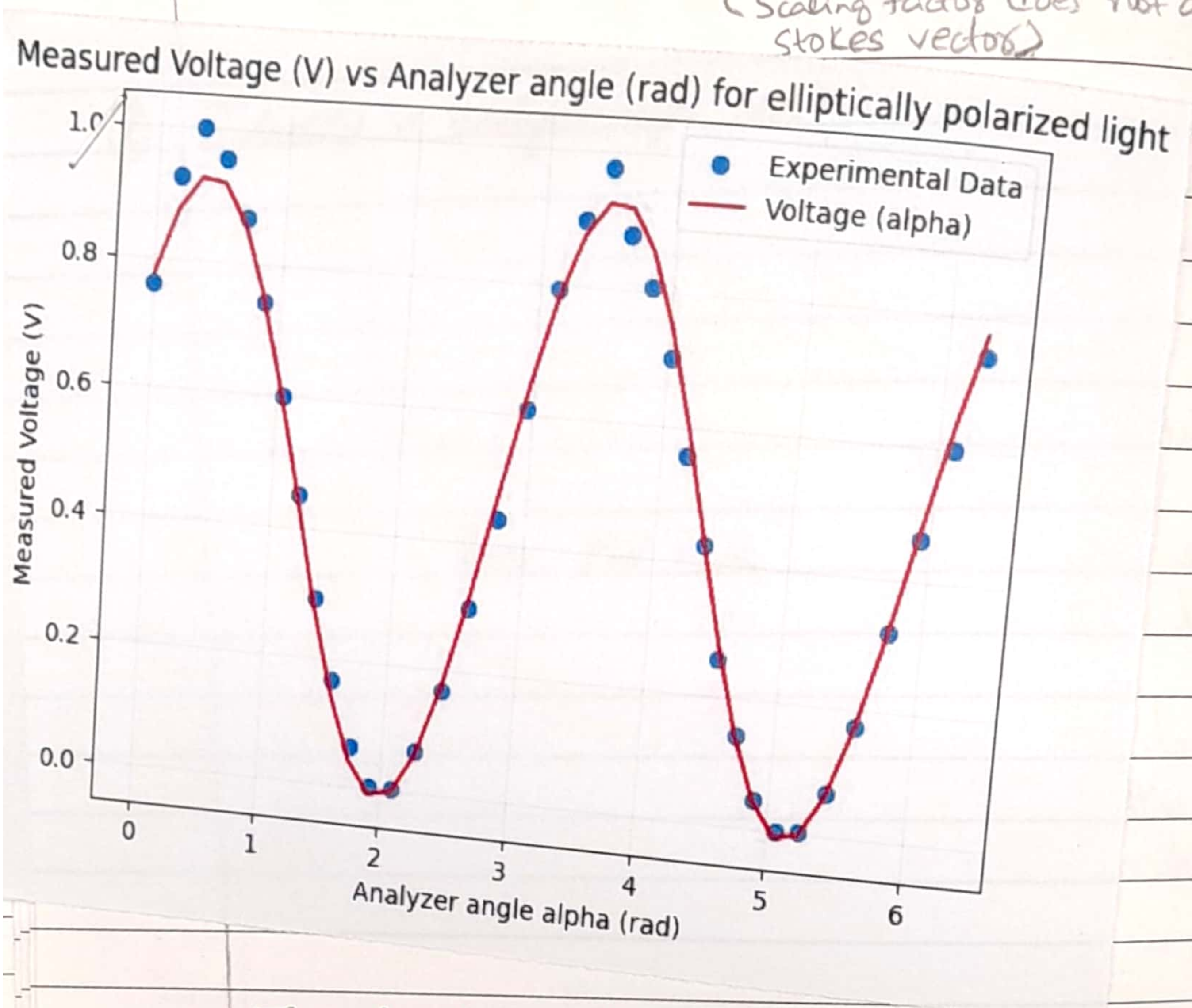
(5) Experimental results:

The experiment was repeated and the angle β was nominally set at 0°. A horizontal polarization was set up and the QWP was set to an arbitrary angle. Intensity readings against multiple configurations were then taken and subsequently plotted.

