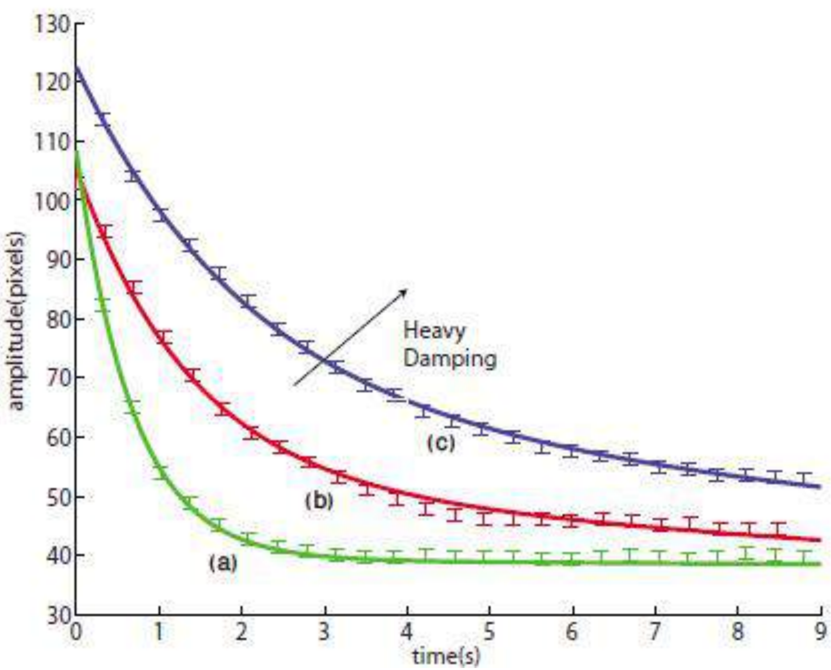
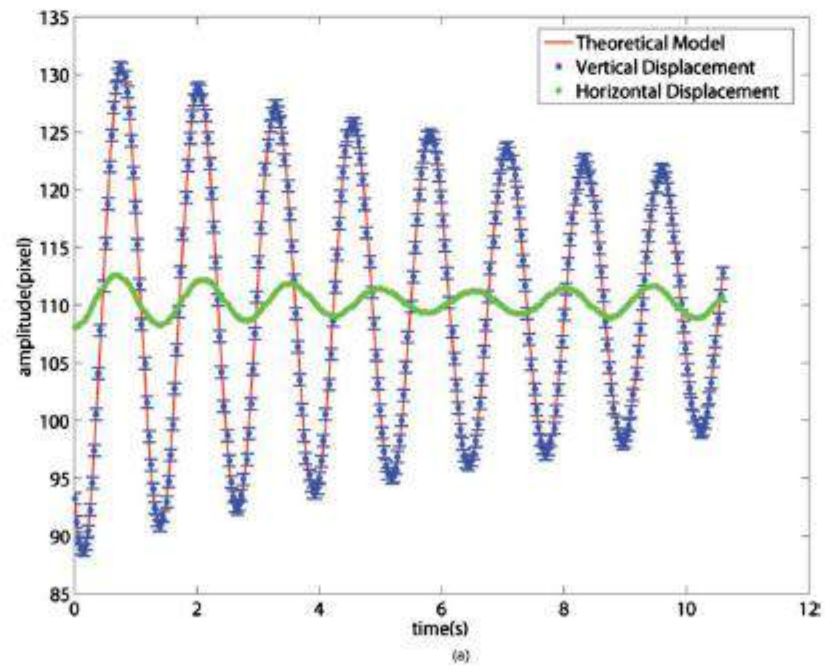
Graphical Presentation of Data





Dr. Muhammad Sabieh Anwar and his Physlab colleagues

Physical Review, 1917

A DIRECT PHOTOELECTRIC DETERMINATION OF PLANCK'S "h."[1]

By R. A. Millikan.

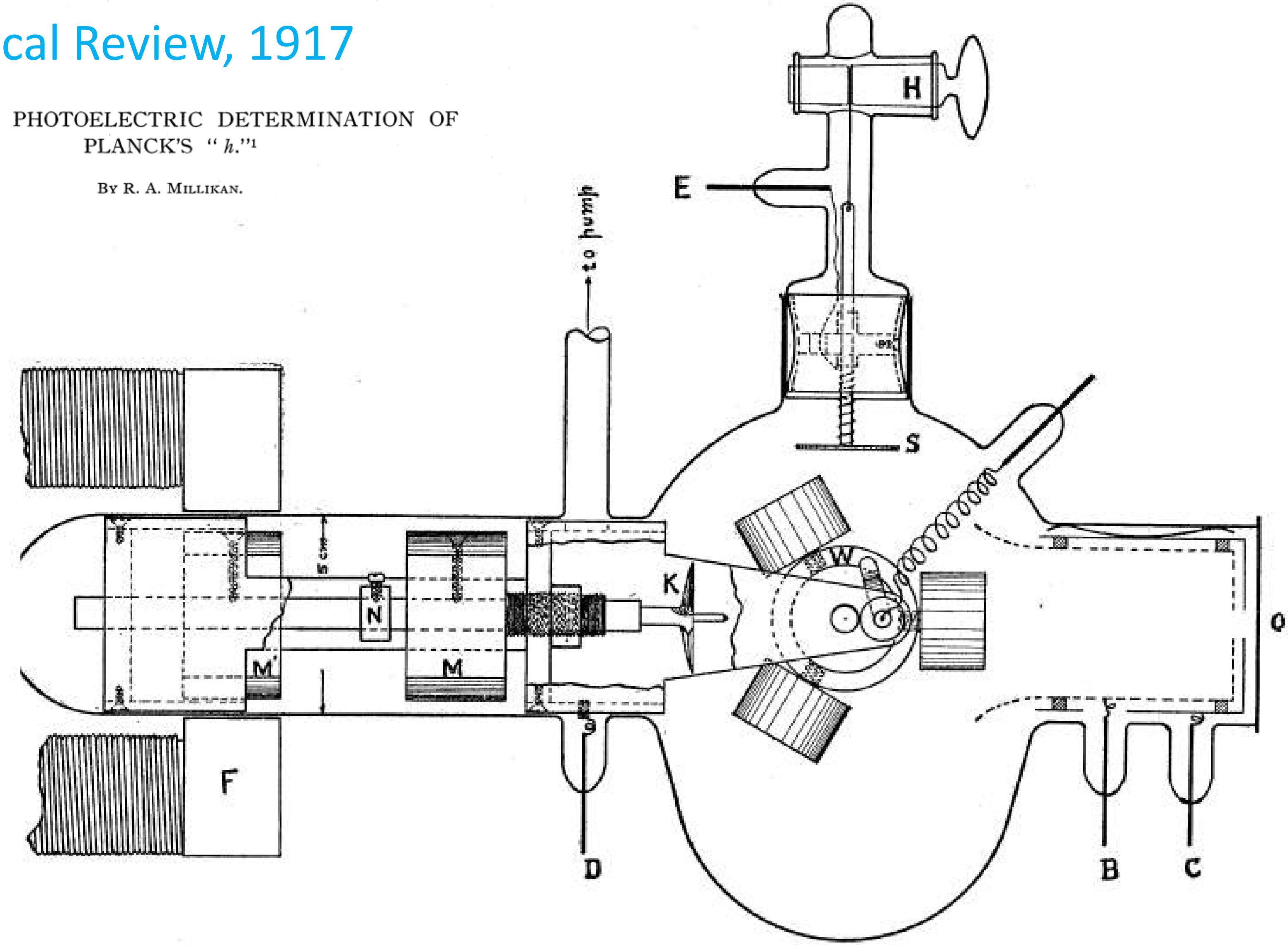
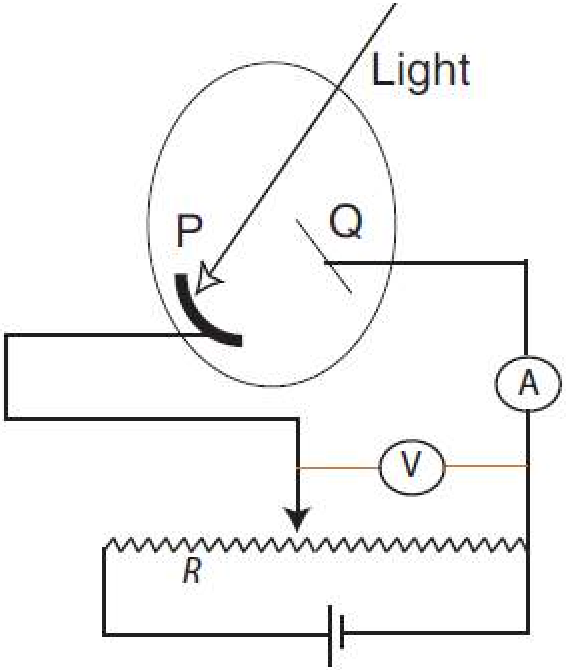


Fig. 2.



$$\frac{1}{2}mv^2 = V_o e = hf - W$$

$$V_o = \frac{h}{e} f - \frac{W}{e}$$

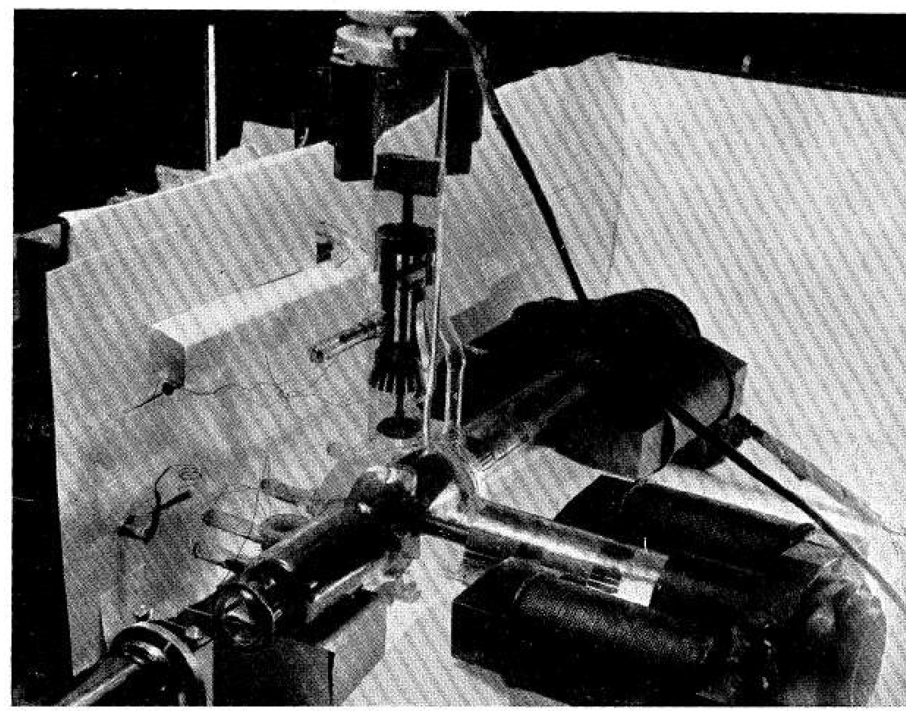


Table I.

5,461.		4,339.		4,047.		3,650.		3,126.		2,535.	
Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts	Def'n., mm.
2.257	28	1.581	44	1.576	82	1.157	67½	.5812	52	−.0576	68
2.205	14	1.629	20	1.524	55	1.105	36	.5288	29	+.0576	38
2.152	7	1.576	10	1.471	36	1.0525	19	.4765	12	+.1620	26
2.100	3	1.524	4	1.419	24	1.0002	11	.4242	5	+.2670	16½
				1.367	3	.9478	4	.3718	2½	+.3720	8

Table III.

4339.		4047.		3650.		3125.		2535.	
Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.	Volts.	Def'n., mm.
−6.00	1,300.0	−6.000	1,950.0	−6.000	6,000.0	−6.000	6,000.0	−6.000	3,500.0
−1.200	10.0	−1.087	21.5	−0.680	19.0	− .077	16.0	+0.736	18.0
−1.153	5.7	−1.027	10.5	−0.628	9.5	.000	6.3	+0.777	10.5
−1.103	2.7	−0.984	6.7	−0.575	5.0	+ .058	2.3	+0.814	6.7
−1.058	1.7	−0.941	3.5	−0.539	2.7	+ .100	1.1	+0.873	4.0
−1.000	1.1	−0.900	2.5	0.495	1.5	+ .151	.5	+3.928	2.0
− .950	0.6	−0.845	1.0	0.437	.6	+ .200	.2	+0.969	1.0
− .870	0.5	−0.793	0.5	0.390	.2			+1.019	0.5
		−0.737	0.25					+1.069	0.0
								+6.000	0.0

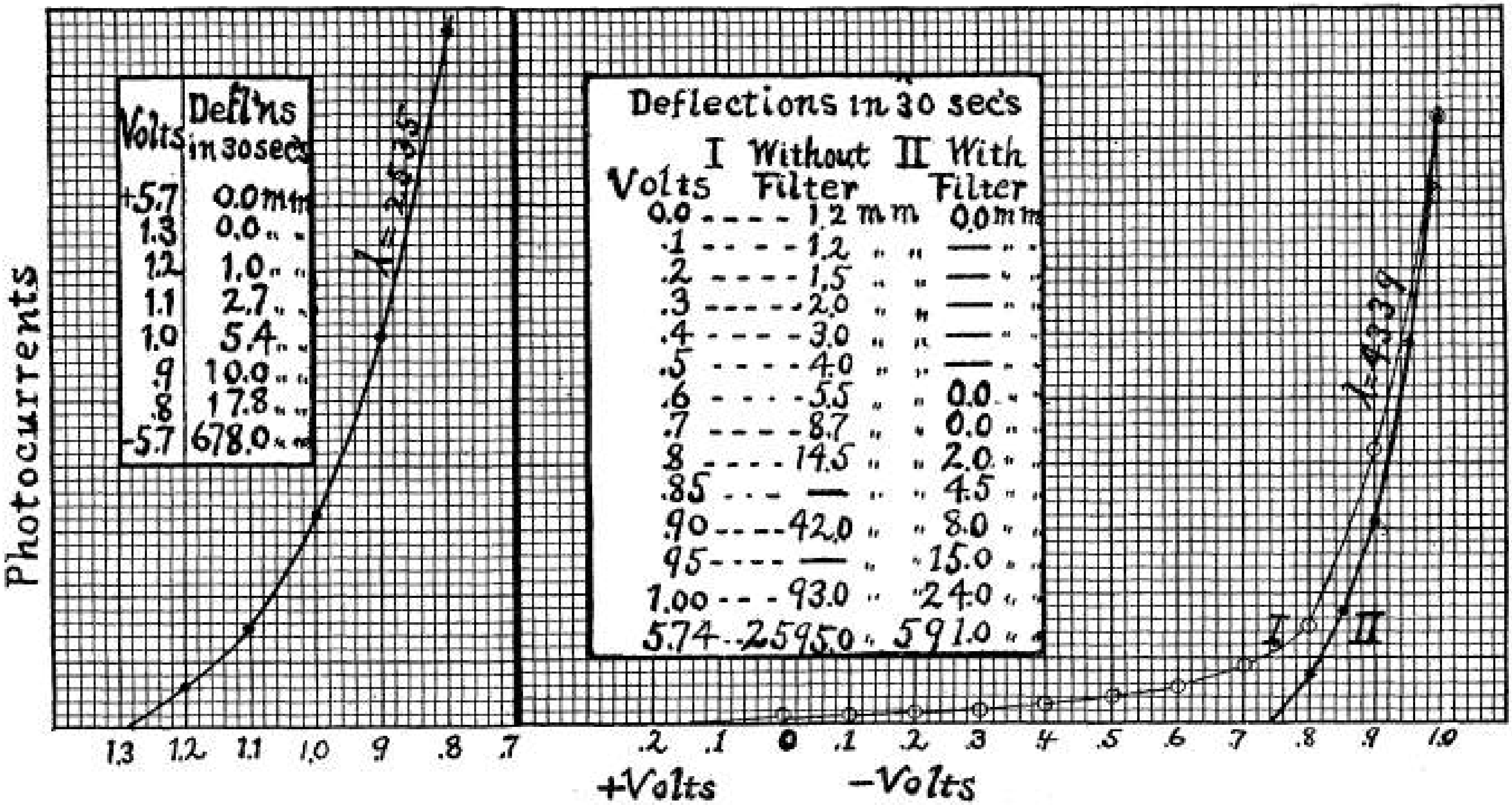
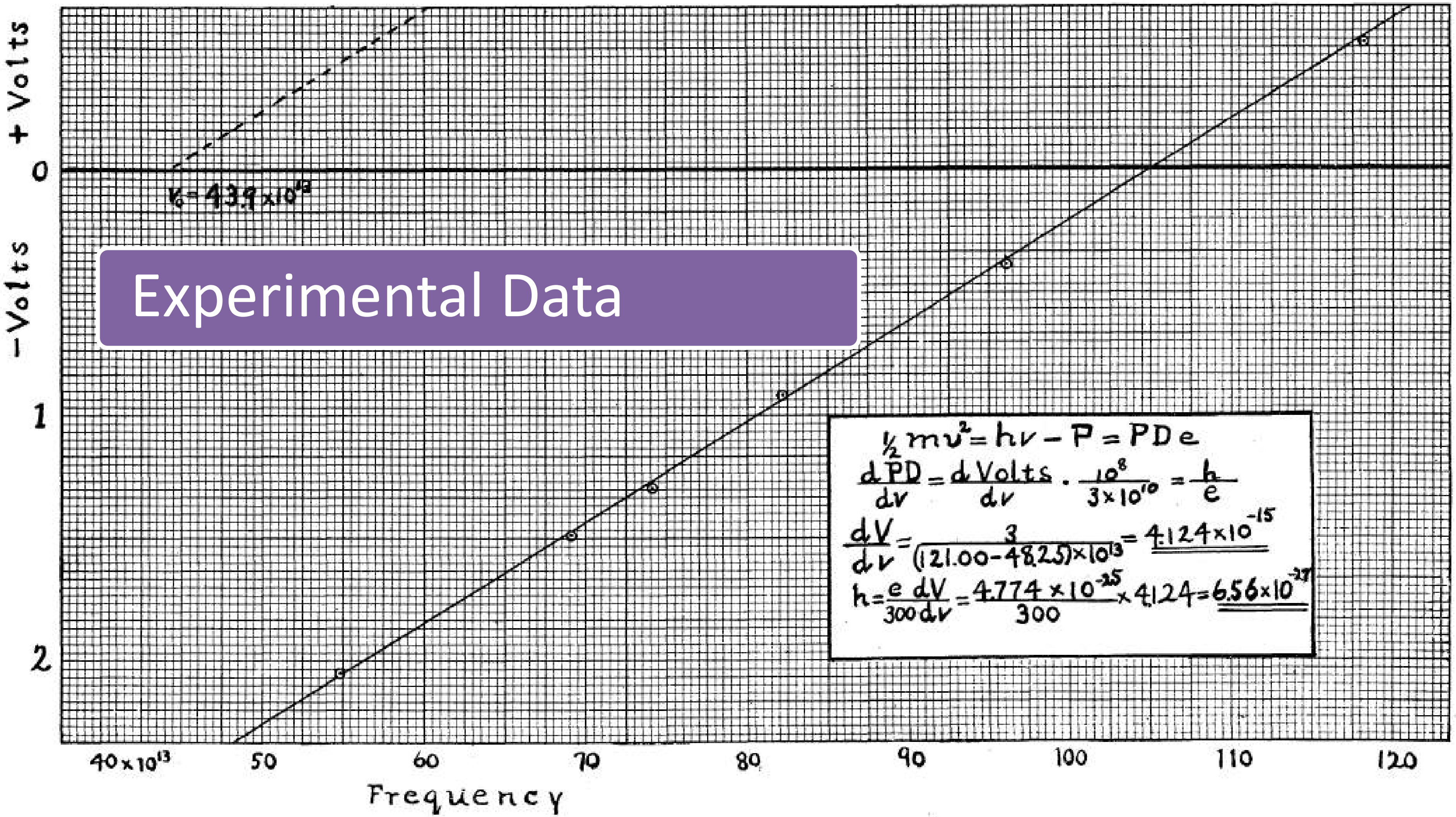

Photocurrents
Volts
Defl'ns in 30 sec's
+5.7 0.0 mm
1.3 0.0 " "
1.2 1.0 " "
1.1 2.7 " "
1.0 5.4 " "
.9 10.0 " "
.8 17.8 " "
-5.7 678.0 " "
λ=2535
Deflections in 30 sec's
I Without Filter
II With Filter
Volts
0.0 ---- 1.2 mm 0.0 mm
.1 ---- 1.2 " " — " "
.2 ---- 1.5 " " — " "
.3 ---- 2.0 " " — " "
.4 ---- 3.0 " " — " "
.5 ---- 4.0 " " — " "
.6 ---- 5.5 " " 0.0 " "
.7 ---- 8.7 " " 0.0 " "
.8 ---- 14.5 " " 2.0 " "
.85 --- — " " 4.5 " "
.90 ---- 42.0 " " 8.0 " "
.95 ---- — " " 15.0 " "
1.00 --- 93.0 " " 24.0 " "
5.74 -- 2595.0 " 591.0 " "
λ=4339
I
II
1.3 1.2 1.1 1.0 .9 .8 .7
.2 .1 0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
+Volts
-Volts

Fig. 4.

