What is the meaning of “same” and “different” in context of measurements?*

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Theory and experiment are two intertwined pillars of science. Theories explain nature by using mathematical arguments while experiments are designed to check their consistency with nature. Experiments validate theories and provide sufficient and adequate information to modify them. They can also give rise to new theoretical framework. Theory gains approbation when theoretical and experimental results match. It is customary to assign experiments or tasks to students to match the predicted and observed results. We have designed a simple experiment using conical shaped pendulums, to see where experiment matches theory.

Suppose we give you a cubical object and ask you to determine its length. Zaid measures it to be 13.32 ± 0.06 cm while Bakr reports the length at 13.462 ± 0.065 cm. He uses a different measuring instrument. Do these results match? The current experiment helps to determine whether measurements from two different experiments or from experiment and theory can qualify as being the “same” or “different”. Clearly the word “identical” has no place in the technical jargon of experiments and must be avoided.

A conical shaped pendulum consists of a rigid object which swings freely about some pivot point in vertical plane. Its period depends on the moment of inertia of the rigid body calculated around the pivot point. The time period is

$$T = 2\pi\sqrt{\frac{I}{MgR_{\mathrm{CM}}}} \tag{1}$$

where I is the moment of inertia, M is the mass of the rigid body, g is the gravitational acceleration and R_{CM} is the distance from center of mass. It is important to note that the time period depends on the mass distribution of the rigid body. Changing the total mass, size, shape and distribution of mass will change the moment of inertia and eventually the time period.

You are provided with different conical shaped pendulums, e.g., upright cone, inverted cone and double cone (diamond). In appendix, the moment of inertia and the center of mass for the solid cylinder is calculated. The appendix is meant to serve as a guideline for determining the moment of inertia I and center of mass R_{CM} for the other pendulums.

The various parameters of the four pendulums are shown in Table (1).

	mass (M,±1g)	radius (r,±0.01cm)	length (l,±0.1cm)
Cylinder	166	1.21	45.4
Double cone	97	1.23	45.4
Cone suspended from base	107	1.25	45.4
Cone suspended from vertex	107	1.27	45.4

Table 1: Parameters of conical shaped pendulums

COMPUTE:
Q 1. Following the same strategy, as it is done for the solid cylinder, compute center of mass, moment of inertia and time period of any two of your favorite shapes.

MEASURE:
Q 2. Experimentally determine the time period of your chosen cone and compare with the theoretical prediction in the previous question. Does your measurement match with the prediction? Comment.

A Appendices

A.1 Time Period of a Pendulum

Consider a rigid body of mass M (solid cylinder in our case), as shown in Figure (1) which oscillates vertically about frictionless horizontal axis passes through a pivot point where l is its height and R_{CM} is the distance from center of mass. When it is displaced through a small angle θ from its equilibrium position, its weight can be resolved into radial and tangential components, that is, $mg\cos\theta$ and $mg\sin\theta$ respectively.

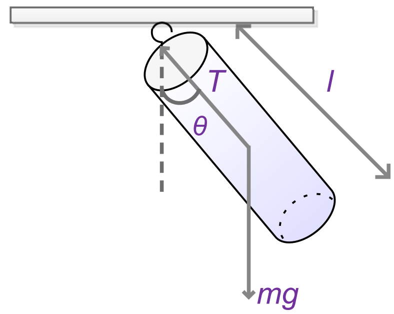

Figure 1: Solid cylinder in oscillation.

Since there is no motion along the radial direction, so tension balances the $mg\cos\theta$ component

$$T = mg\cos\theta \tag{2}$$

and $mg\sin\theta$ is responsible only for the oscillation around mean position and provides the necessary restoring force to produce the torque:

$$\tau = -(mg\sin\theta)R_{\mathrm{CM}}. \tag{3}$$

For small angles $\sin\theta \approx \theta$, yielding

$$\tau = -mg\theta R_{\mathrm{CM}}. \tag{4}$$

Since $\tau = I\frac{d^2\theta}{dt^2}$, we have

$$\frac{d^2\theta}{dt^2} + \frac{mgR_{\mathrm{CM}}}{I}\theta = 0 \tag{5}$$

which can be compared with the standard form of simple pendulum motion

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \tag{6}$$

leading to the frequency and the time period

$$\omega = \sqrt{\frac{mgR_{\mathrm{CM}}}{I}} \tag{7}$$

$$T = 2\pi\sqrt{\frac{I}{mgR_{\mathrm{CM}}}}. \tag{8}$$

A.2 Center of Mass of a Uniform Solid Cylinder

Consider the solid cylinder from Figure (2) oriented along the z–axis. Mass is evenly distributed.

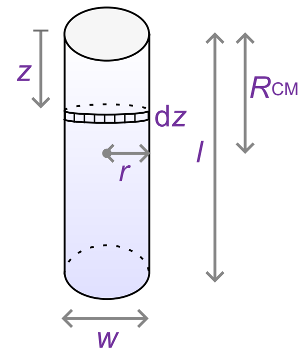

Figure 2: Geometry of the solid cylinder for center of mass's location and moment of inertia.

The mass density is ρ.

$$M = \int dm = \int \rho dV, \tag{9}$$

If r is the radius of the cylinder, the volume of the infinitesimal element shown as a shaded disk will be

$$dV = \pi r^2 dz. \tag{10}$$

The center of mass along the z-axis can be calculated as:

$$R_{\mathrm{CM}} = \tfrac{1}{M}\int z\, dm \tag{11}$$

$$= \frac{\int_0^l z\rho dV}{\int_0^l \rho dV}$$

$$= \frac{\int_0^l z\pi r^2 dz}{\int_0^l \rho\pi r^2 dz}$$

$$= \frac{\int_0^l z dz}{\int_0^l dz} = \frac{\left|\frac{z^2}{2}\right|_0^l}{|z|_0^l} = \frac{l}{2} \tag{12}$$

which is the required result.

A.3 Moment of Inertia of a Solid Cylinder

Consider once again, the solid cylinder from Figure (2). The moment of inertial of the small disk is:

$$\begin{aligned} dI &= z^2 dm \\ &= z^2 \rho \pi r^2 dz \\ I &= \rho \pi r^2 \int_0^l z^2 dz \\ I &= \frac{\rho \pi r^2 l^3}{3} = \frac{1}{3} M l^2 \end{aligned} \tag{13}$$

where $r = w/2$. It yields the time period

$$T = 2\pi \sqrt{\frac{2l}{3g}}. \tag{14}$$