Plots have been created with Voltage (V) instead of Intensity to minimize uncertainty propagation

Intensity = Voltage * 0.43 * 10^{-5} convert acc to this if you prefer

(Scaling factor does not affect stokes vectors)



The intensity varied sinusoidally as a function of the analyser angle α (°).

This sin + cos(2α) proportionality was expected based on our Jones Calculus manipulation earlier.

Fitted function:

$I(\alpha) = 0.4585 + 0.3161\cos(2\alpha) + 0.3481\sin(2\alpha)$

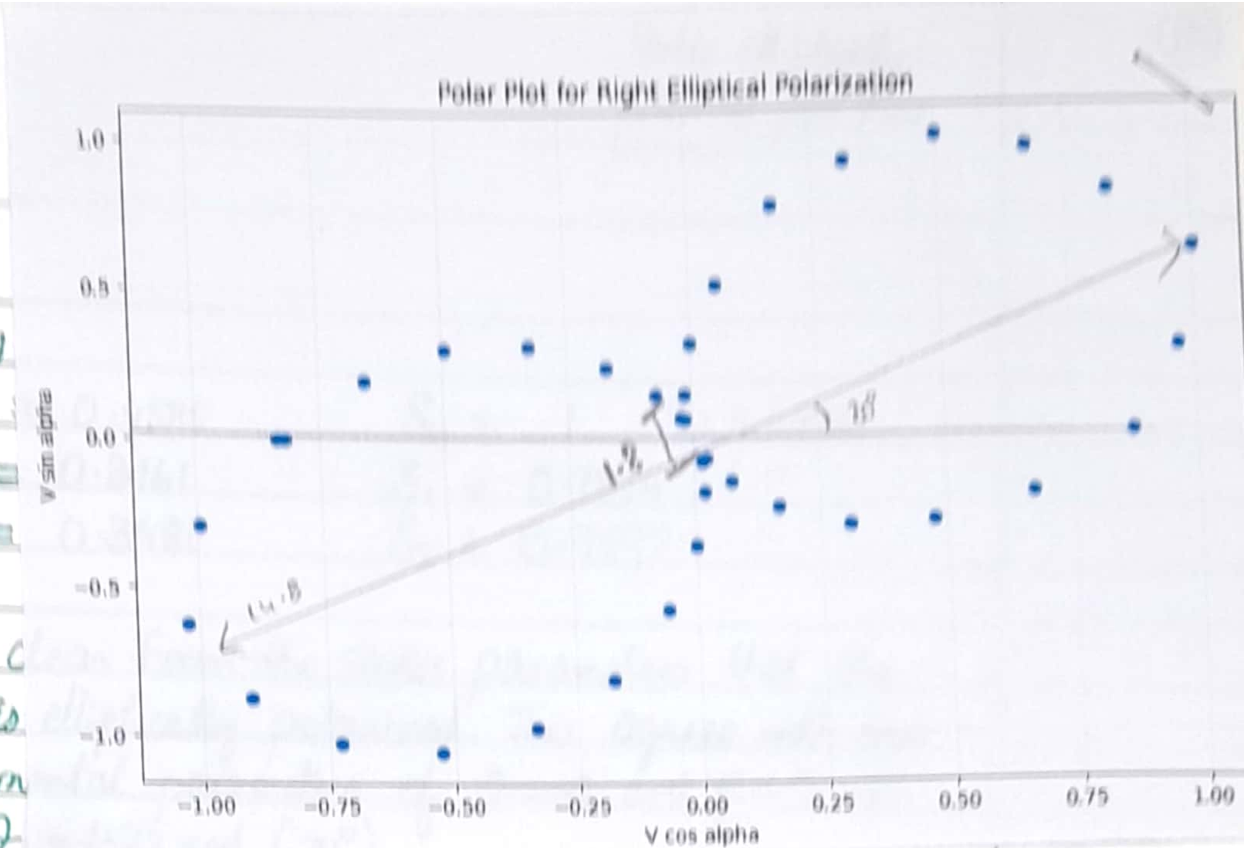
Uncertainty in Four Coeffs = ±0.00005

now

a_0

$a_2 =$

$b_2 =$



It is c

light is

experim

and θ

(ii) $\tau = 20^\circ \pm 0.5^\circ$

$$\varepsilon = \tan^{-1}\left(\sqrt{\frac{1.8}{14.8}}\right) = 19.2^\circ \pm 0.05^\circ$$