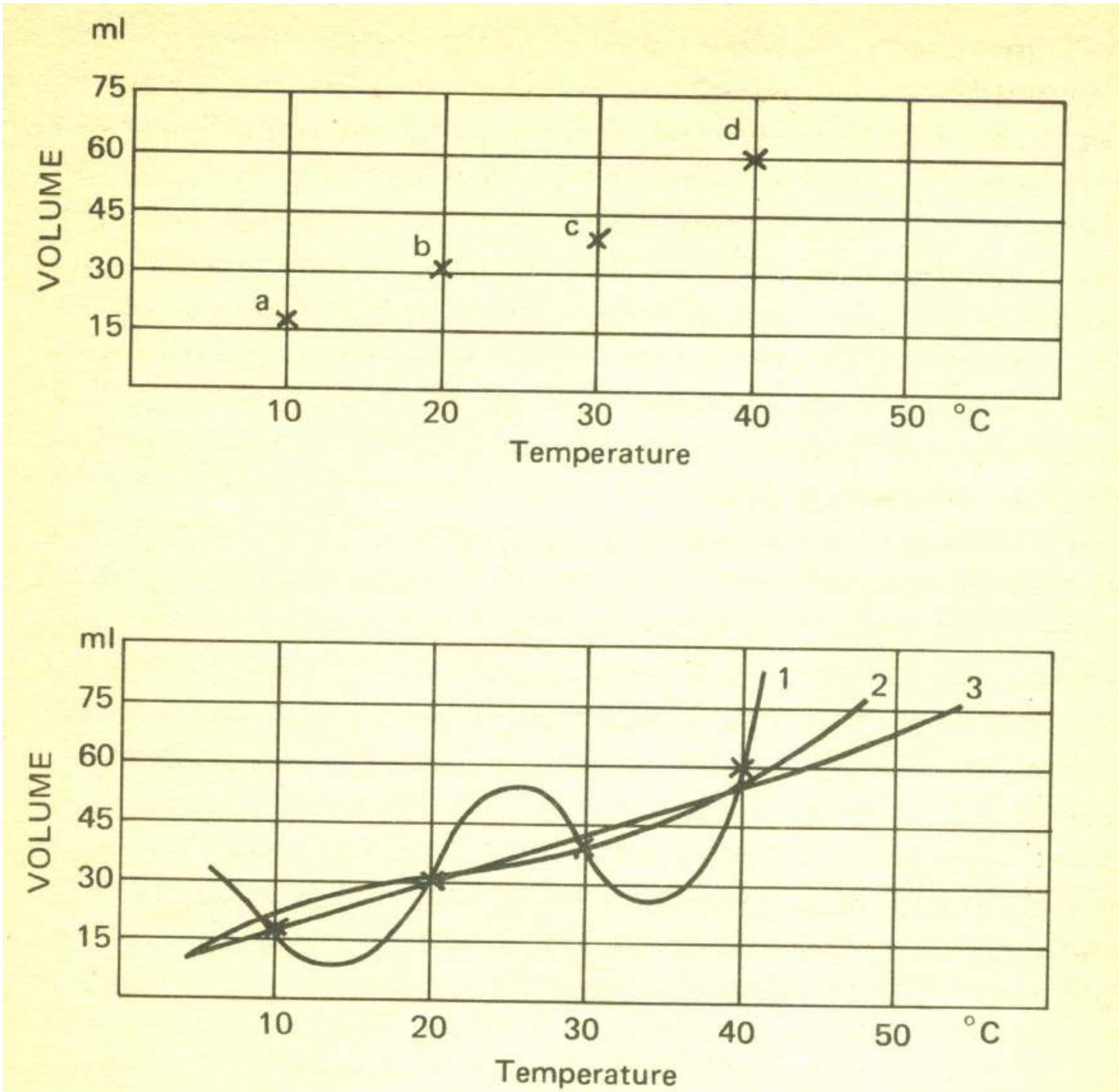
$$V_o = \frac{h}{e} f - \frac{W}{e}$$

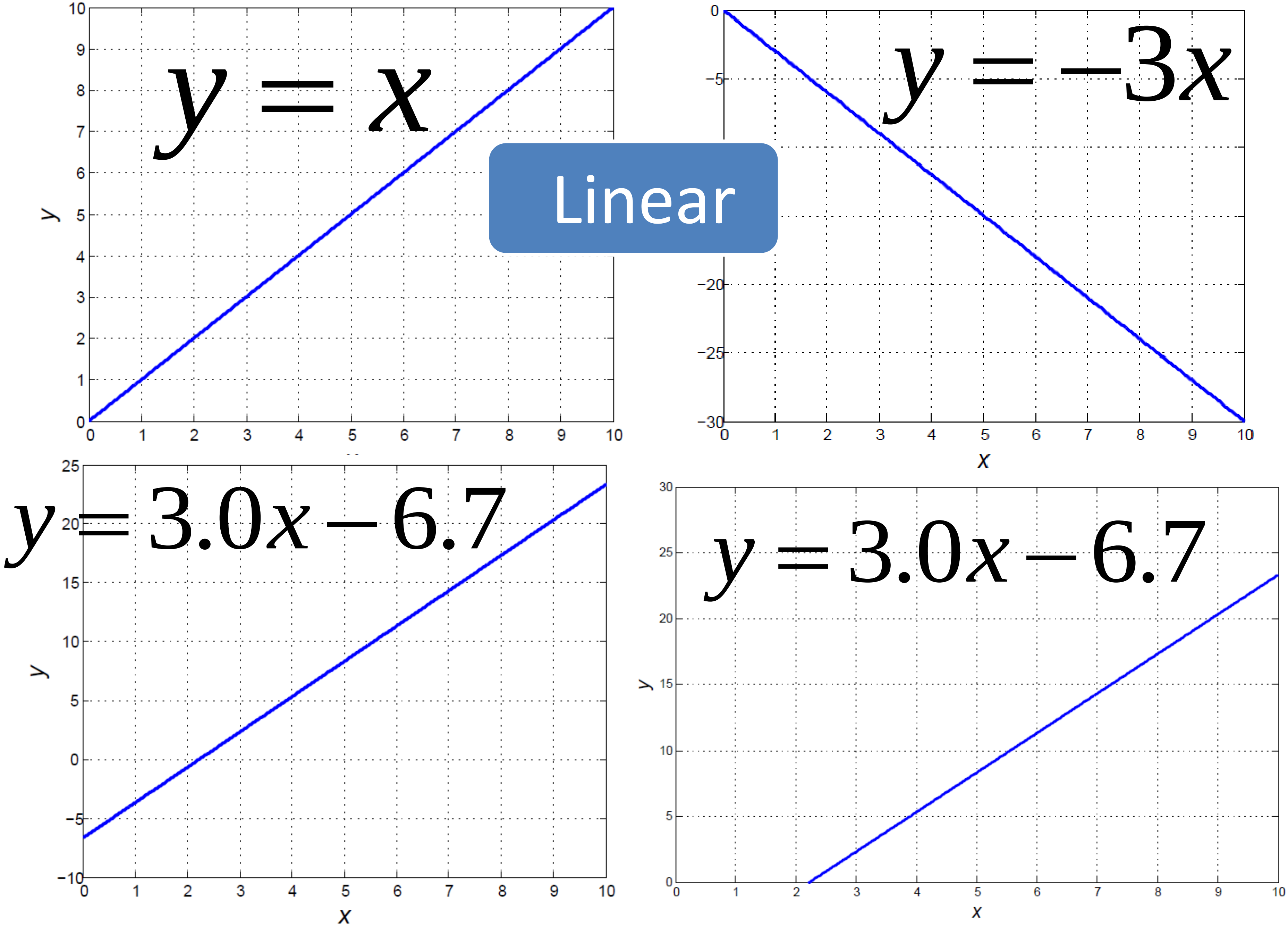
Model

Experimental Data



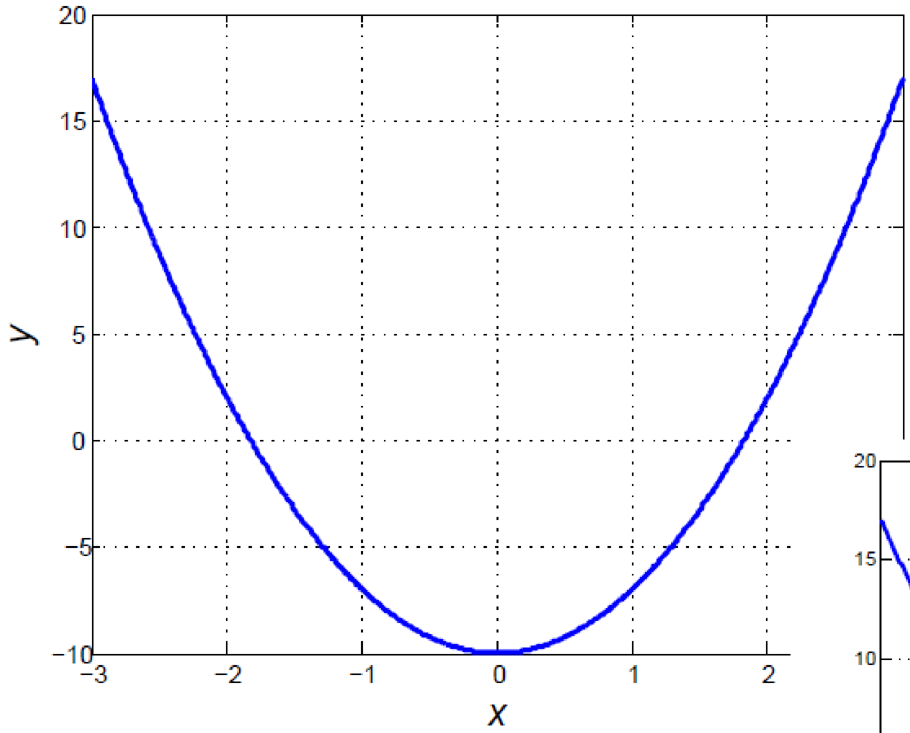
Typical models that fit typical experimental data



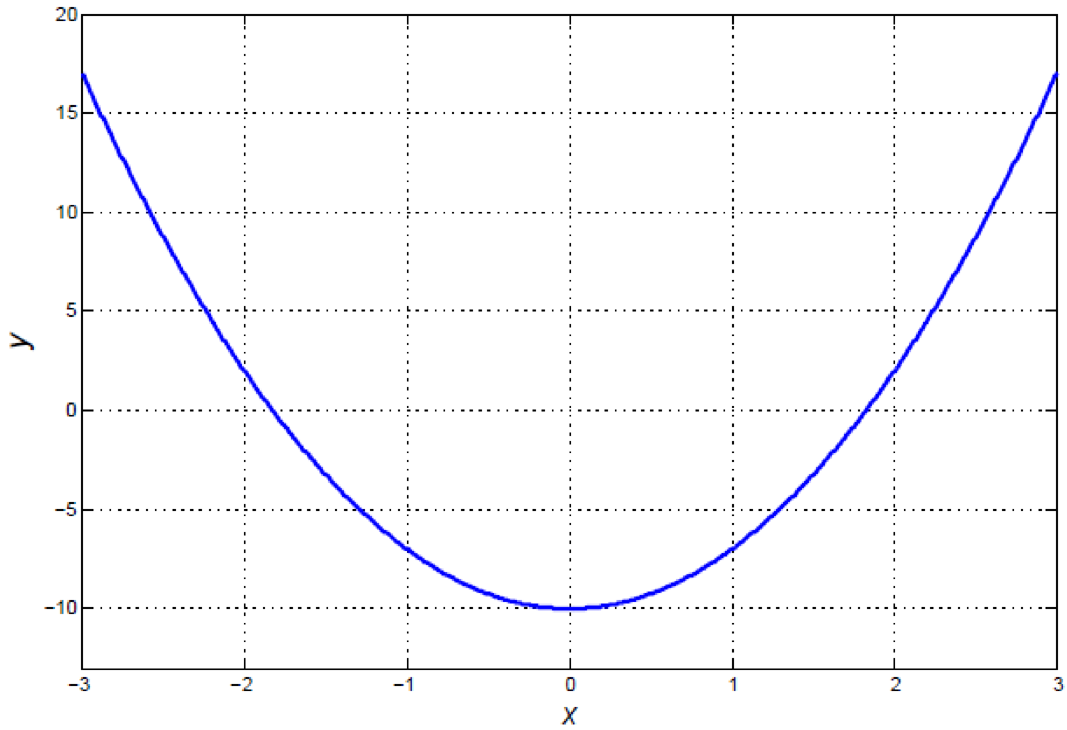


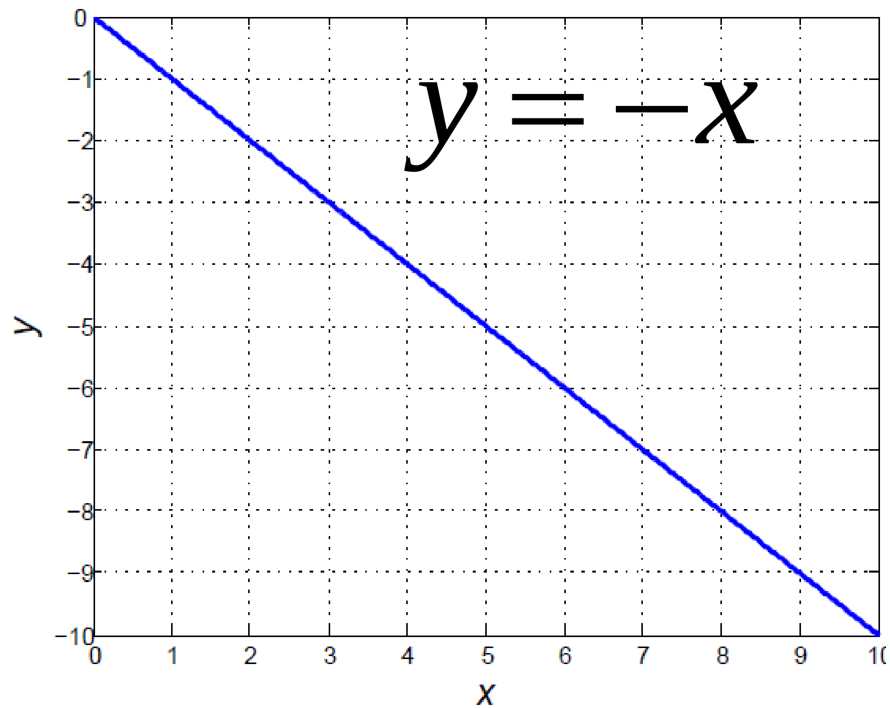
Linear
$y = x$
$y = -3x$
$y = 3.0x - 6.7$
$y = 3.0x - 6.7$
x
y

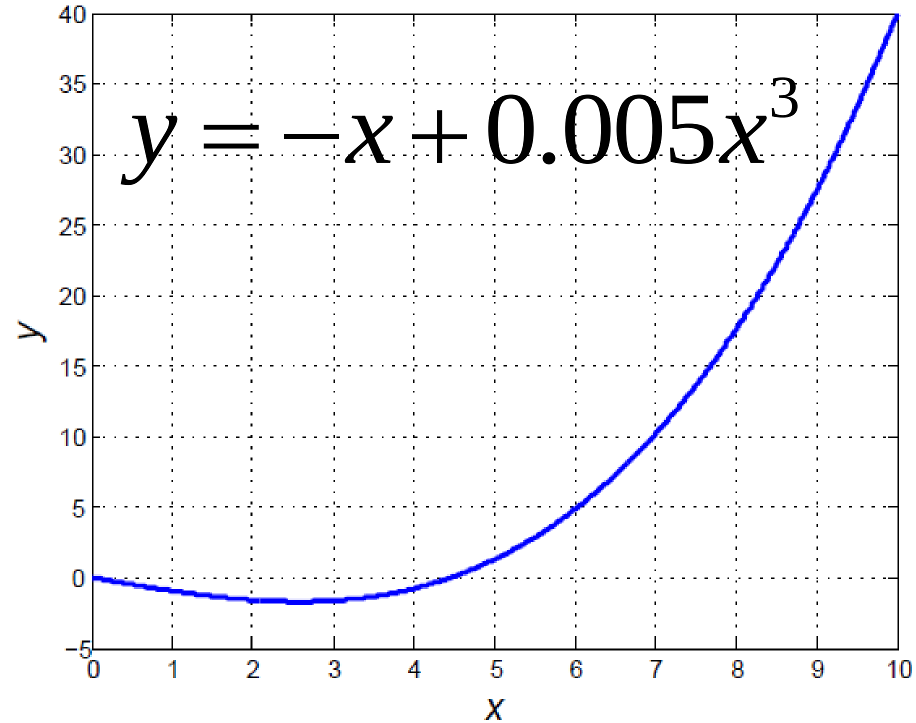
$$y = 3x^2 - 10$$

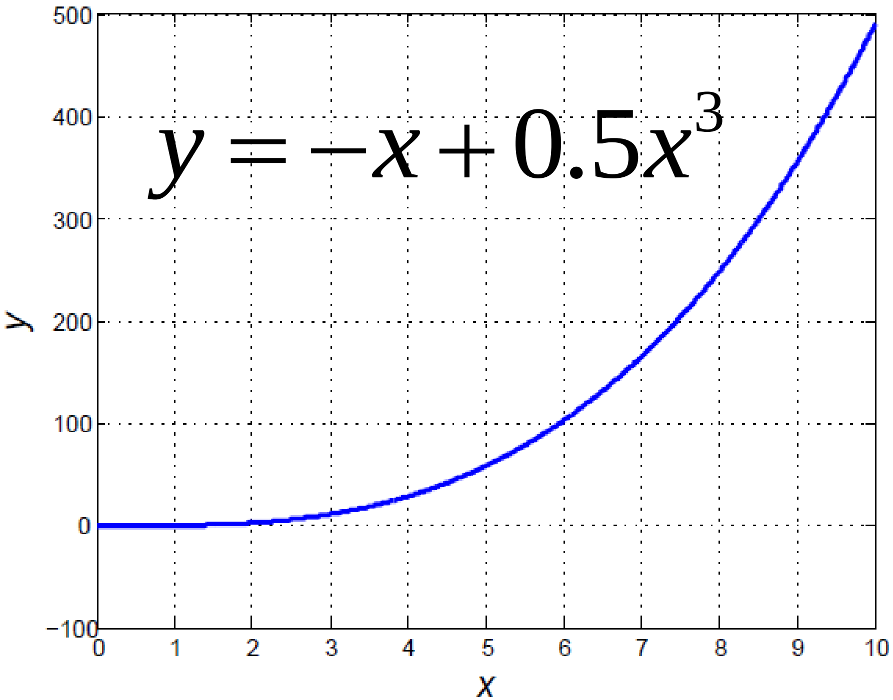


Quadratic

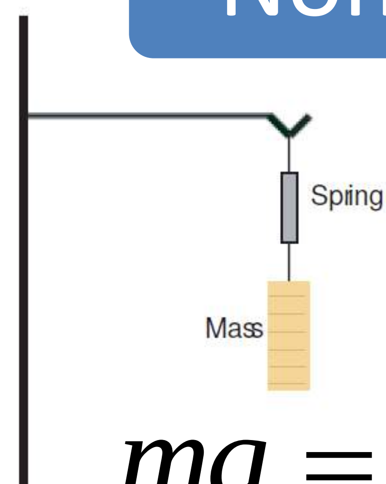






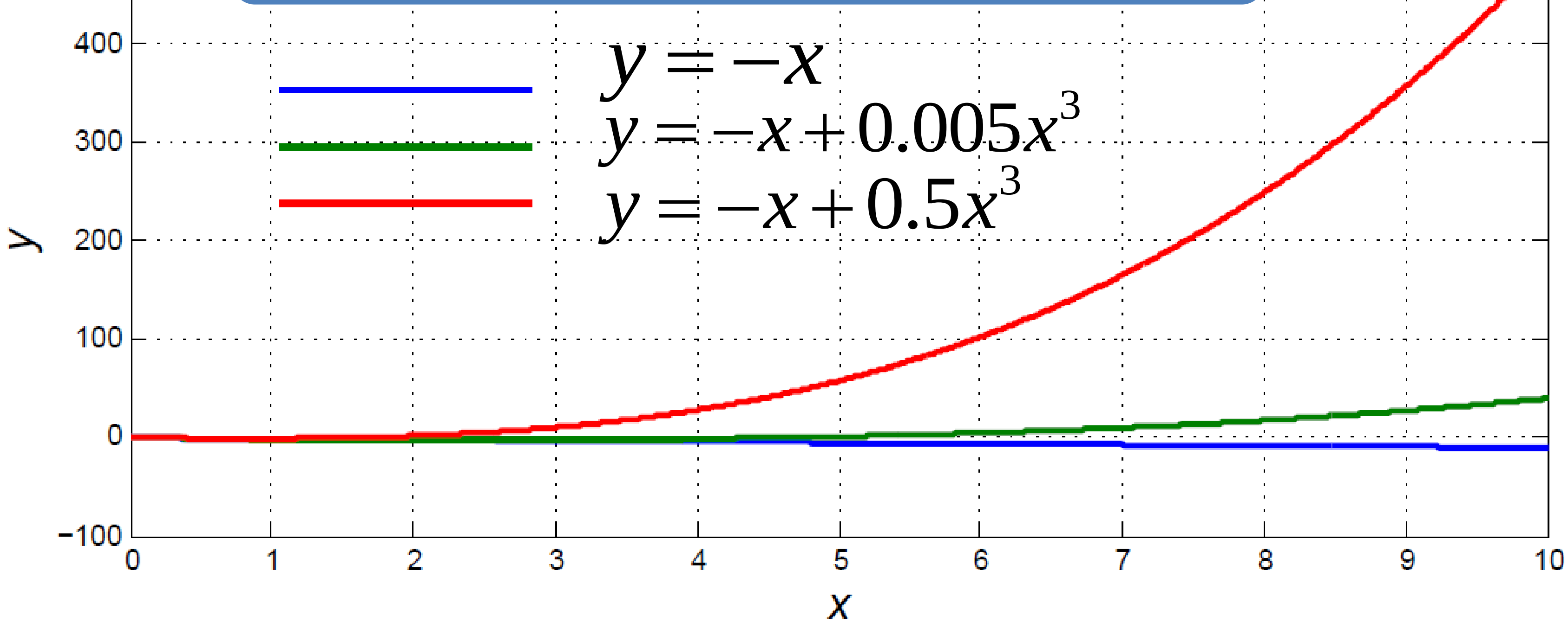


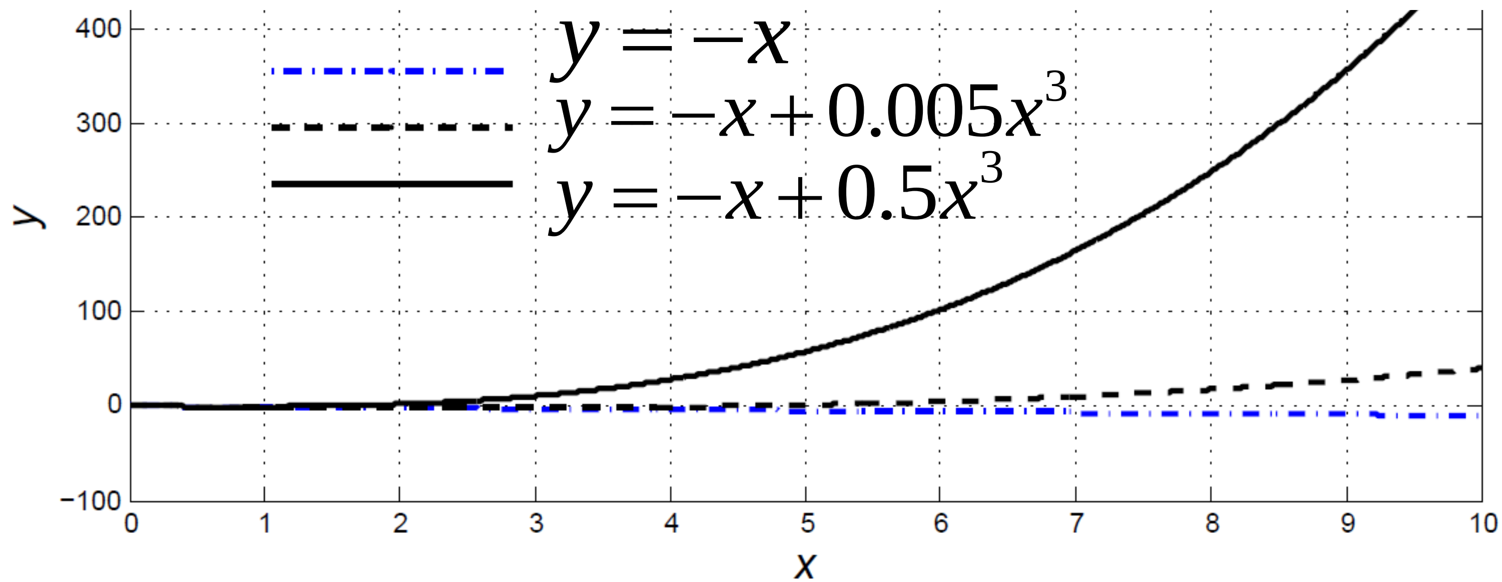
Nonlinearity



$$mg = -kx + \beta x^3$$

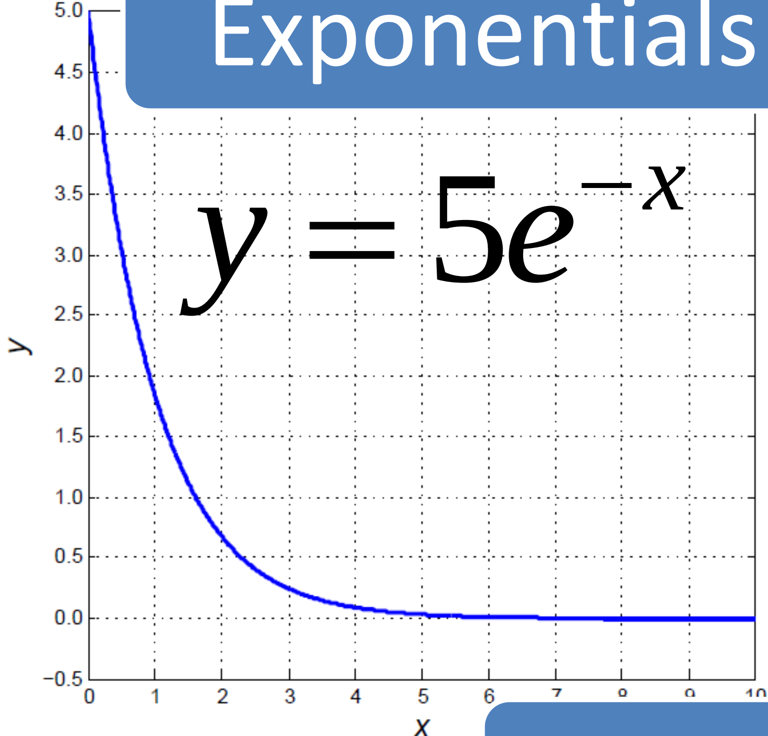
Superposing graphs



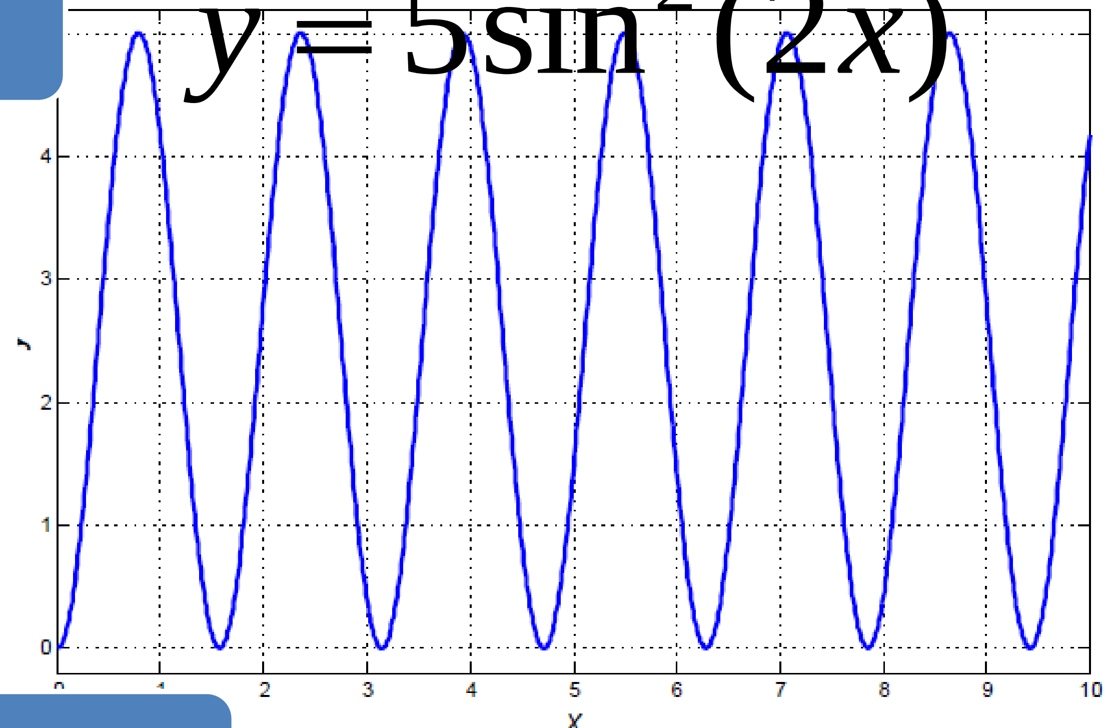


Exponentials

$$y = 5e^{-x}$$

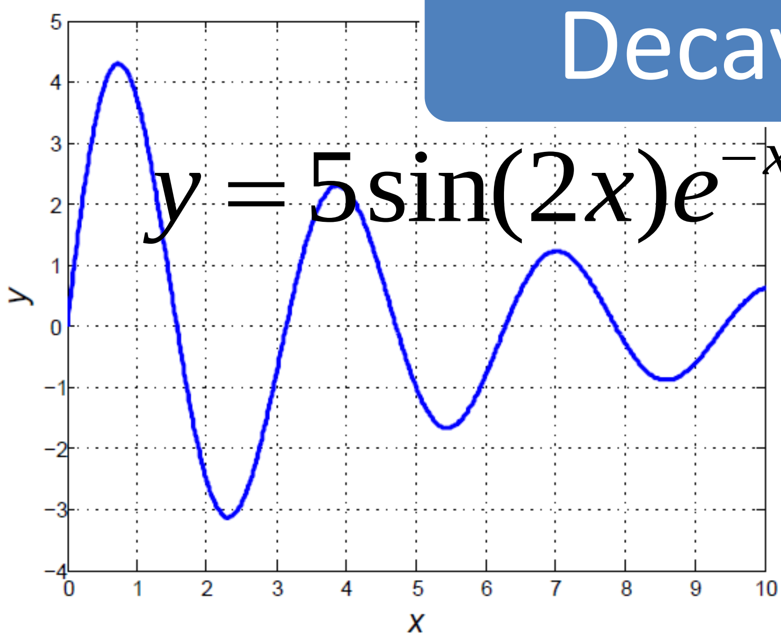


$$y = 5\sin^2(2x)$$

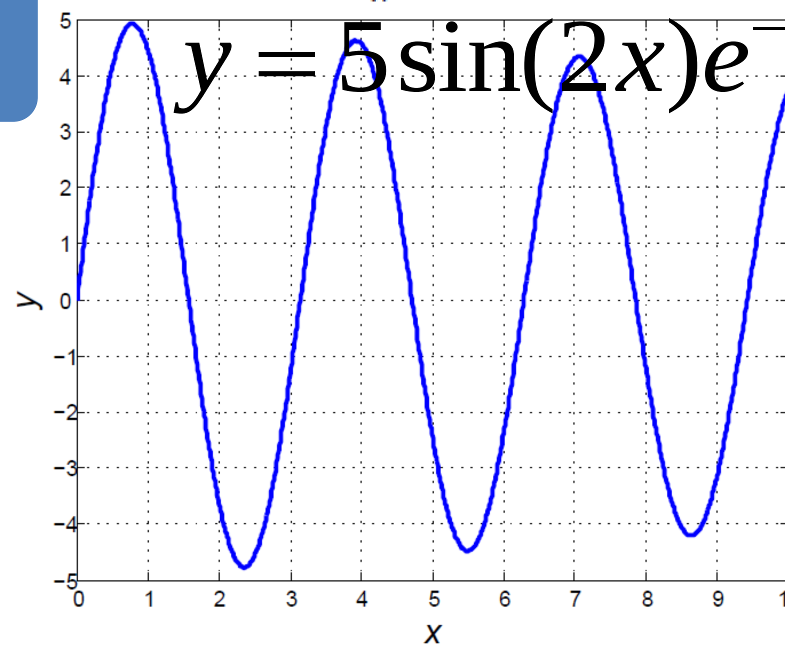


Decay

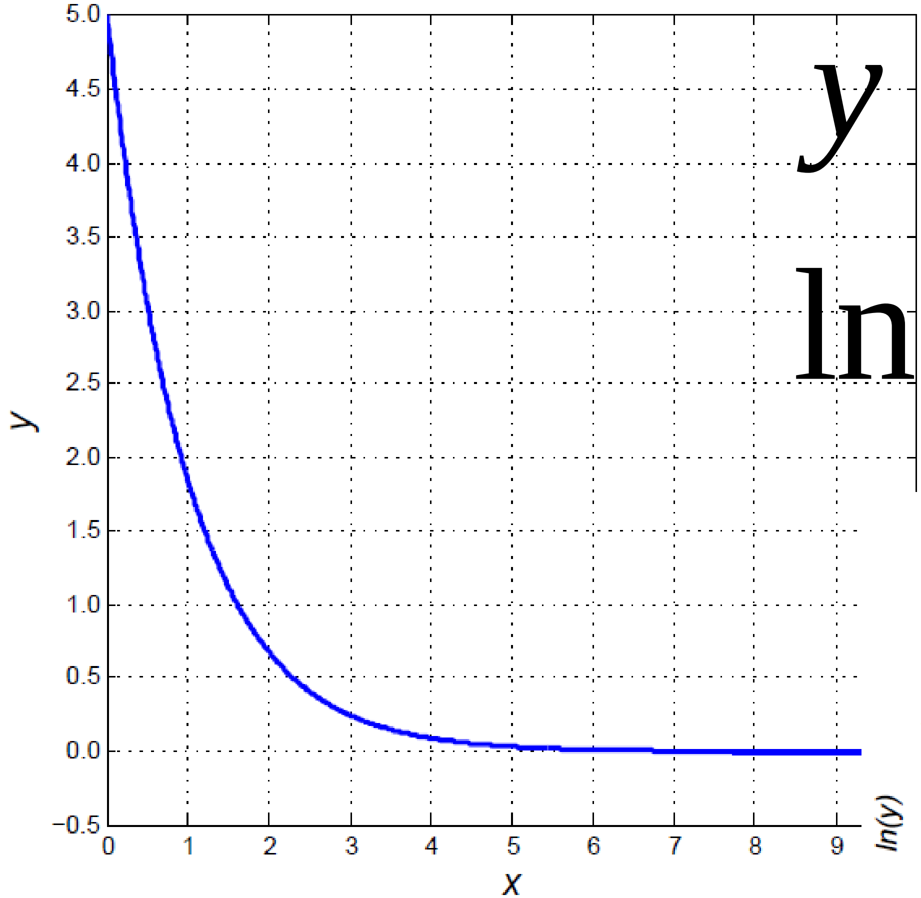
$$y = 5\sin(2x)e^{-x/5}$$



$$y = 5\sin(2x)e^{-x/50}$$

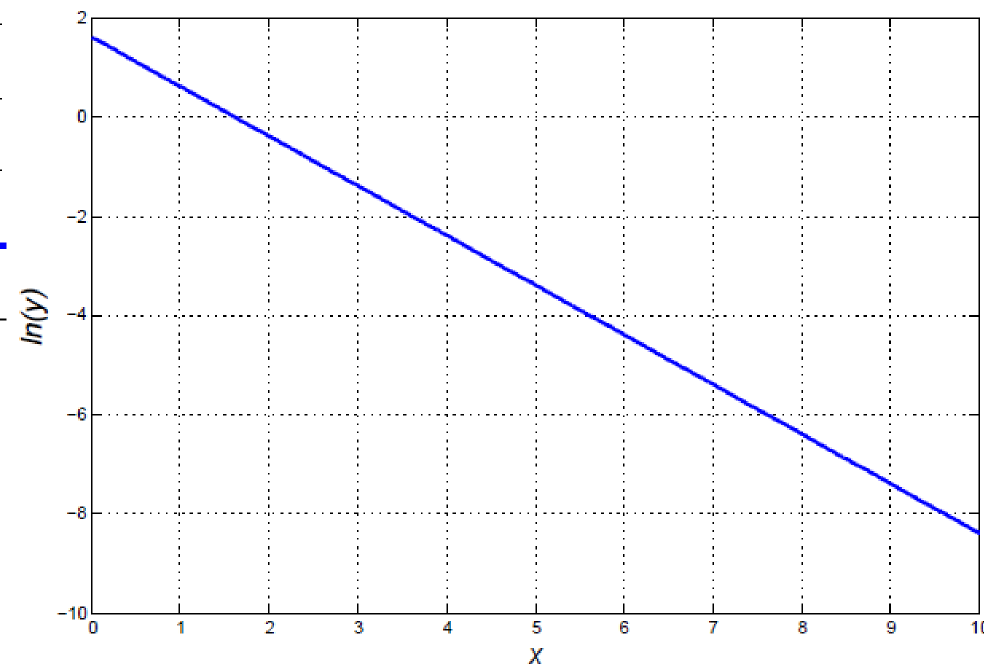


Linearization



$$y = 5e^{-x}$$

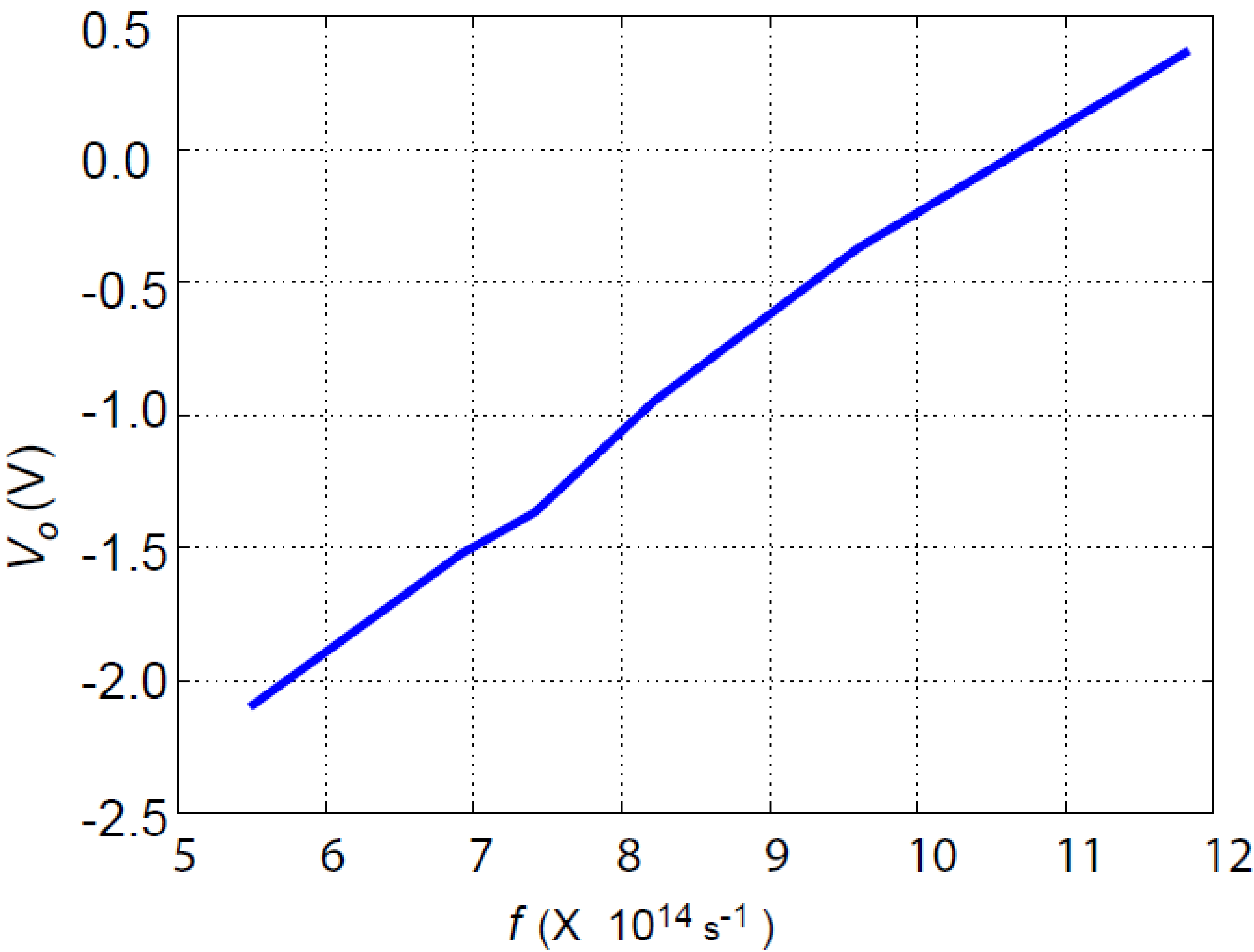
$$\ln(y) = \ln(5) - x$$



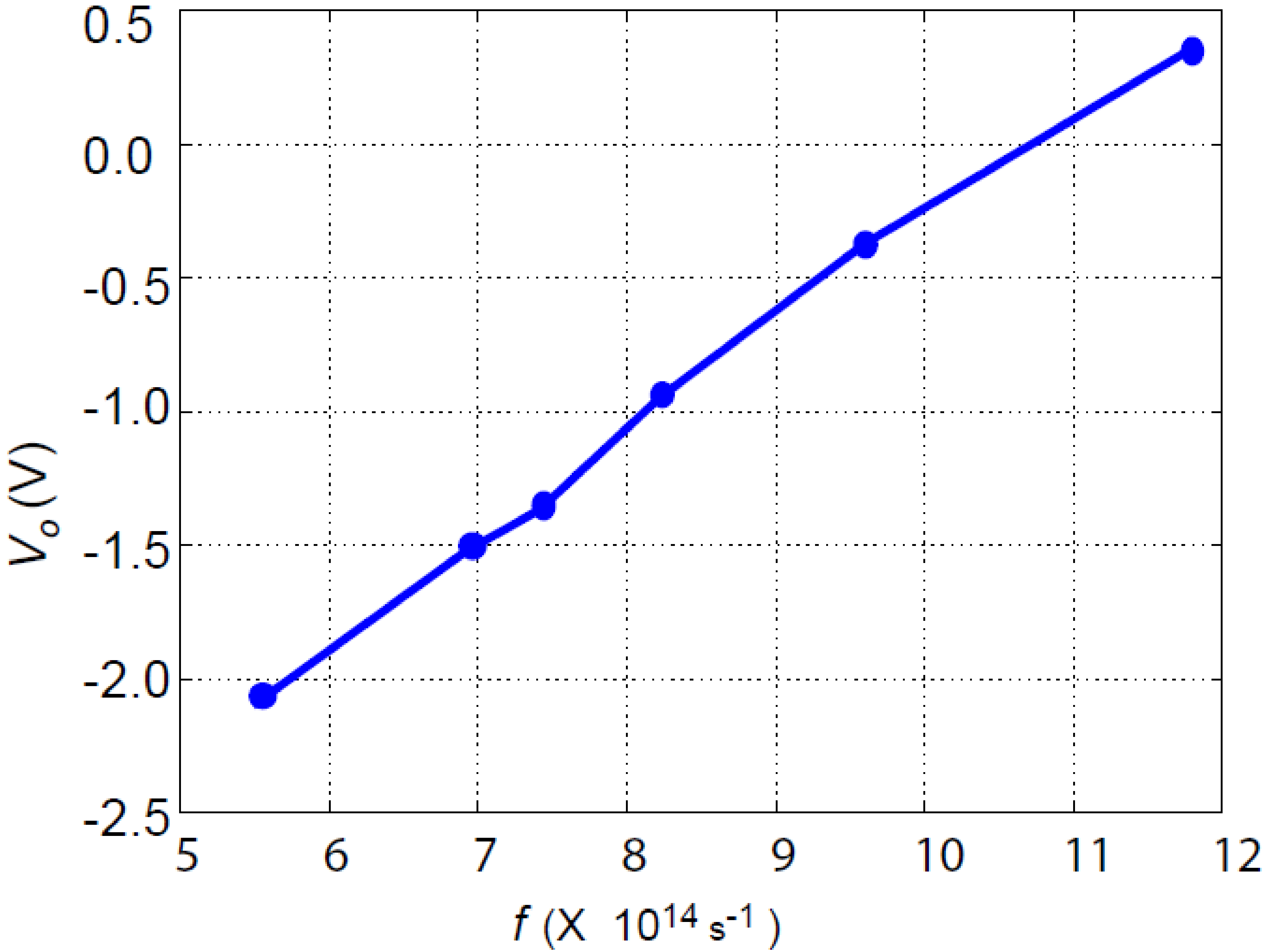
Plotting Experimental Data

Stopping voltage $V_o(\mathrm{V})$	**Wavelength** $\lambda\ (\mathring{\mathrm{A}})$	**Frequency** $f = c/\lambda$ $(\times 10^{14} s^{-1})$
-2.100	5466	5.488
-1.524	4339	6.914
-1.367	4047	7.413
-0.9478	3650	8.219
-0.3718	3126	9.597
+0.3720	2535	11.834