From Stokes parameters:

$$\varepsilon = \frac{1}{2}\tan^{-1}\left(\frac{0.7592}{0.7874}\right) \times \frac{180}{\pi} = 21.9^\circ \pm 0.05^\circ$$

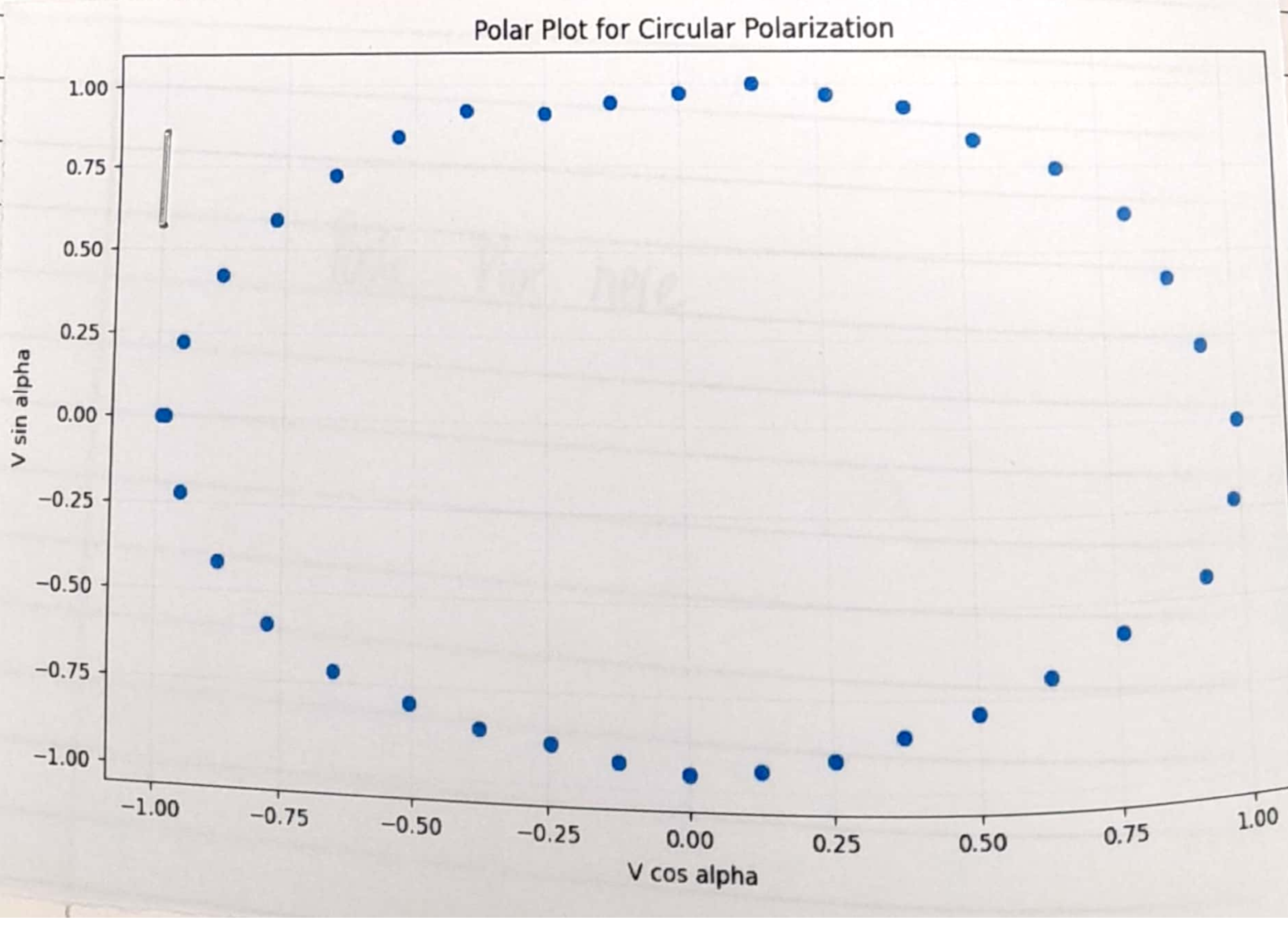
% error = 12.3% ± 0.05% Abs error = 2.7° ± 0.05°

There is a strong correlation with the values previously calculated!