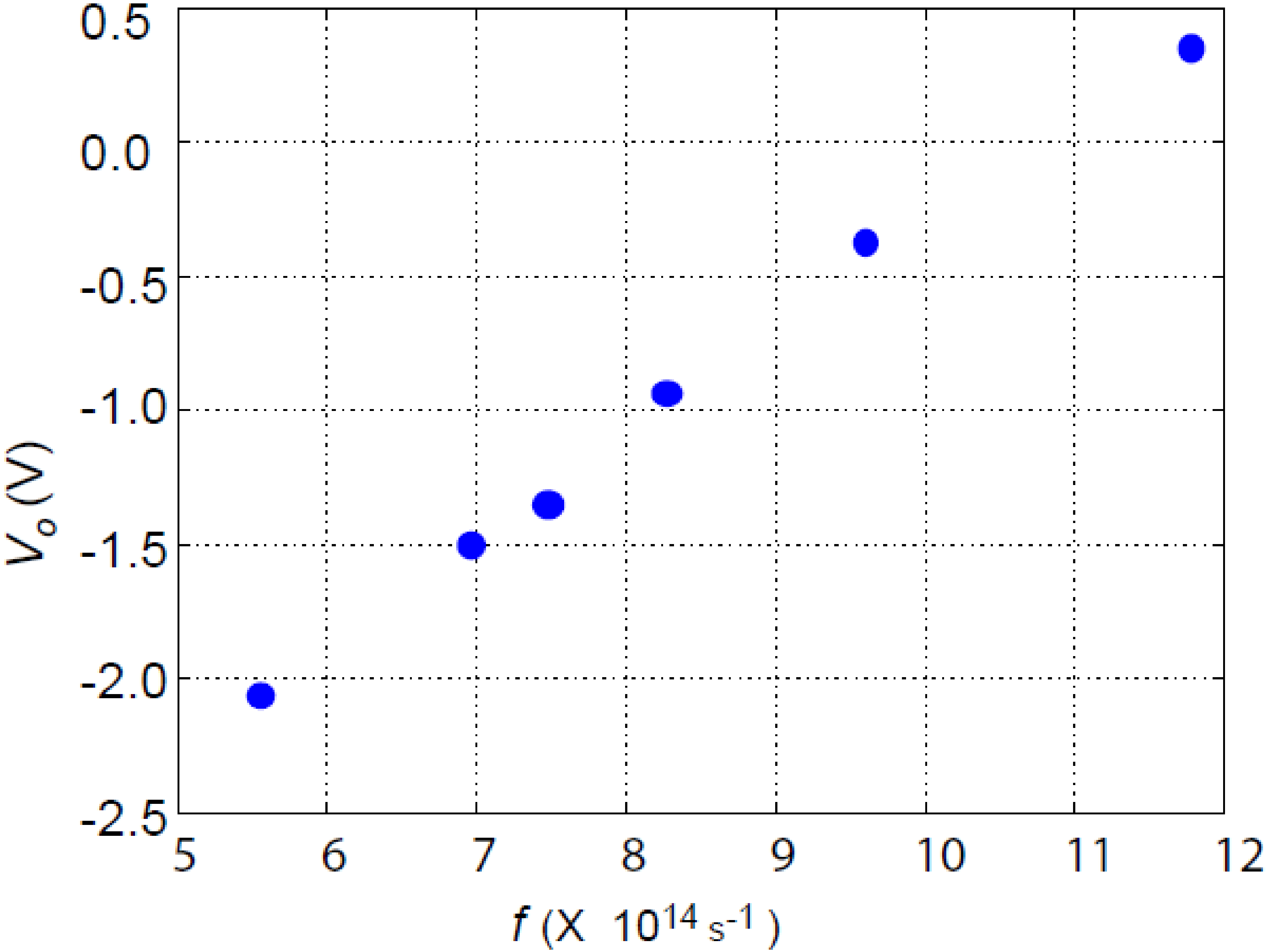
Possible Ways of Plotting
(a) Not acceptable



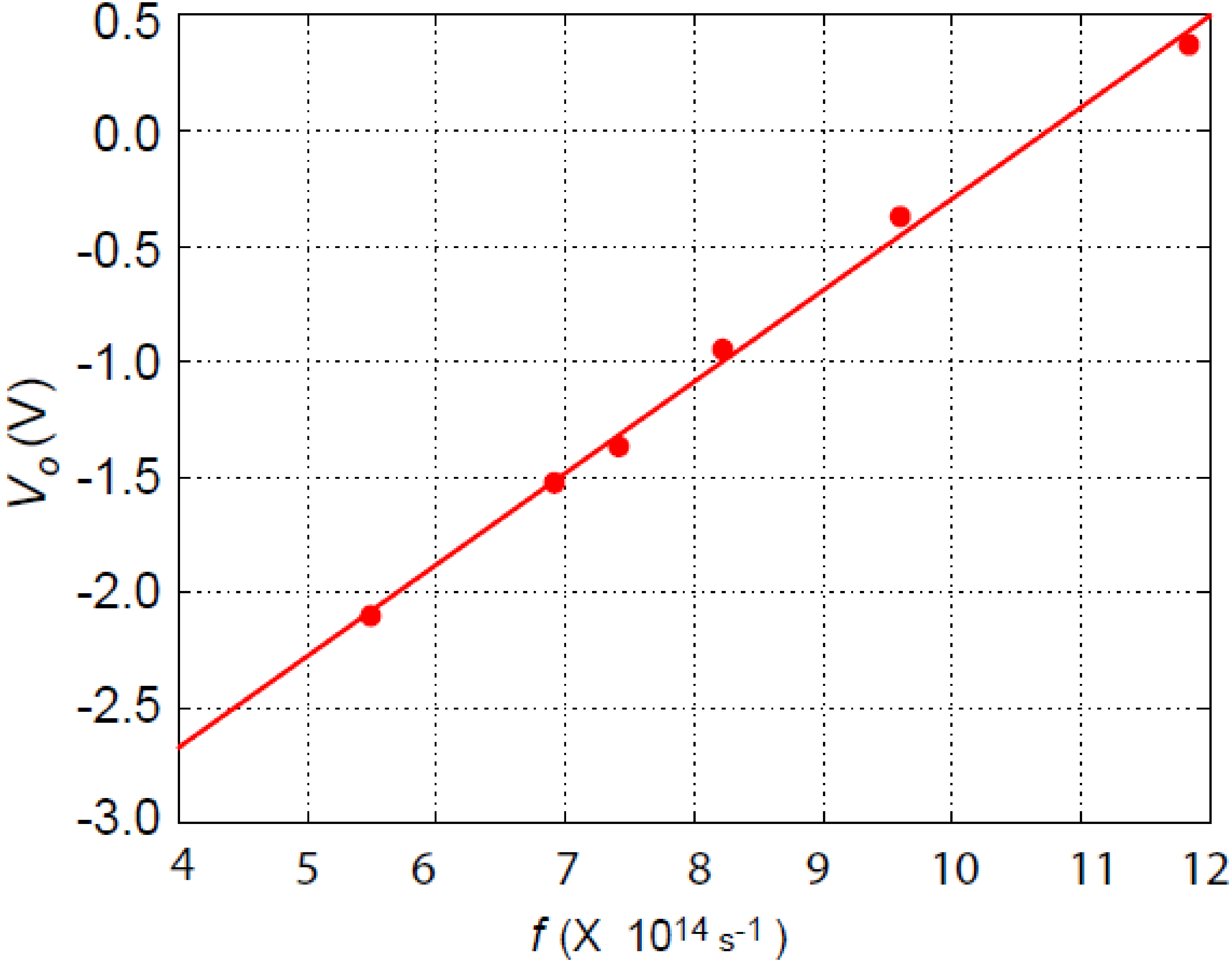
(b) Barely acceptable

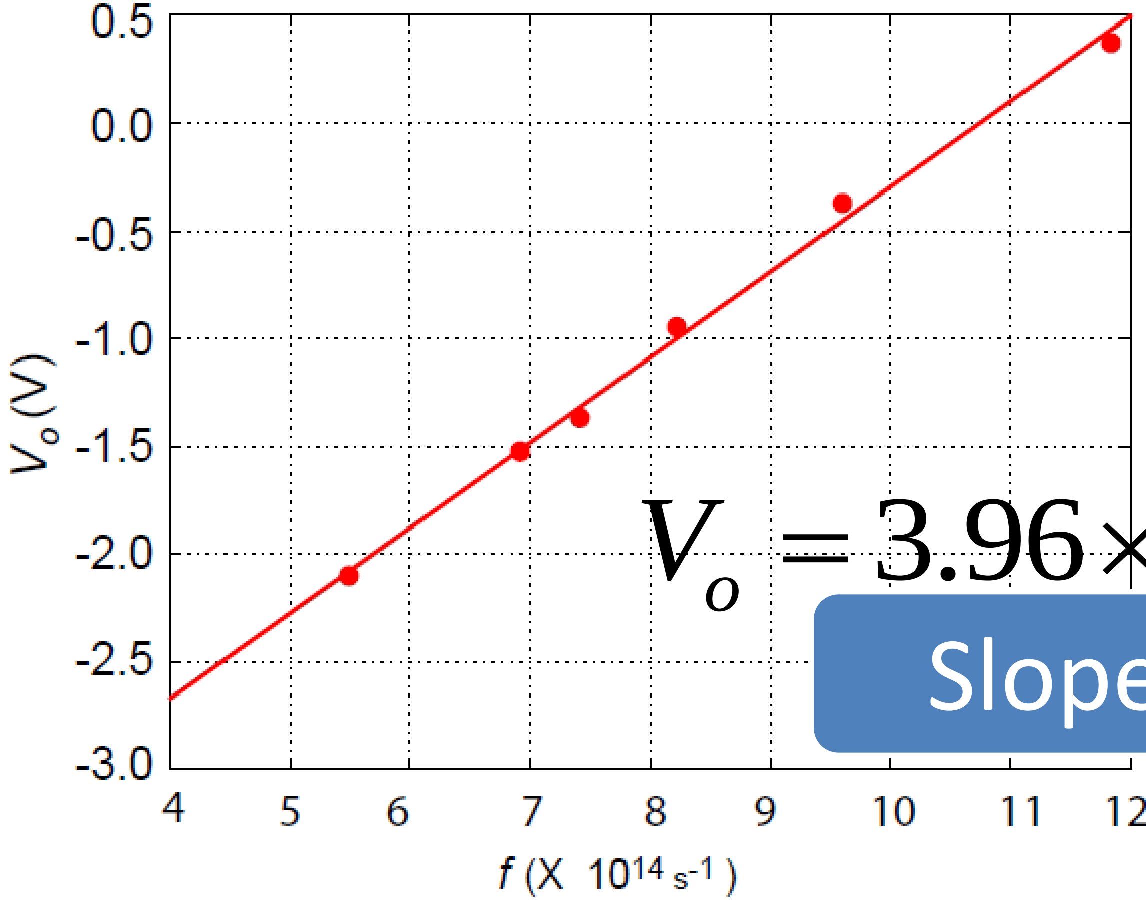


(c) Better



(d) Acceptable, good in all respects





$$V_o = \frac{h}{e} f - \frac{W}{e}$$

$$V_o = 3.96 \times 10^{-15} f - 4.25$$

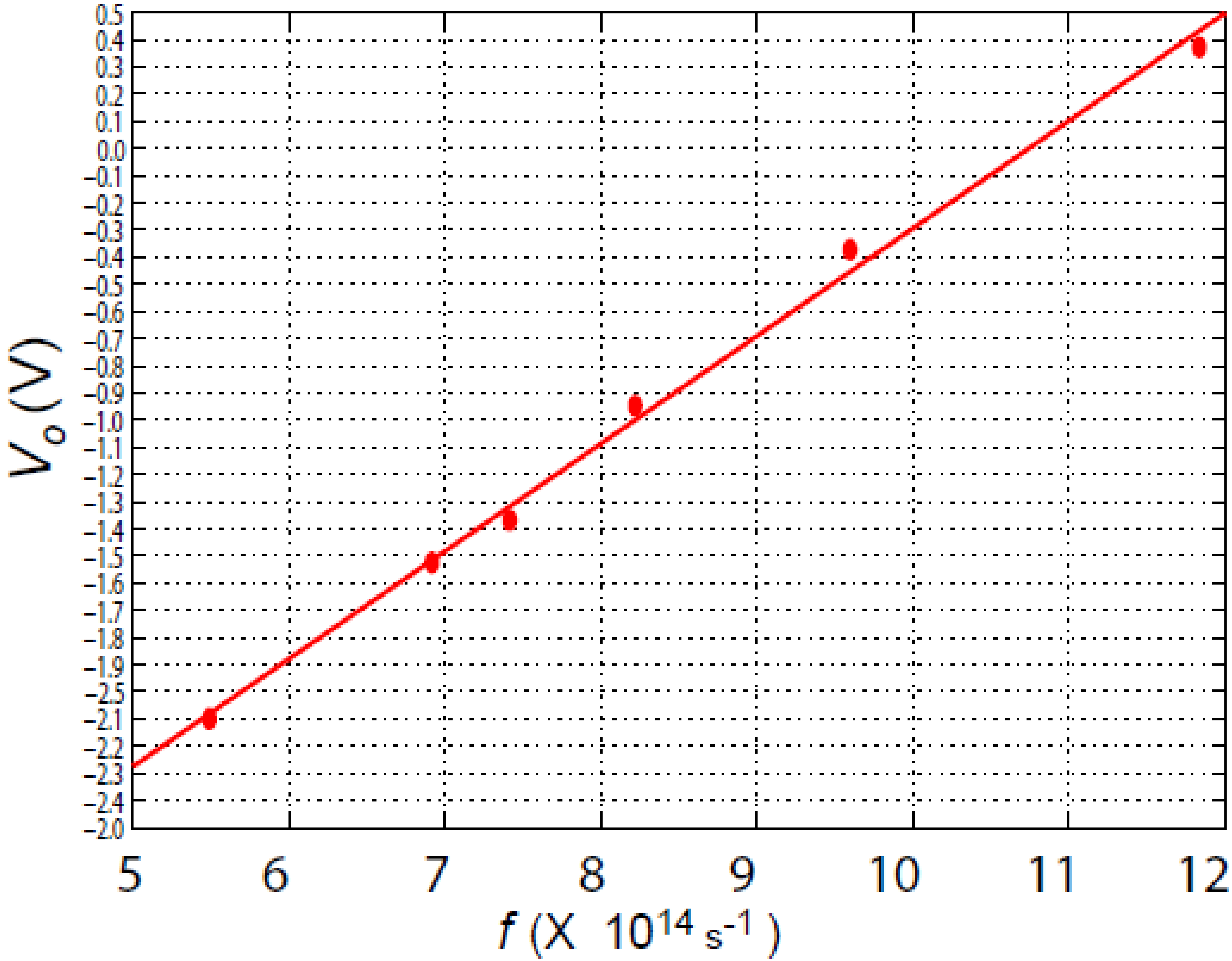
Slope

Intercept

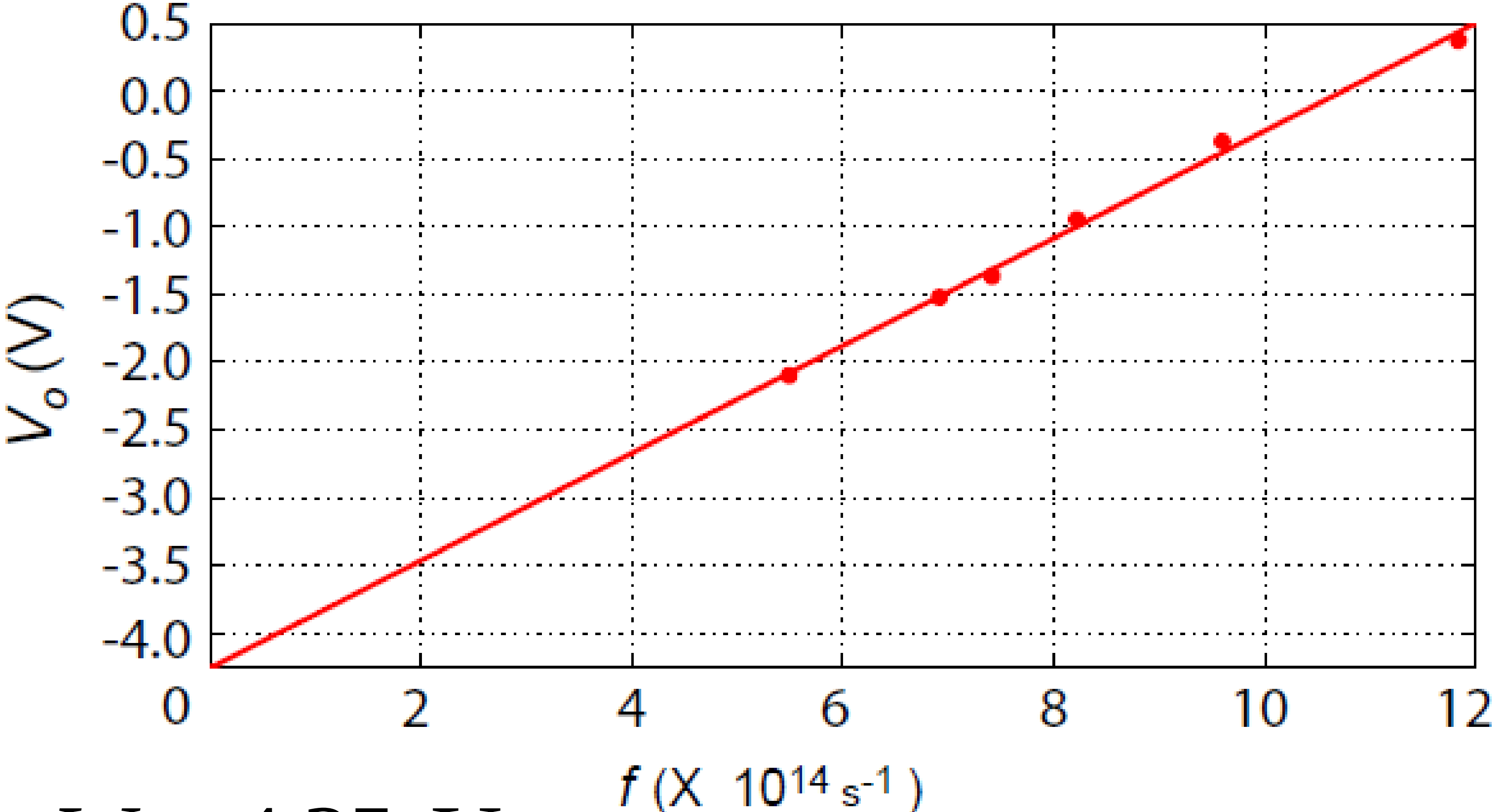
$$h = 6.34 \times 10^{-34}\,\mathrm{Js}$$

$$W = 4.25\,\mathrm{eV}$$

(e) Unnecessary detail or embellishment

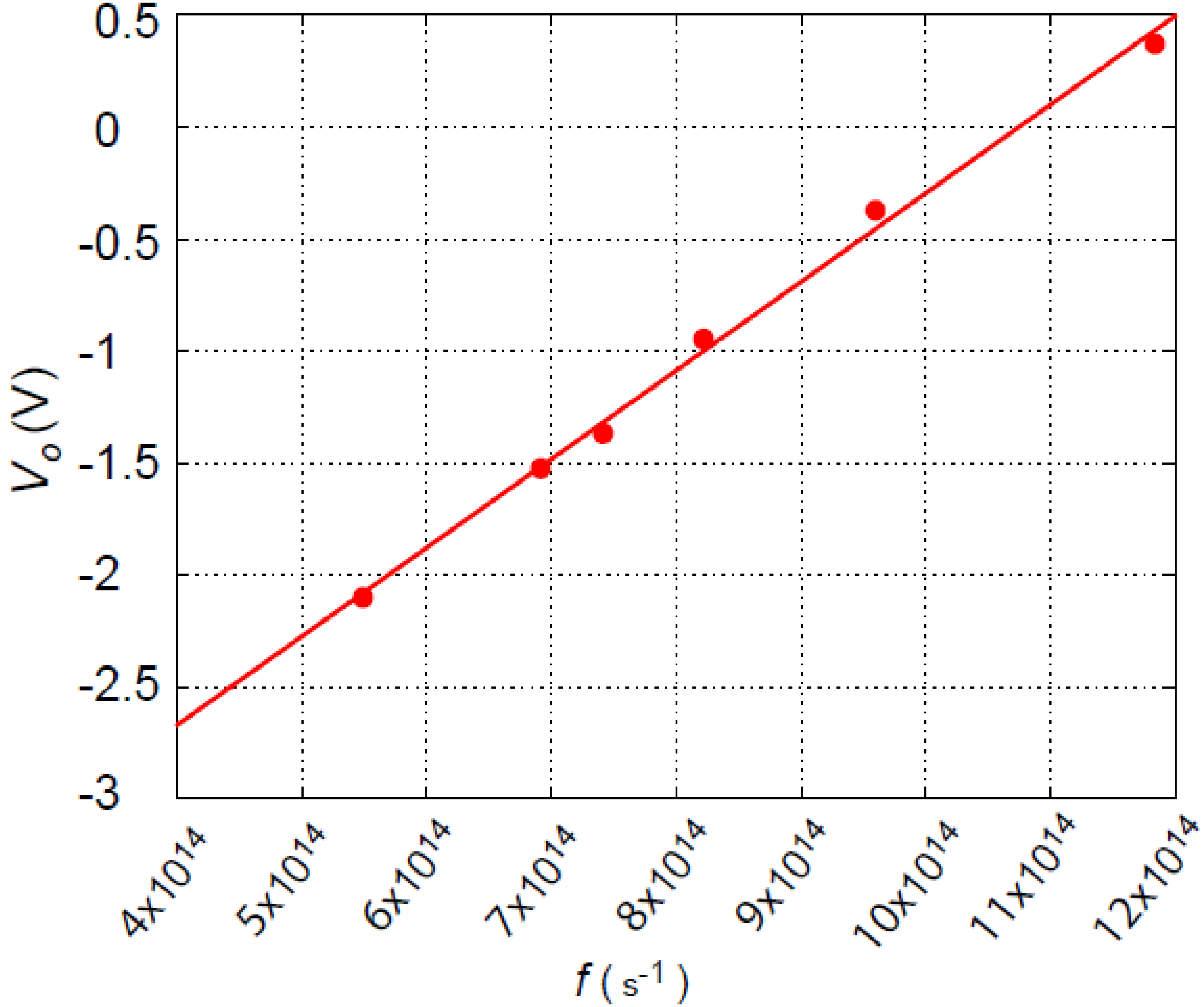


(f) Axes too long

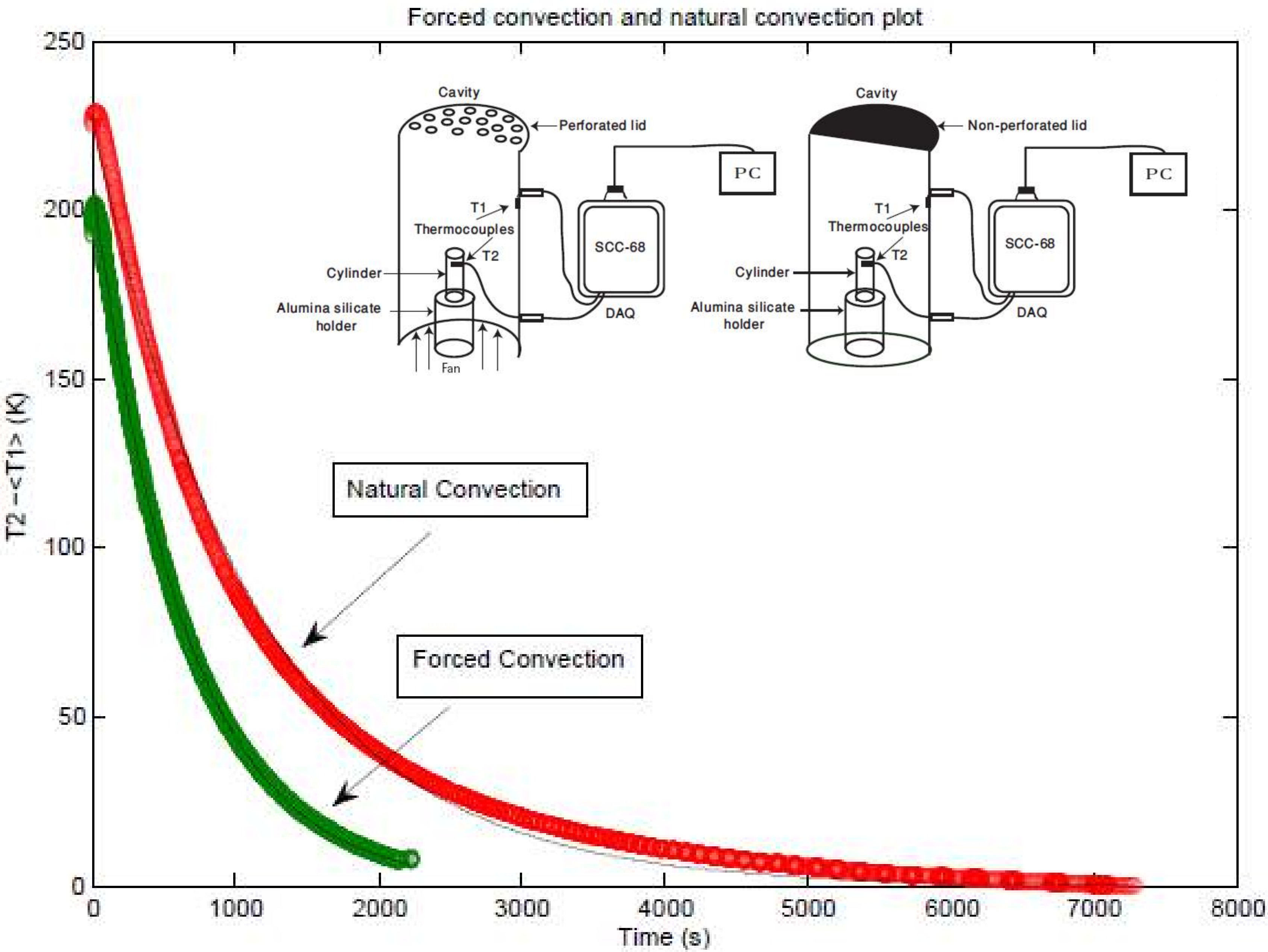


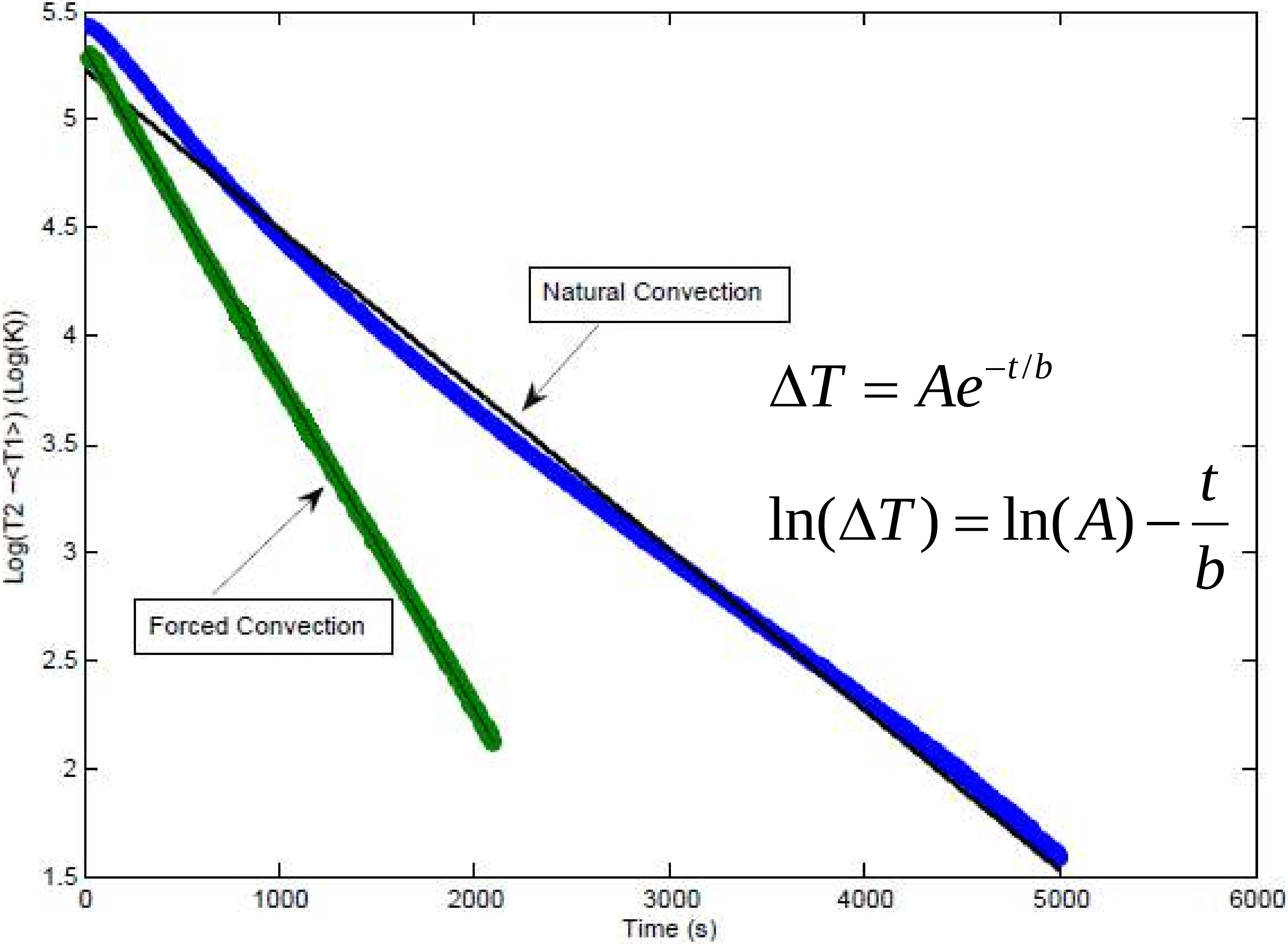
$$W = 4.25\text{eV}$$

(f) Axes tick marks are inconsistent and clumsy



Linearizing Plots: Cooling Objects

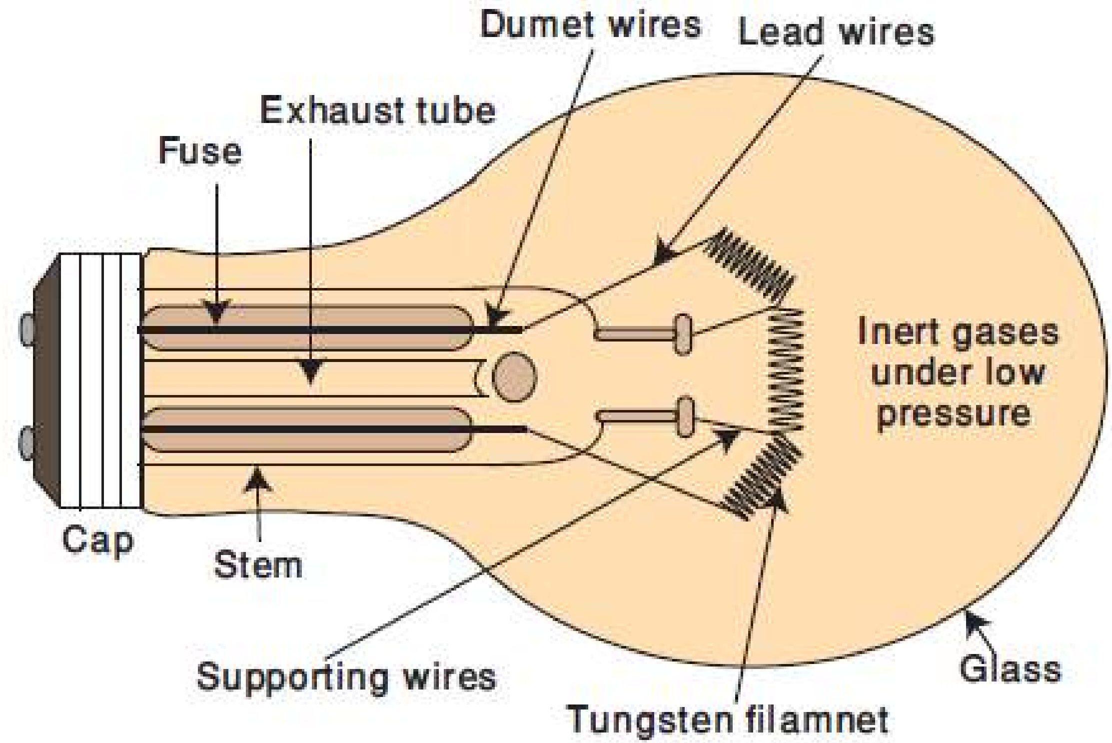




$$\Delta T = Ae^{-t/b}$$

$$\ln(\Delta T) = \ln(A) - \frac{t}{b}$$

Light Bulb (Power Law)

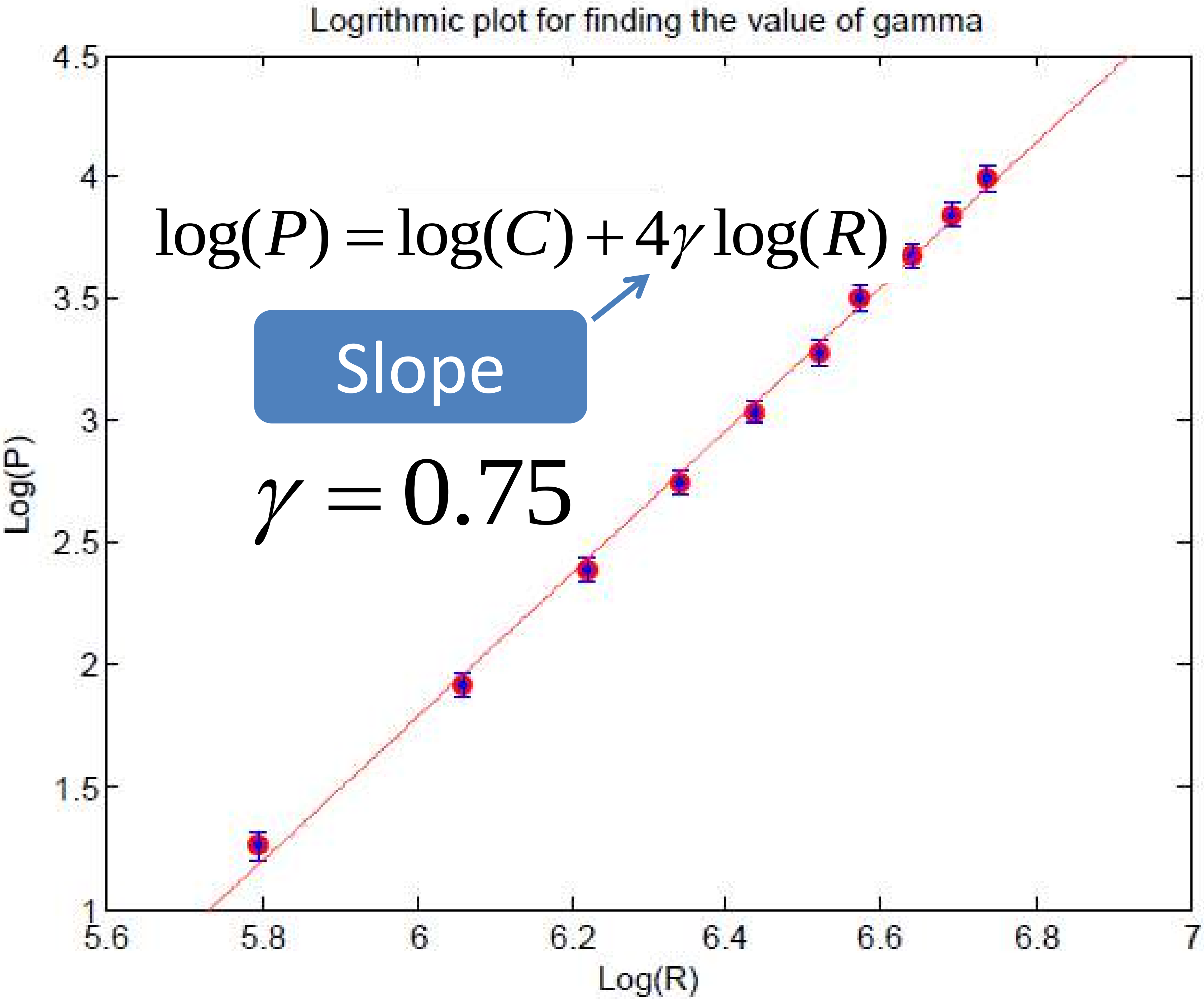


$$P_{\mathrm{elec}} = \sigma A T^4$$

$$T = R^{\gamma}$$

$$P_{\mathrm{elec}} = \sigma A R^{4\gamma} = C R^{4\gamma}$$

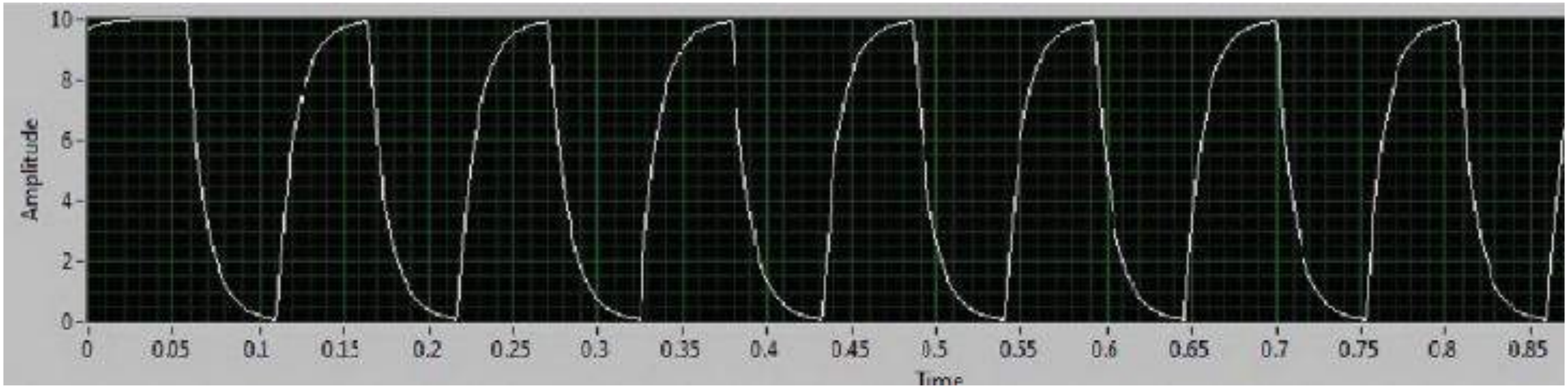
$$\log(P) = \log(C) + 4\gamma \log(R)$$

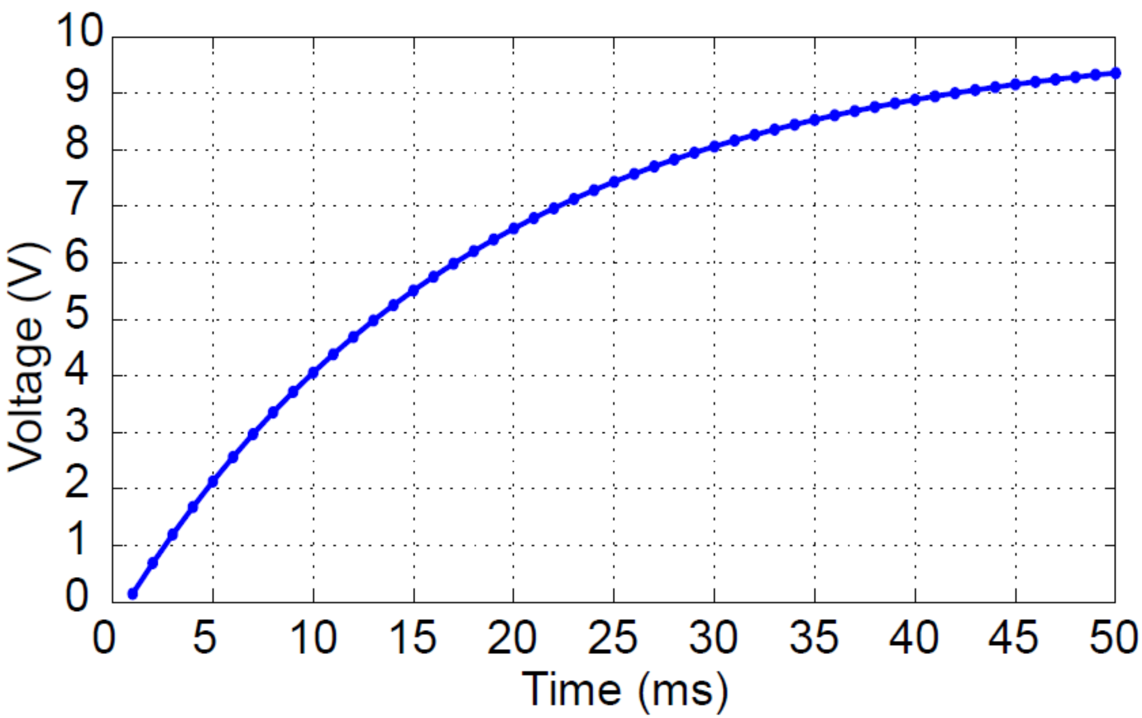


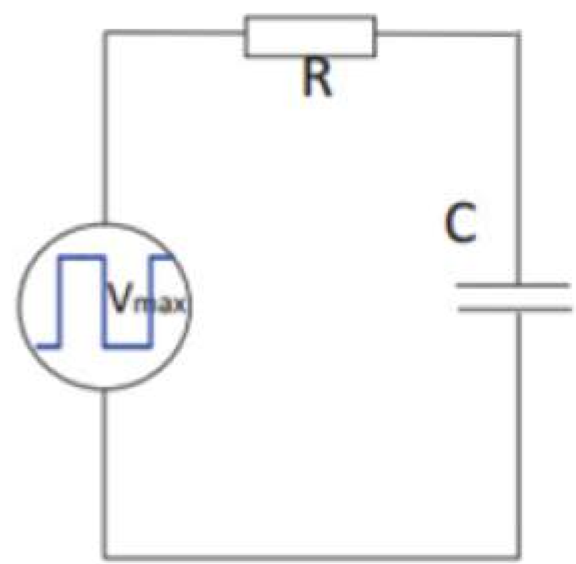
$$\log(P) = \log(C) + 4\gamma \log(R)$$

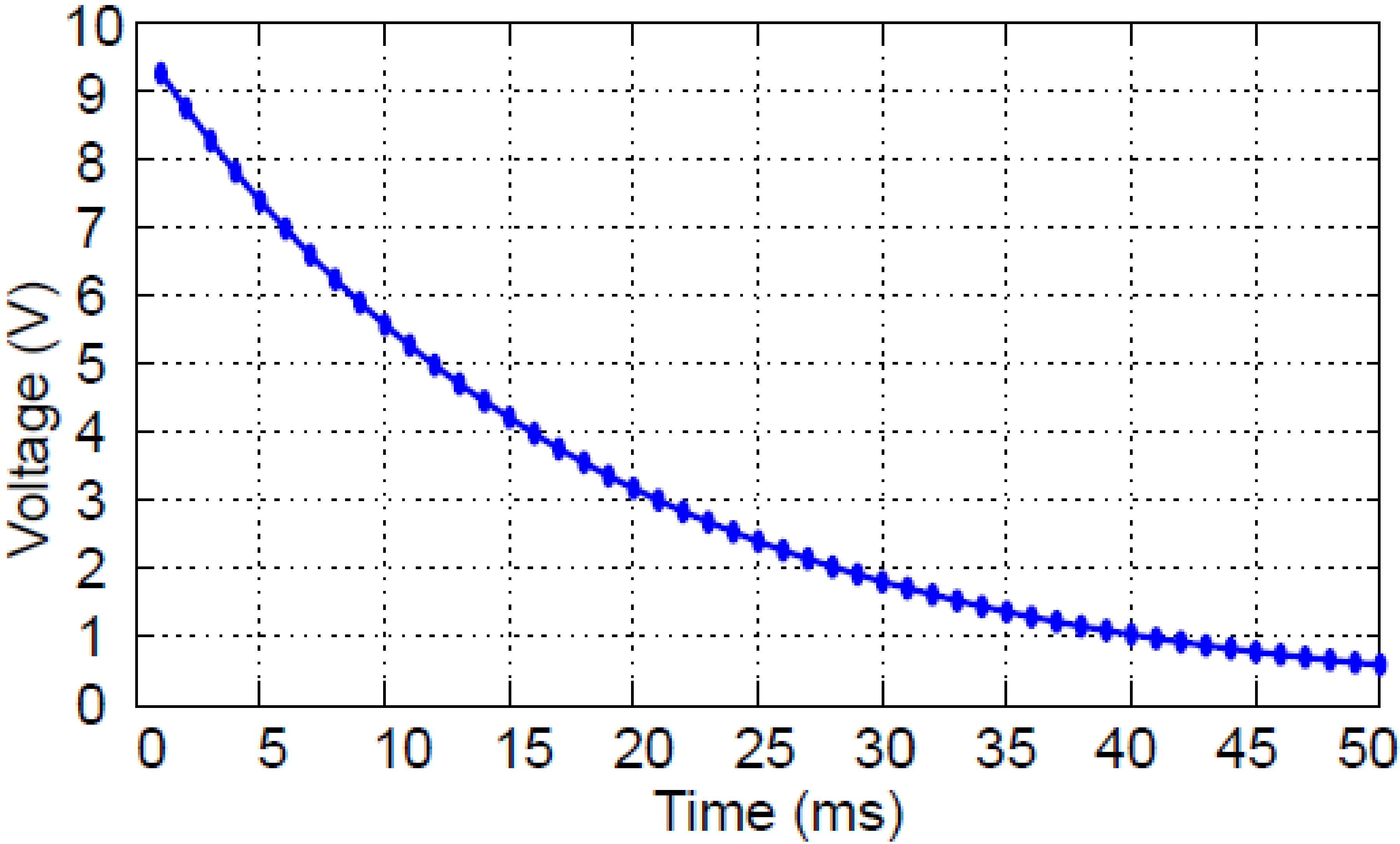
$$\gamma = 0.75$$

Piecewise graphs (voltage across capacitor)