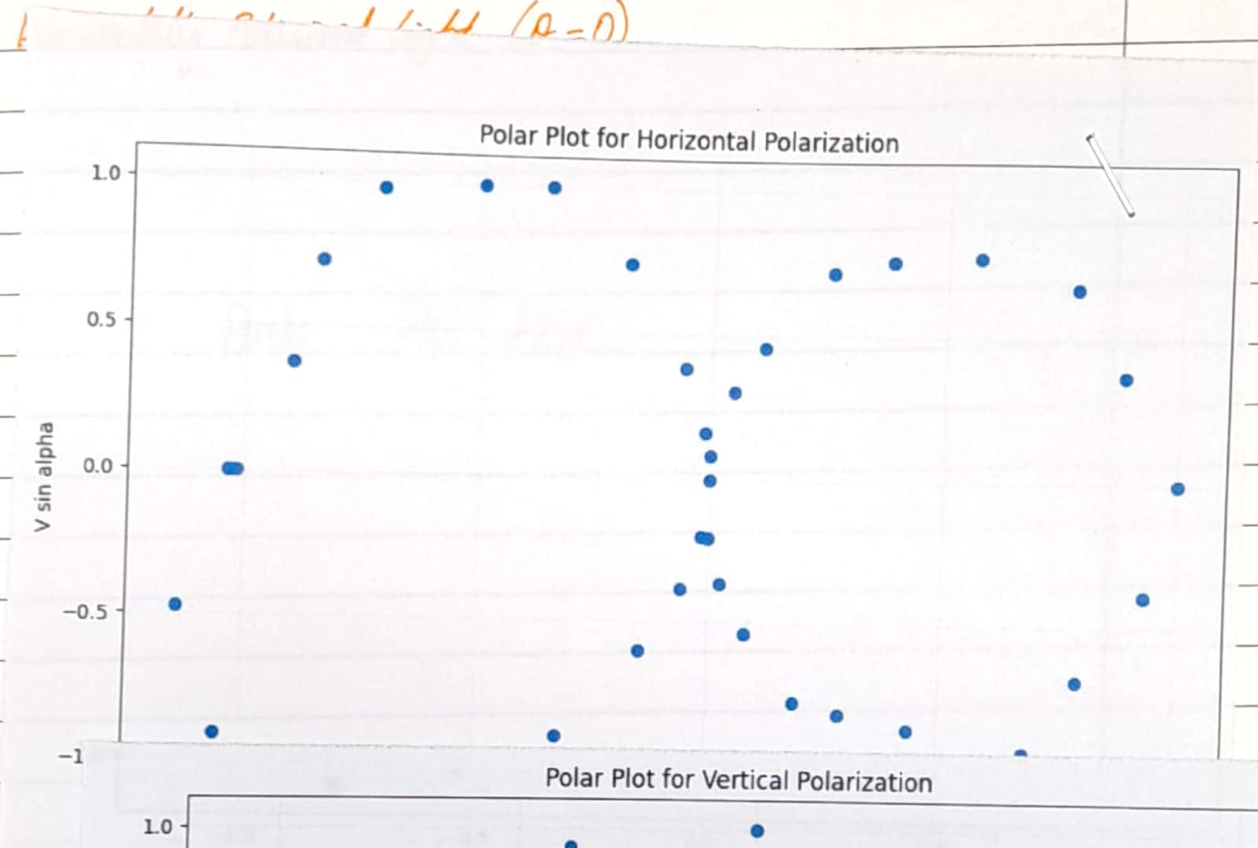
Since there is no convenient way for us to measure the 4th Stokes parameter right now, it is difficult to determine the handedness of the elliptical and circular polarizations.

⇒ Here are the polar plots generated with experimental data for the other degenerate polarization states.

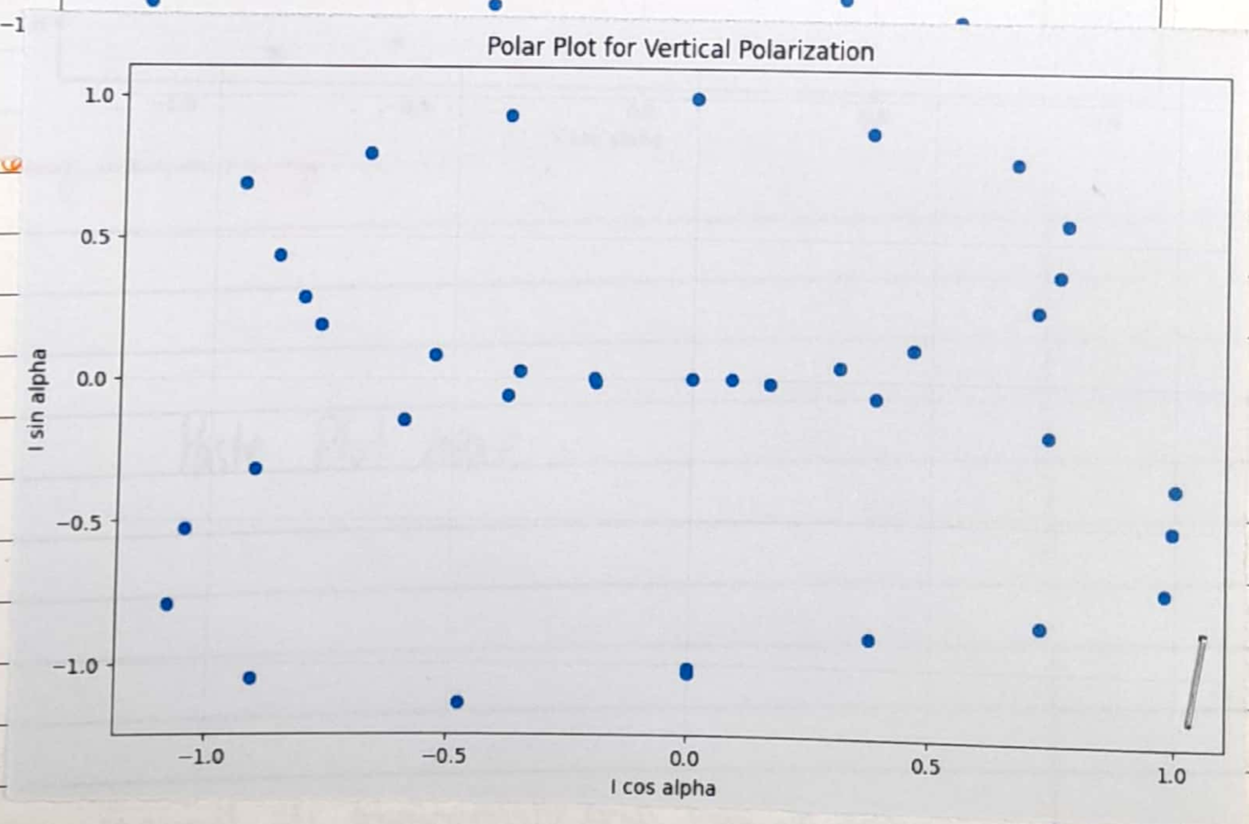
(iii) Circularly Polarized Light ($\theta = \pi/4$)



(iv)


Polar Plot for Horizontal Polarization
V sin alpha
1.0
0.5
0.0
−0.5
−1

(v)


Polar Plot for Vertical Polarization
I sin alpha
I cos alpha
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0

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⑥ Conclusion:

Over the course of this experiment, and others conducted before this, we have successfully identified, studied and generated arbitrary polarizations of light using simple lab equipment such as polarizers, quarter wave plates, half wave plates, Photo diodes, operational amplifiers and Neutral Density Filters.

We applied Jones calculus to formulate fourier expansion coefficients for our voltage function parametrized by the analyzer angle. We also determined that these coefficients were highly correlated with the Stokes vector construction of arbitrary angled polarizations.

Experimental data was collected (twice - BE CAREFUL WHILE ALIGNING YOUR LASER) for a multitude of polarization angles and the Stokes vector formulation and polarization ellipse parameters were within 10% error of the true polarization angle (pretty good result!)

Future work would entail studying partial polarization of light and determining its Poincaré sphere projection.