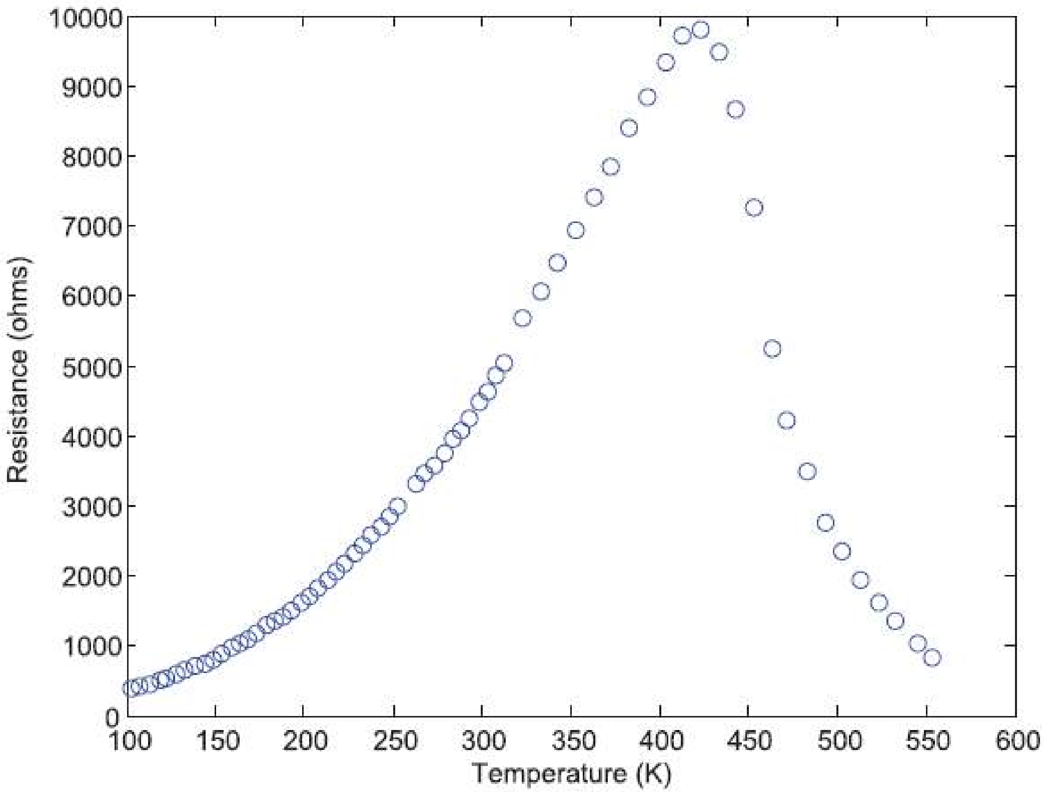


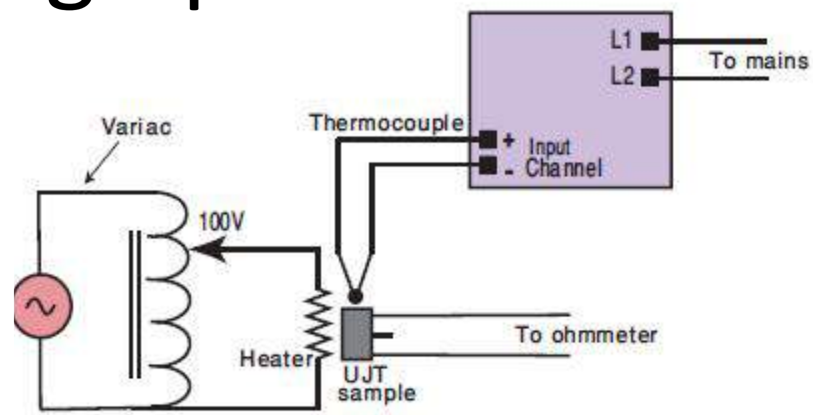




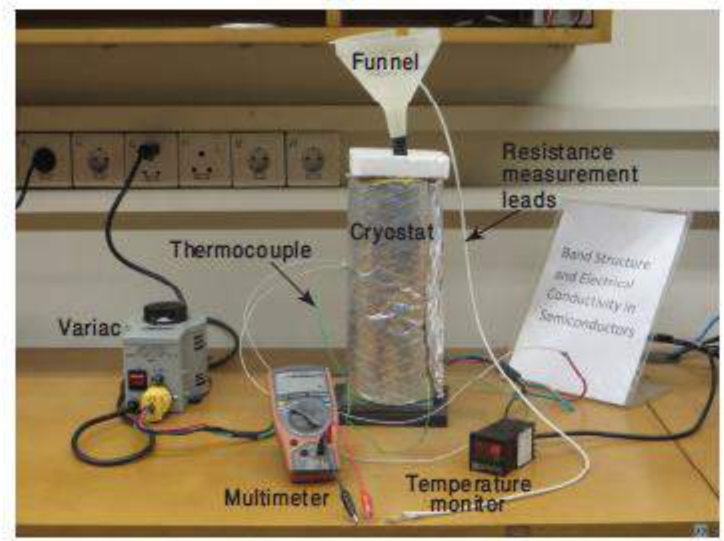

Voltage (V)
10
9
8
7
6
5
4
3
2
1
0
0
5
10
15
20
25
30
35
40
45
50
Time (ms)

Extracting regimes of diverse behavior from a graph

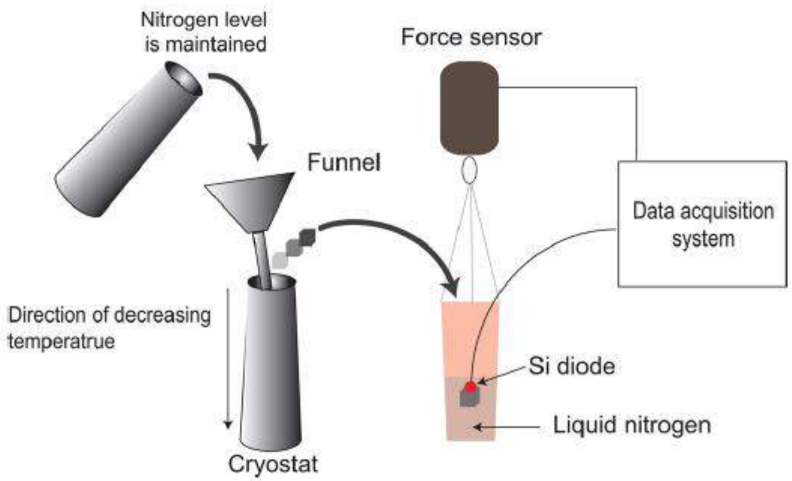


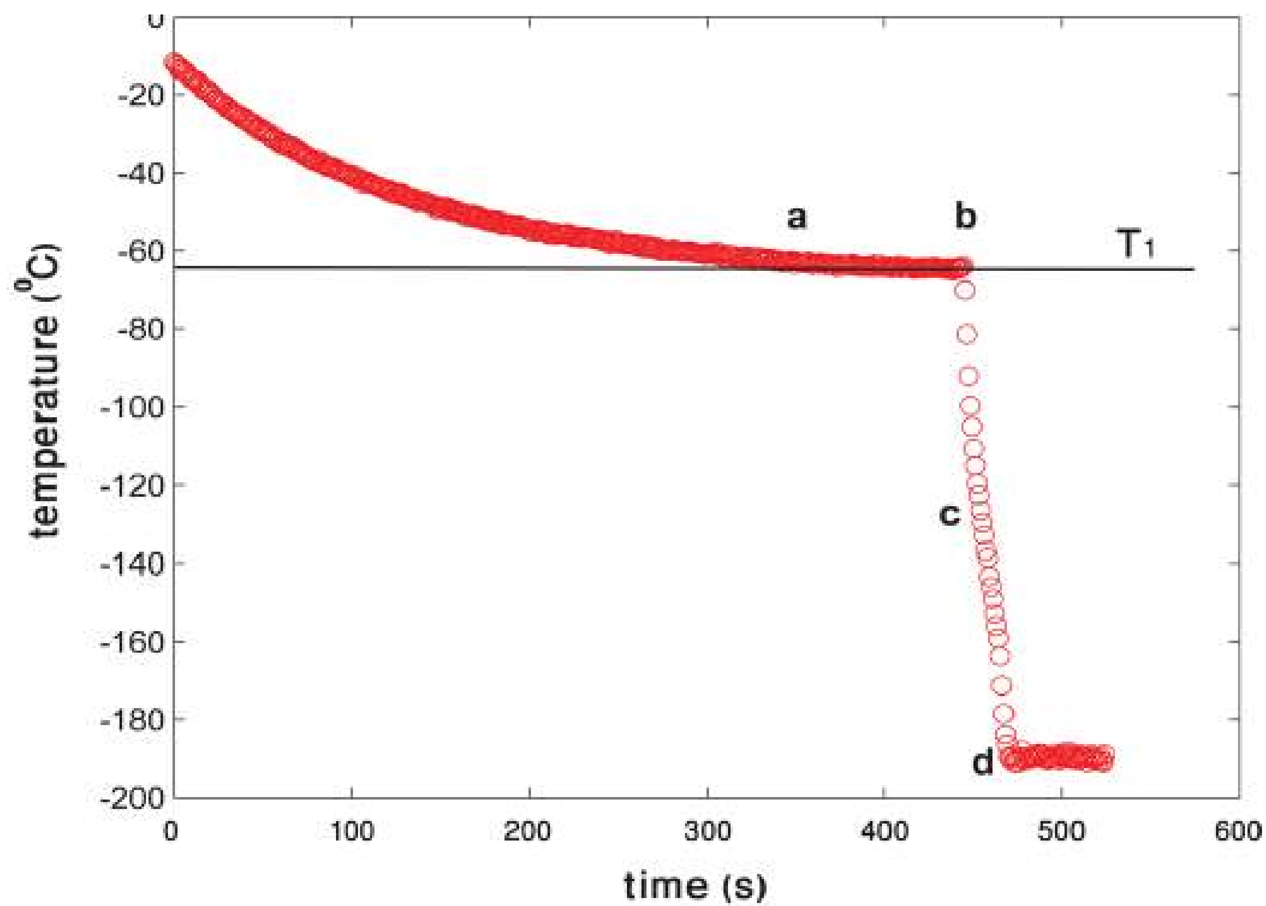


(b)

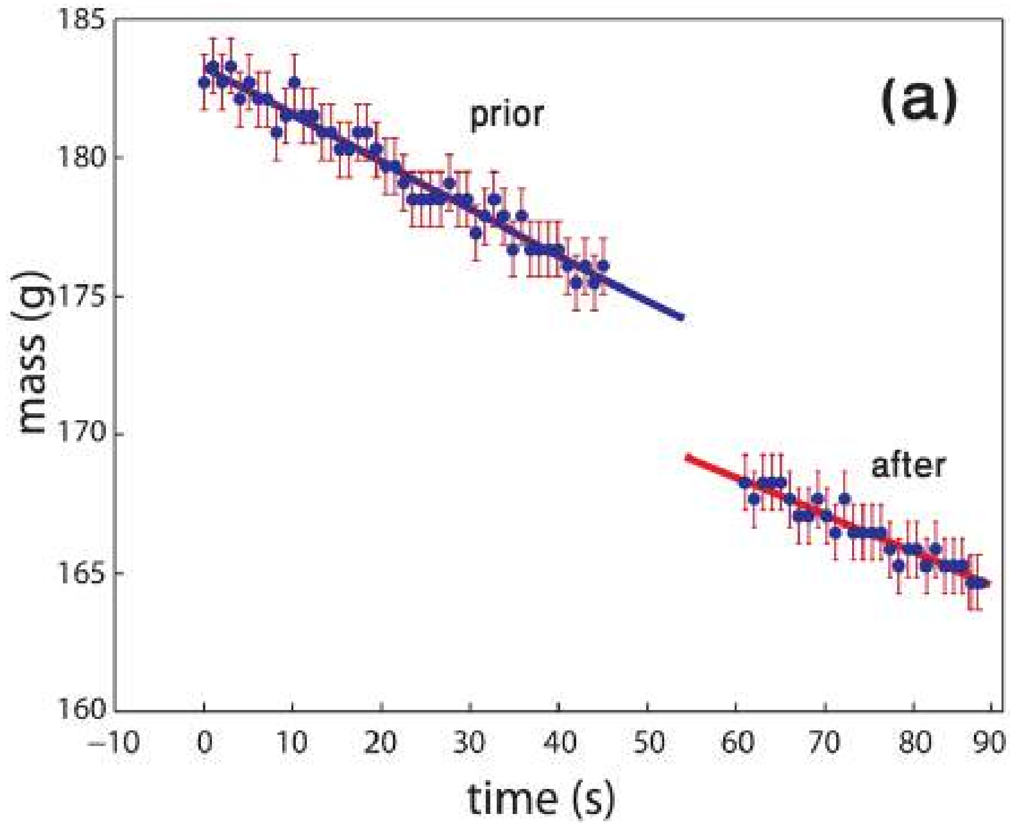


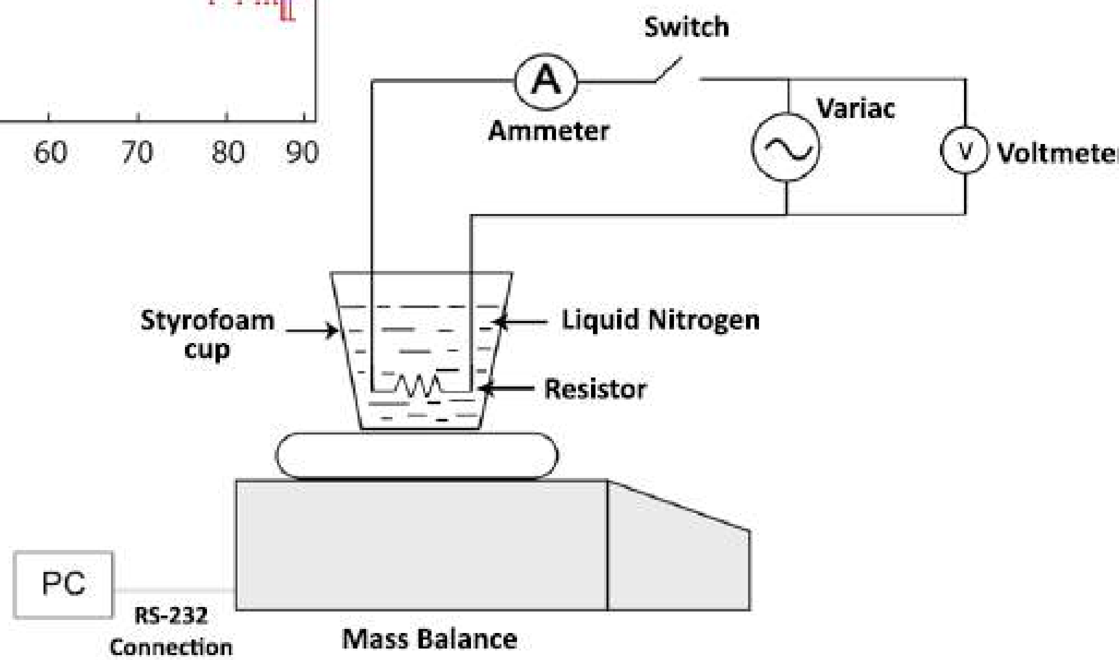
Controlled Discontinuities in experiments (example 1)



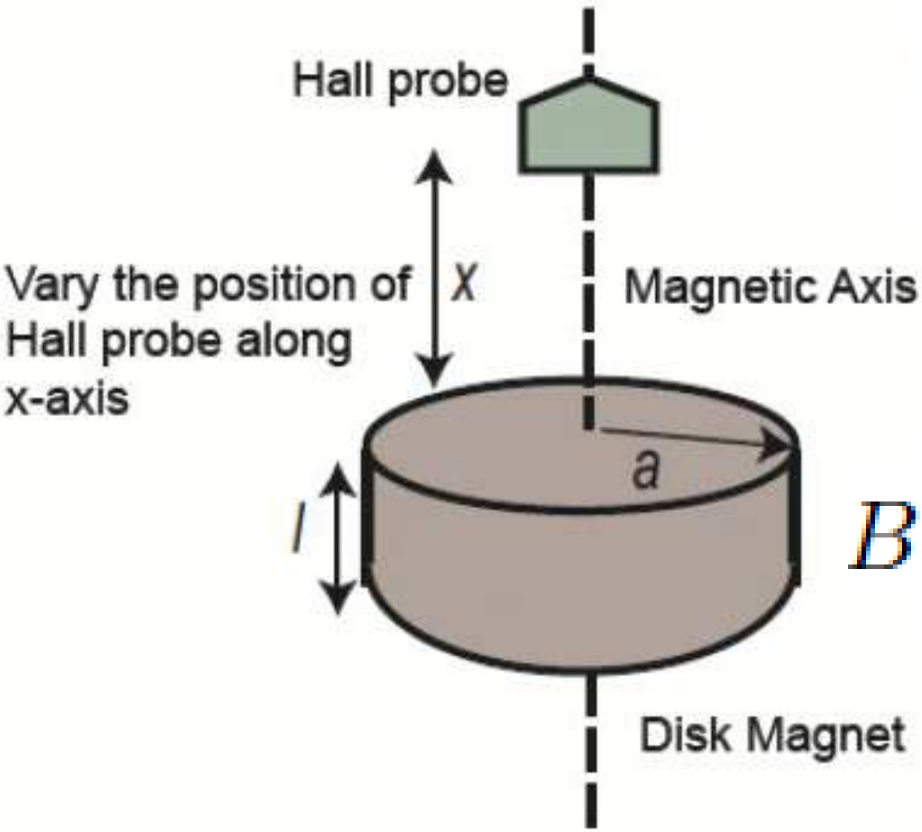


Controlled Discontinuities in experiments (example 2)





When does a model breakdown?

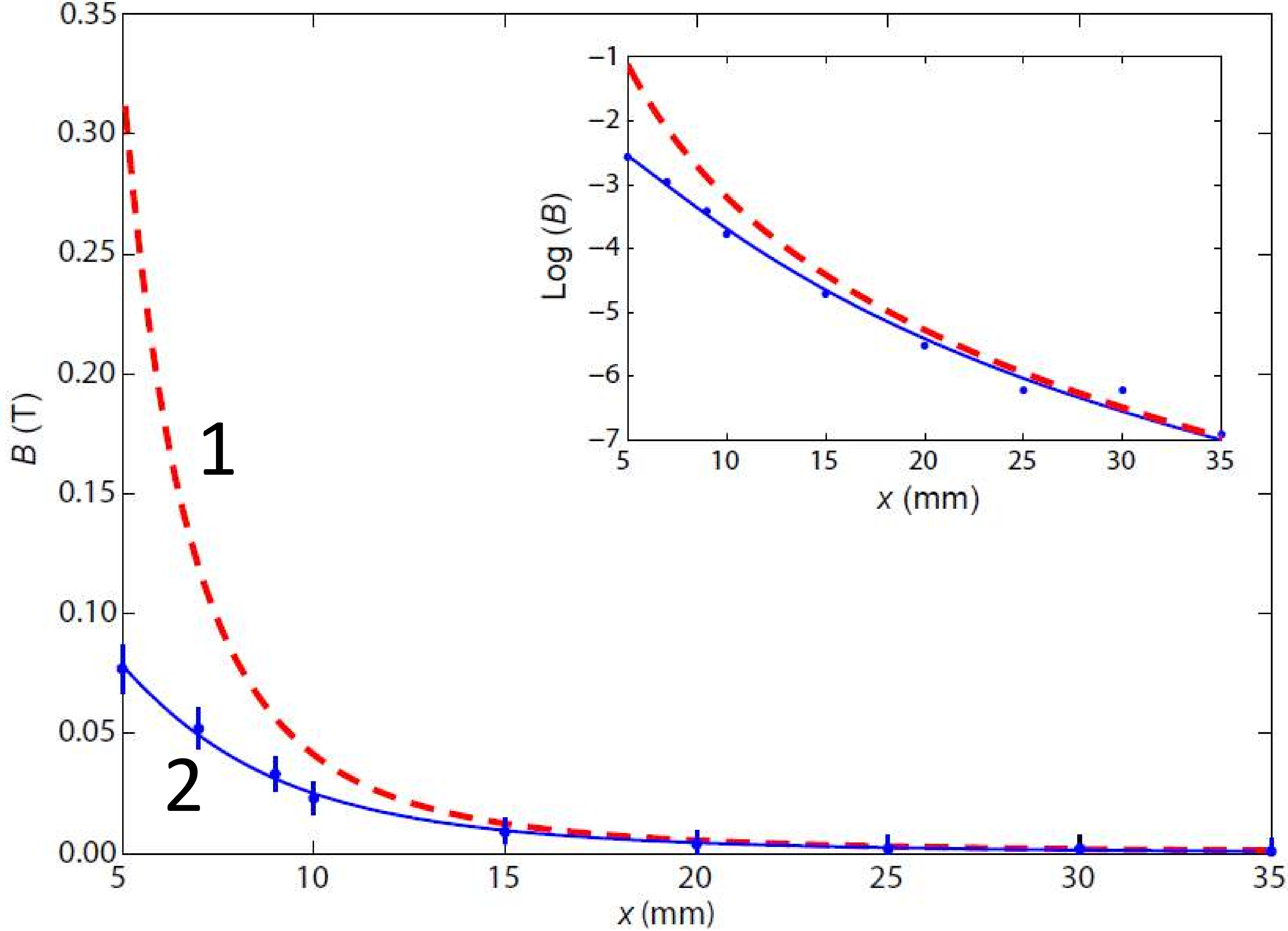


$$B = \frac{\mu_o m}{2\pi x_o^3}$$

Model 1

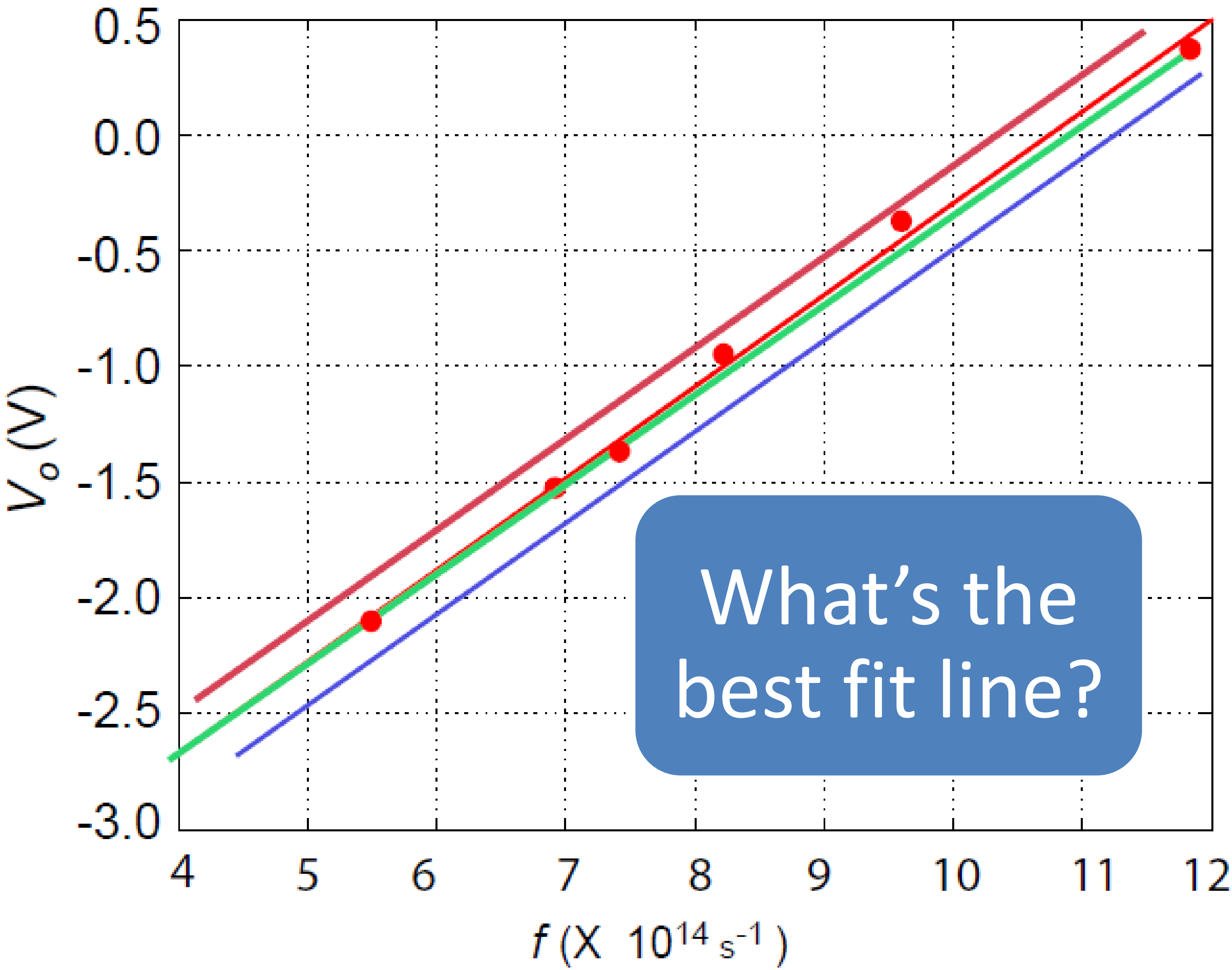
$$B = \frac{\mu_o}{2} M \left[\frac{x_o + d/2}{\left(a^2 + (x_o + d/2)^2\right)^{1/2}} - \frac{x_o - d/2}{\left(a^2 + (x_o - d/2)^2\right)^{1/2}} \right]$$

Model 2



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
B (T)
5
10
15
20
25
30
35
x (mm)
1
2
−1
−2
−3
−4
−5
−6
−7
Log (B)
5
10
15
20
25
30
35
x (mm)

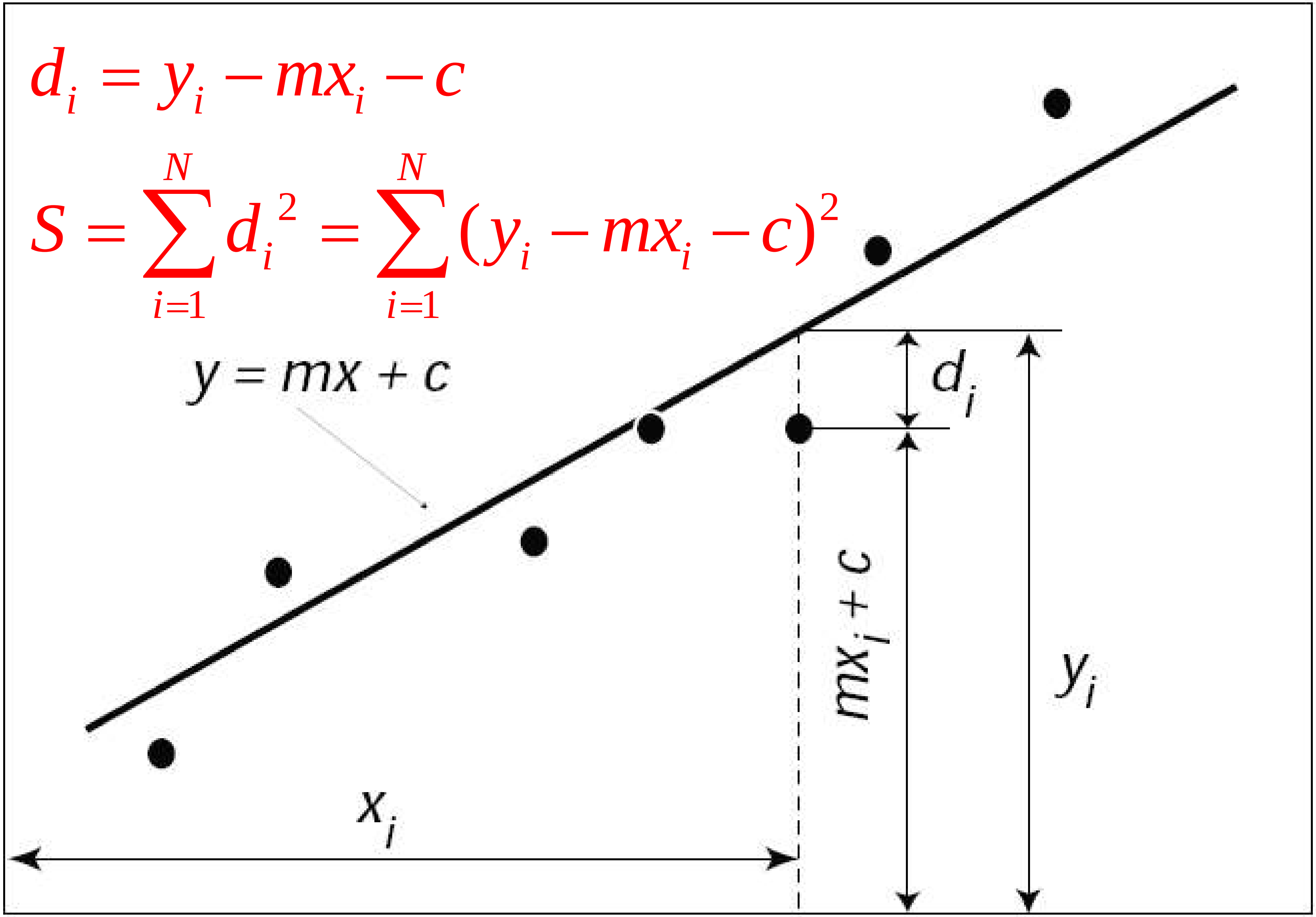
Fitting Experimental Data to a Model



Best fit in the least squares sense

$$d_i = y_i - mx_i - c$$

$$S = \sum_{i=1}^{N} d_i^{\,2} = \sum_{i=1}^{N} (y_i - mx_i - c)^2$$



origin

Least squares curve fitting is minimization of S

$$\frac{\partial S}{\partial m} = -2\sum[x_i(y_i - mx_i - c)] = 0$$

$$\frac{\partial S}{\partial c} = -2\sum(y_i - mx_i - c) = 0$$

$$m = \frac{\sum y_i(x_i - \bar{x})}{\sum(x_i - \bar{x})^2}$$

$$c = \bar{y} - m\bar{x}$$

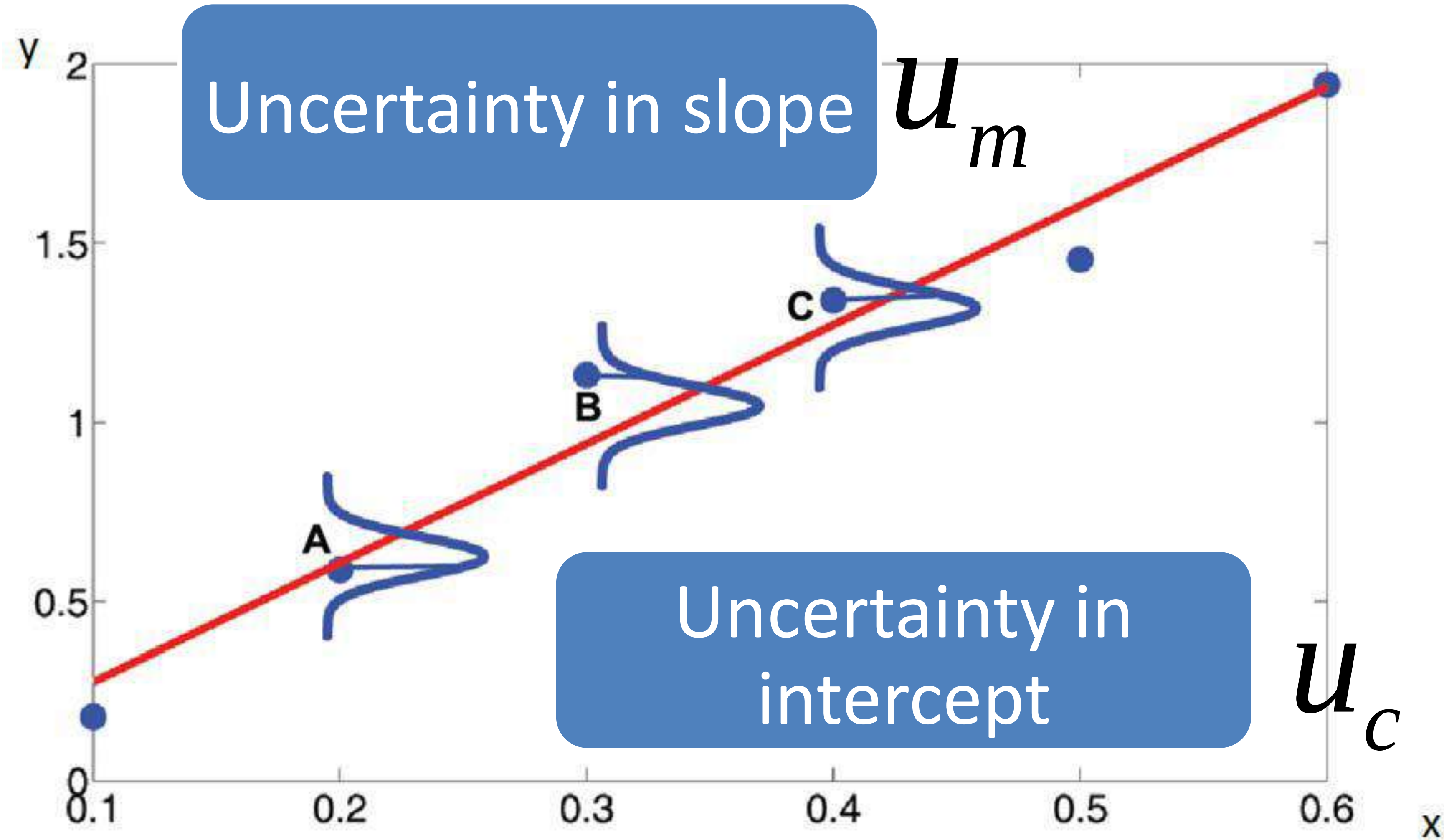
Minimization

Simultaneous Equations

Least-squares fit

Detailed derivation (Available in the Data Processing Guide on the Physlab website).

Uncertainty in the least-squares fit



Uncertainty in slope and intercept

Uncertainty in slope

$$u_m = \sqrt{\frac{\sum_i^N d_i^2}{D(N-2)}}$$

Uncertainty in intercept

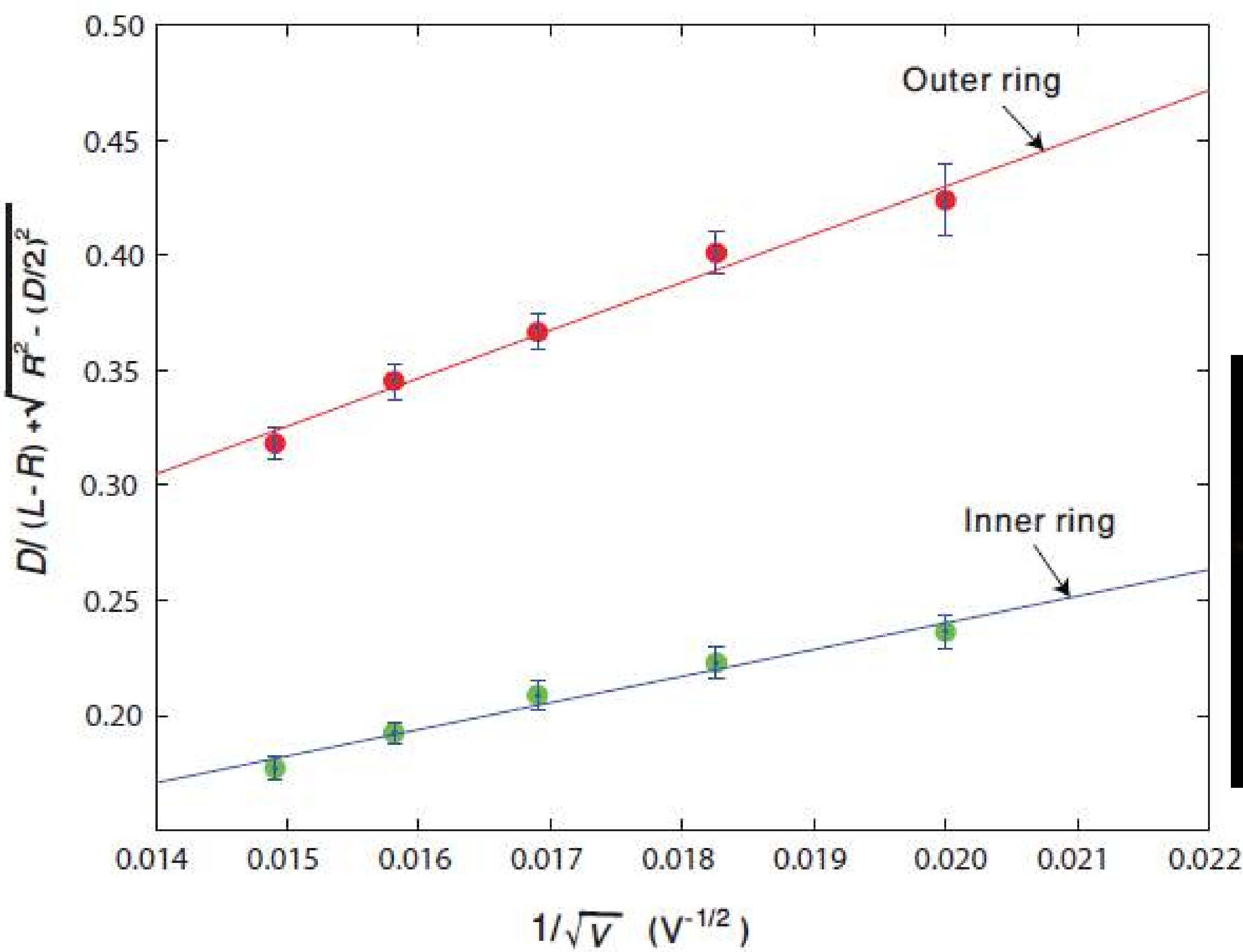
$$u_c = \sqrt{\left(\frac{1}{N} + \frac{\bar{x}^2}{D}\right)\left(\frac{\sum_i^N d_i^2}{(N-2)}\right)}$$

$$d_i = y_i - mx_i - c,$$

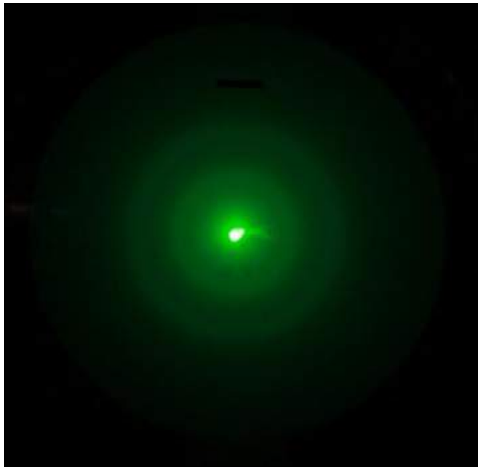
$$D = \sum_i^N (x_i - \bar{x})^2.$$

How Does Uncertainty in Data Creep in?

$$\left[\frac{D}{\left(L - R + \sqrt{R^2 - (D/2)^2}\right)}\right] = \left[\frac{2(1.23 \times 10^{-9}\ \mathrm{m})}{d}\right]\frac{1}{\sqrt{V}}$$

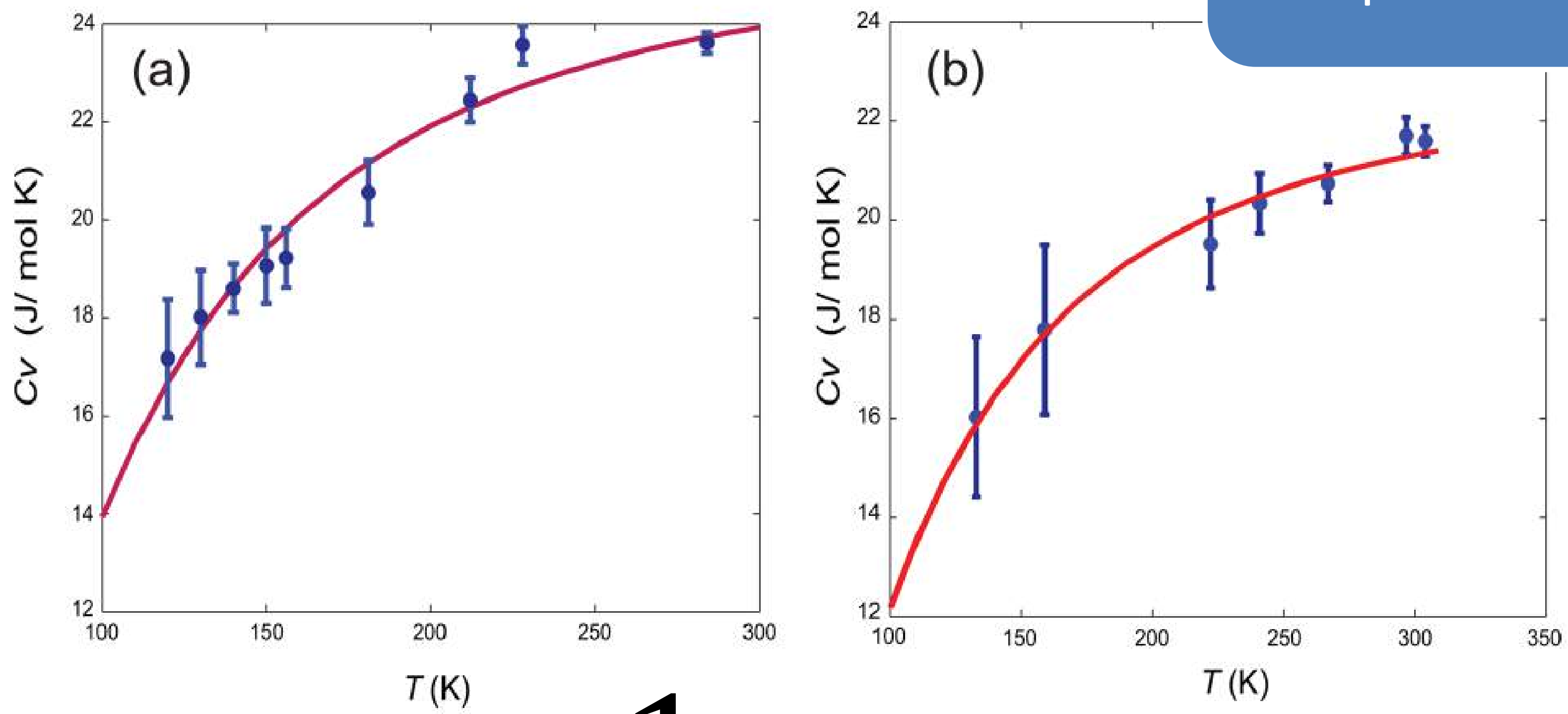


Diffraction of electrons from graphite



Concept of weighted fits

Measurement of heat capacities



$$w_i = \frac{1}{{u_i}^2}$$

Each data point is weighted differently.

Formulas for weighted fits

Slope and intercept

$$m = \frac{\Sigma_i w_i \, \Sigma_i w_i (x_i y_i) - \Sigma_i (w_i x_i) \, \Sigma_i (w_i y_i)}{\Sigma_i w_i \, \Sigma_i (w x_i^2) - (\Sigma_i w_i x_i)^2},$$

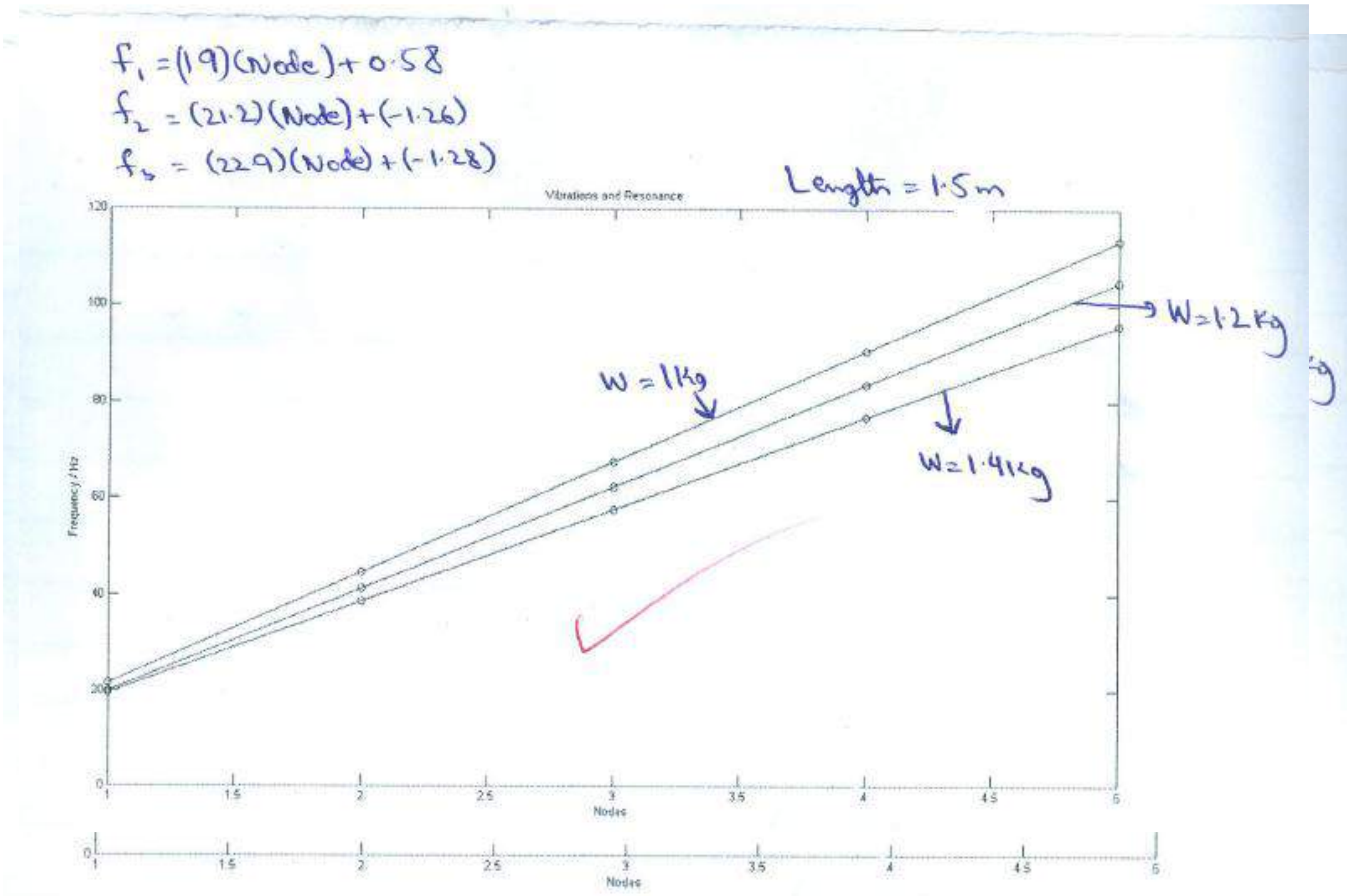
$$c = \frac{\Sigma_i (w_i x_i^2) \, \Sigma_i (w_i y_i) - \Sigma_i (w_i x_i) \, \Sigma_i (w_i x_i y_i)}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}$$