Final readings

Taken after realigning laser properly

Theta (degrees)	Alpha (degrees)	PD out (V)
0	0	0.84
	10	1.22
	20	1.5
	30	1.65
	40	1.71
	50	1.67
	60	1.55
	70	1.37
	80	1.13
	90	0.89
	100	0.64
	110	0.42
	120	0.22
	130	0.09
	140	0.07
	150	0.12
	160	0.28
	170	0.48
	180	0.81
	190	1.04
	200	1.33
	210	1.43
	220	1.52
	230	1.56
	240	1.43
	250	1.34
	260	1.07
	270	0.88
	280	0.62
	290	0.38
	300	0.2
	310	0.08
	320	0.07
	330	0.15
	340	0.32
	350	0.52
	360	0.82

Theta (degrees)	Alpha (degrees)	PD out (V)
45	0	1.14
	10	1.22
	20	1.2
	30	1.08
	40	0.95
	50	0.84
	60	0.72
	70	0.65
	80	0.6
	90	0.6
	100	0.64
	110	0.69
	120	0.77
	130	0.81
	140	0.88
	150	1.01
	160	1.04
	170	1.1
	180	1.13
	190	1.07
	200	1.03
	210	0.93
	220	0.84
	230	0.78
	240	0.68
	250	0.63
	260	0.55
	270	0.57
	280	0.59
	290	0.65
	300	0.76
	310	0.87
	320	0.93
	330	1.03
	340	1.07
	350	1.08
	360	1.15

Theta (degrees)	Alpha (degrees)	PD out (V)
70	0	1.4
	10	1.69
	20	1.82
	30	1.74
	40	1.59
	50	1.37
	60	1.12
	70	0.86
	80	0.58
	90	0.36
	100	0.18
	110	0.08
	120	0.08
	130	0.19
	140	0.36
	150	0.59
	160	0.84
	170	1.14
	180	1.46
	190	1.64
	200	1.78
	210	1.81
	220	1.48
	230	1.3
	240	1.05
	250	0.82
	260	0.52
	270	0.32
	280	0.15
	290	0.07
	300	0.08
	310	0.19
	320	0.37
	330	0.63
	340	0.88
	350	1.12
	360	1.37

1st readings

Theta (degrees)	Alpha (degrees)	PD out (V)
0	0	1.16
	10	1.13
	20	1.09
	30	1.05
	40	0.99
	50	0.92
	60	0.88
	70	0.84
	80	0.84
	90	0.85
	100	0.85
	110	0.86
	120	0.87
	130	0.91
	140	0.96
	150	1.03
	160	1.15
	170	1.17
	180	1.17
	190	1.11
	200	1.09
	210	1.08
	220	1.03
	230	0.96
	240	0.95
	250	0.91
	260	0.9
	270	0.86
	280	0.85
	290	0.86
	300	0.94
	310	0.97
	320	1
	330	1.04
	340	1.09
	350	1.13
	360	1.15

Theta (degrees)	Alpha (degrees)	PD out (V)
90	0	0.95
	10	0.99
	20	0.92
	30	0.88
	40	0.84
	50	0.82
	60	0.8
	70	0.78
	80	0.77
	90	0.75
	100	0.75
	110	0.77
	120	0.84
	130	0.88
	140	0.98
	150	1.03
	160	1.07
	170	1.08
	180	1.03
	190	1.02
	200	1.02
	210	0.93
	220	0.93
	230	0.91
	240	0.87
	250	0.85
	260	0.86
	270	0.86
	280	0.83
	290	0.88
	300	0.87
	310	0.89
	320	0.94
	330	0.99
	340	1.01
	350	1
	360	1.01

Theta (degrees)	Alpha (degrees)	PD out (V)
45	0	1.25
	10	0.87
	20	0.54
	30	0.29
	40	0.1
	50	0.01
	60	0.03
	70	0.14
	80	0.39
	90	0.65
	100	0.94
	110	1.17
	120	1.44
	130	1.65
	140	1.83
	150	1.88
	160	1.79
	170	1.55
	180	1.23
	190	0.92
	200	0.58
	210	0.31
	220	0.11
	230	0.01
	240	0.01
	250	0.16
	260	0.37
	270	0.64
	280	0.89
	290	1.18
	300	1.62
	310	1.71
	320	1.82
	330	1.83
	340	1.69
	[illegible]	1.45
	360	1.22