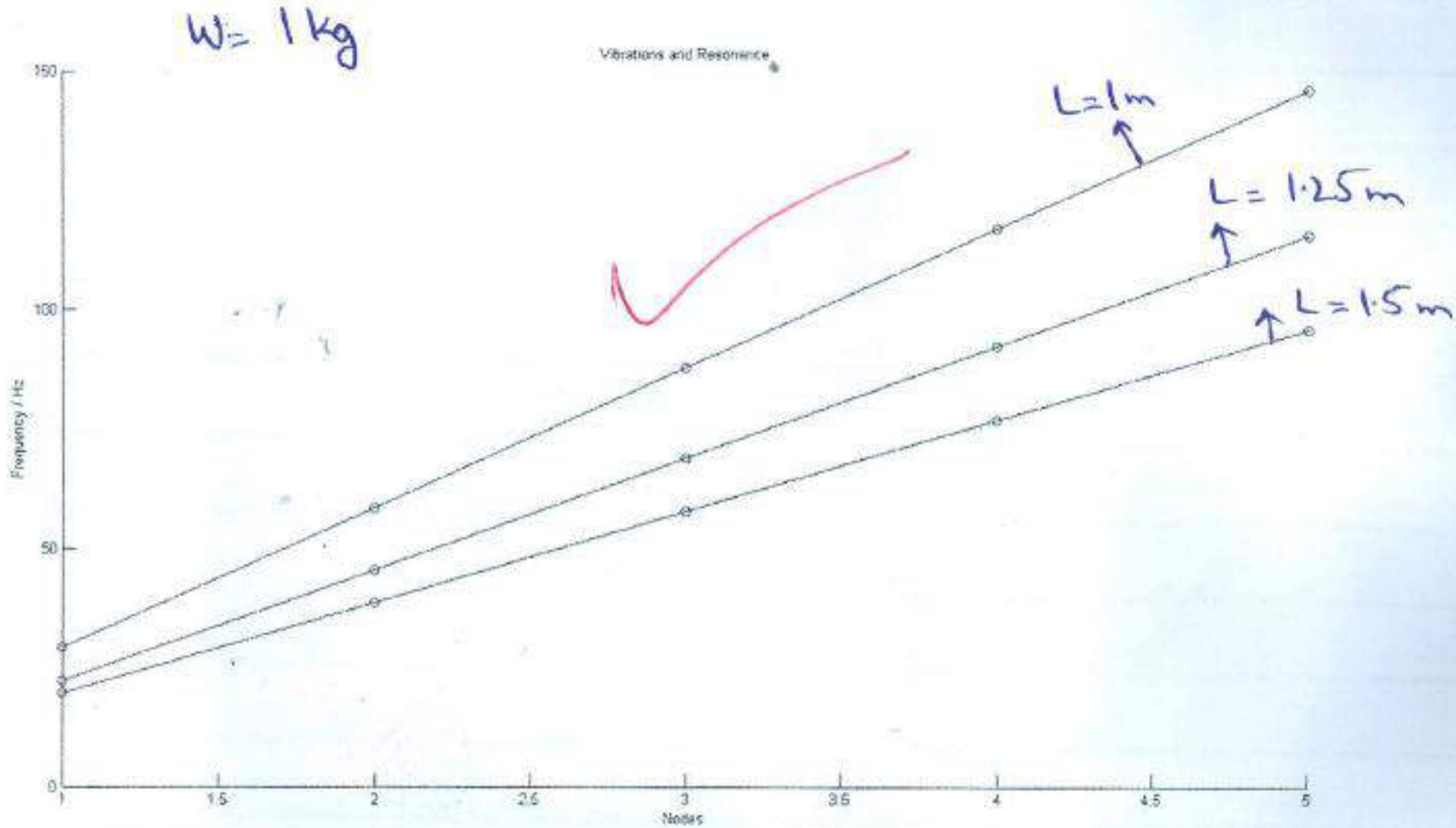
Uncertainties therein

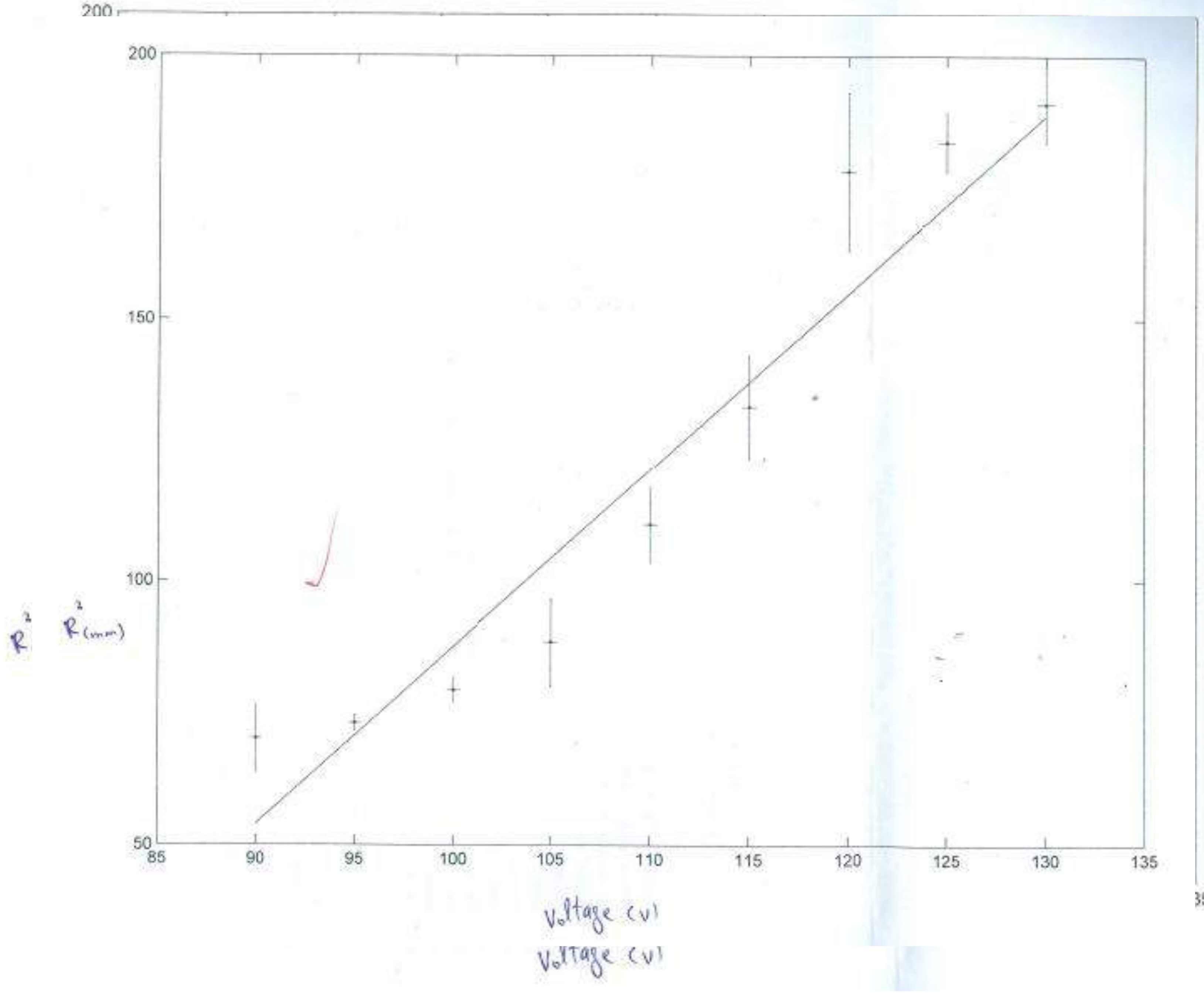
$$u_m = \sqrt{\frac{\Sigma_i w_i}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}},$$

$$u_c = \sqrt{\frac{\Sigma_i (w_i x_i^2)}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}}.$$

Sample Work from Students of PHY100/200




W = 1 kg
Vibrations and Resonence
L=1m
L = 1·25 m
L=1·5 m
Frequency / Hz
Nodes
150
100
50
0
1
1.5
2
2.5
3
3.5
4
4.5
5


200
150
100
50
85
90
95
100
105
110
115
120
125
130
135
R² (mm)
Voltage (v)

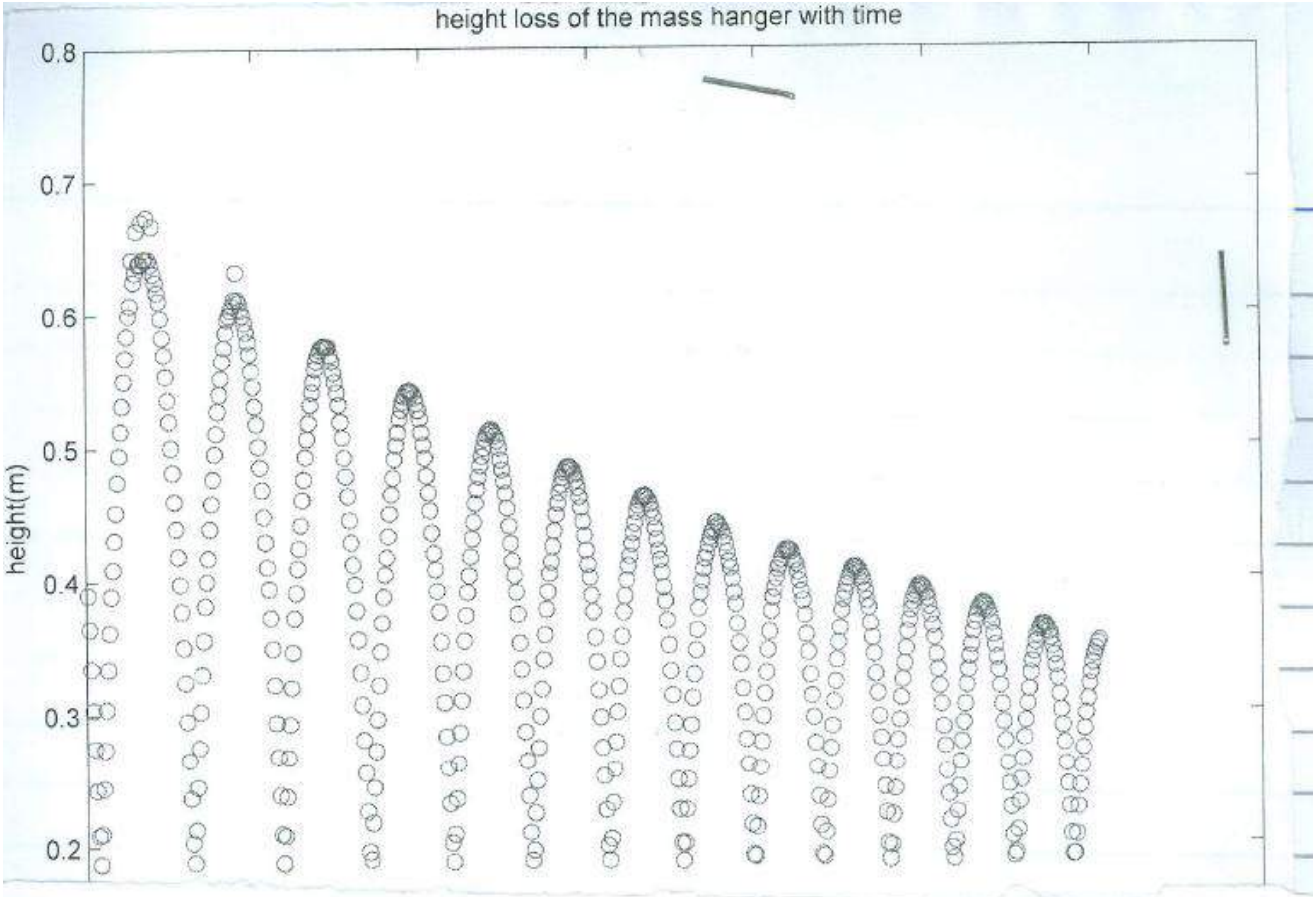
```
clc;
clear all;
close all
v=[90:5:130];
r= [8.379,8.555,8.908,9.408,10.525,11.554,13.3476,13.5534, 13.818];
r2 = r.^2;
plot(v,r2,'.')
dr = [0.382,0.0882,0.1323,0.441,0.3381,0.4263,0.5630,0.20433,0.26019]; %uncertainities
in r
a = v.*0.01;                                  %Um ratings of v
p = sqrt((0.3).^2 + (a).^2);                   %uncertainities in v
q= (2.*r).*dr;                      %uncertainities in r square by formula deltaq = 2rdeltar
Utrans = 0.32.*p;                   %Utrans 0.32 is slope from graph
Utotal = sqrt((q).^2+(Utrans).^2); %Uncertainities of rsquare after transformation
w = 1./(Utotal.^2)                  %weight of uncertainities

%calculation of final slope fm

v2 = v.*2;           %v square
sumw = sum(w);       %sum of weights
pr_vr2 = v.*r2;      %product of xy i.e v and r square
pr_w_vr2 = w.*pr_vr2; %product of weight and product of xy i.e v and r square
sumpr_w_vr2 = sum(pr_w_vr2); %sum of product of above two
pr_wv = w.*v ;              % product of weight and voltage
sumpr_wv = sum(pr_wv);  % sum of product of weight and voltage
pr_wr2 = w.*r2 ;        % product of weight and r square
sumpr_wr2 = sum(pr_wr2); % sum of p[roduct of weight and r square
pr_wv2 = w.*v2;            % product of weight and v square
sumpr_wv2 = sum(pr_wv2);    % sum of product of weight and v square
sumpr_wv_2 = (sumpr_wv).^2; % square of sum of product of weight and voltage


nume=((sum(w)).*(sum(w.*(v.*r2))))-((sum(w.*v)).*sum(w.*r2))
denom=((sum(w)).*(sum(w.*(v.^2))))-((sum(w.*v)).^2)

um=sqrt(sum(w)./denom)

final_m = nume/denom

% calculating c
nume_c = (((sum(w.*(v.^2))).*sum(w.*r2))-((sum(w.*v)).*(sum(w.*(v.*r2)))))
final_c = nume_c/denom

uc = sqrt((sum(w.*(v.^2)))/denom)

y = ((final_m).*(v))+ final_c;
hold on
```

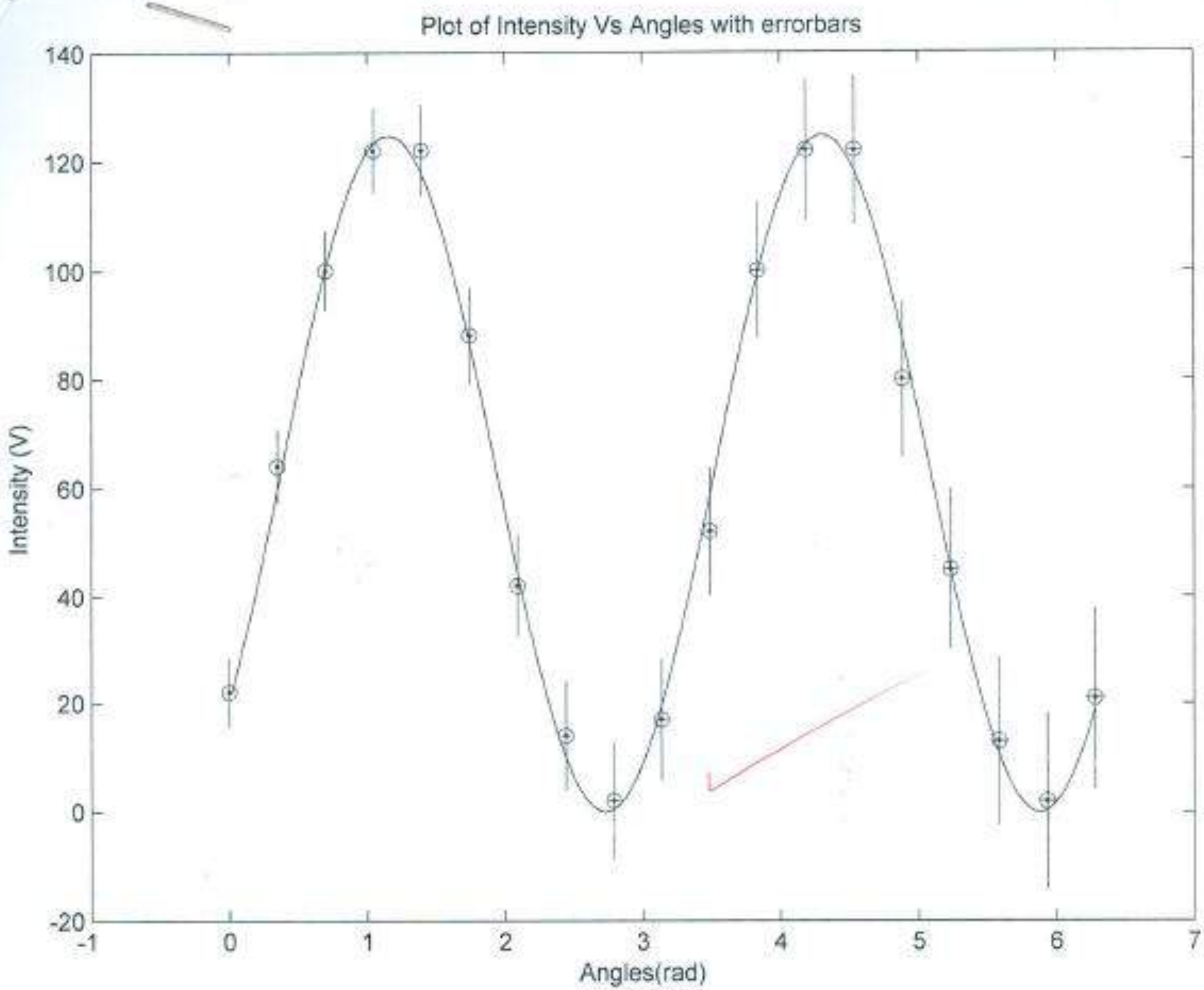
CALCULATIONS

cycle	Height (m)	Delta h (m)	N	Delta E (Joules)	W avg = (rad/sec^2)
1	0.61420	0	7.797	0	5.065
2	0.57090	0.04300	7.247	0.04250	4.816
3	0.53480	0.03610	6.789	0.03540	4.646
4	0.50840	0.02640	6.454	0.02590	4.676
5	0.47060	0.03780	5.974	0.03710	4.267
6	0.45560	0.01500	5.784	0.01470	4.148
7	0.42510	0.03050	5.396	0.02990	1.904
8	0.41010	0.01500	5.206	0.01470	4.148
9	0.39180	0.01830	4.974	0.01800	3.569
10	0.37480	0.01700	4.758	0.01670	3.220

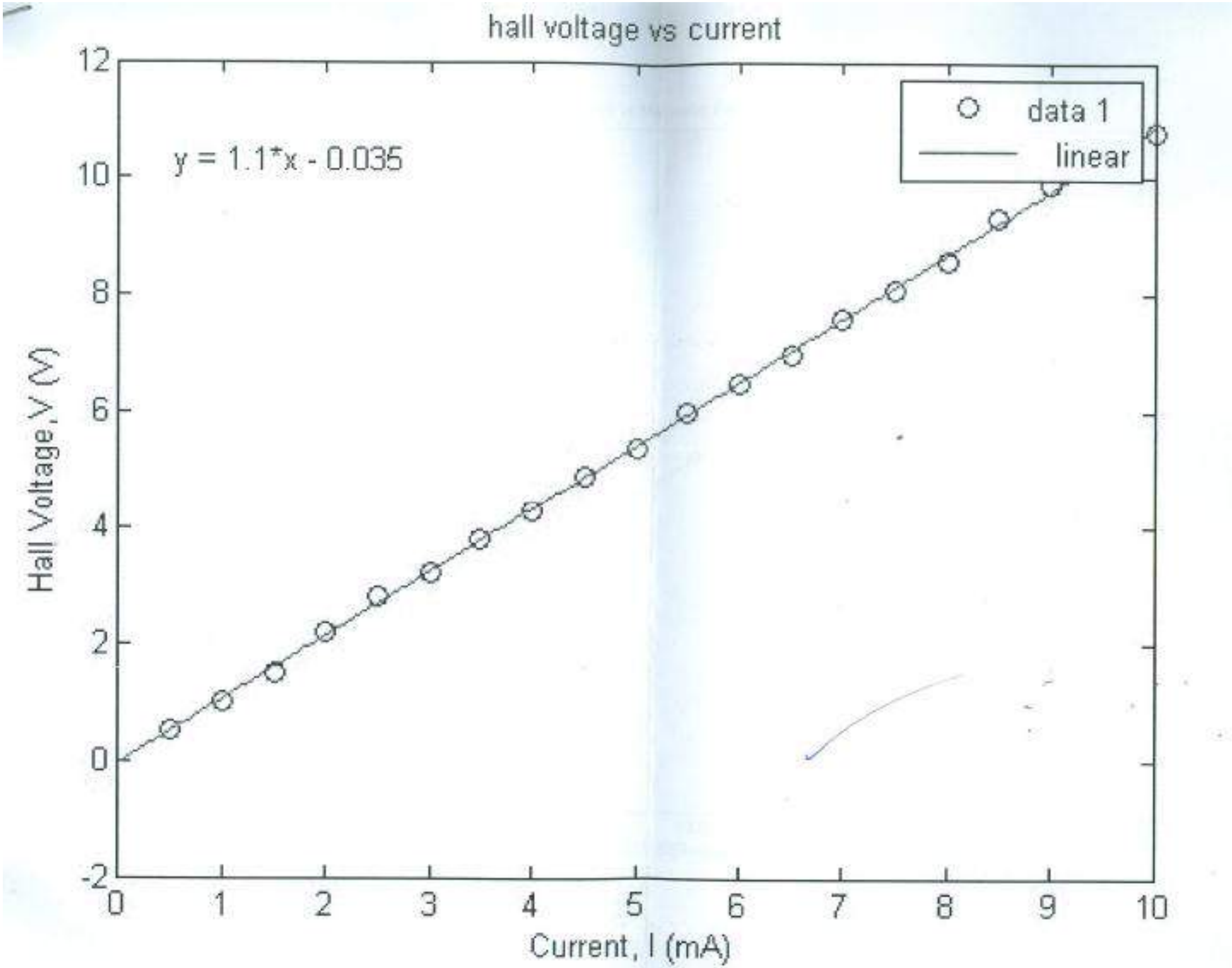
Uncertainty in height= 0.00005/(6)^0.5 (Analog instrument)

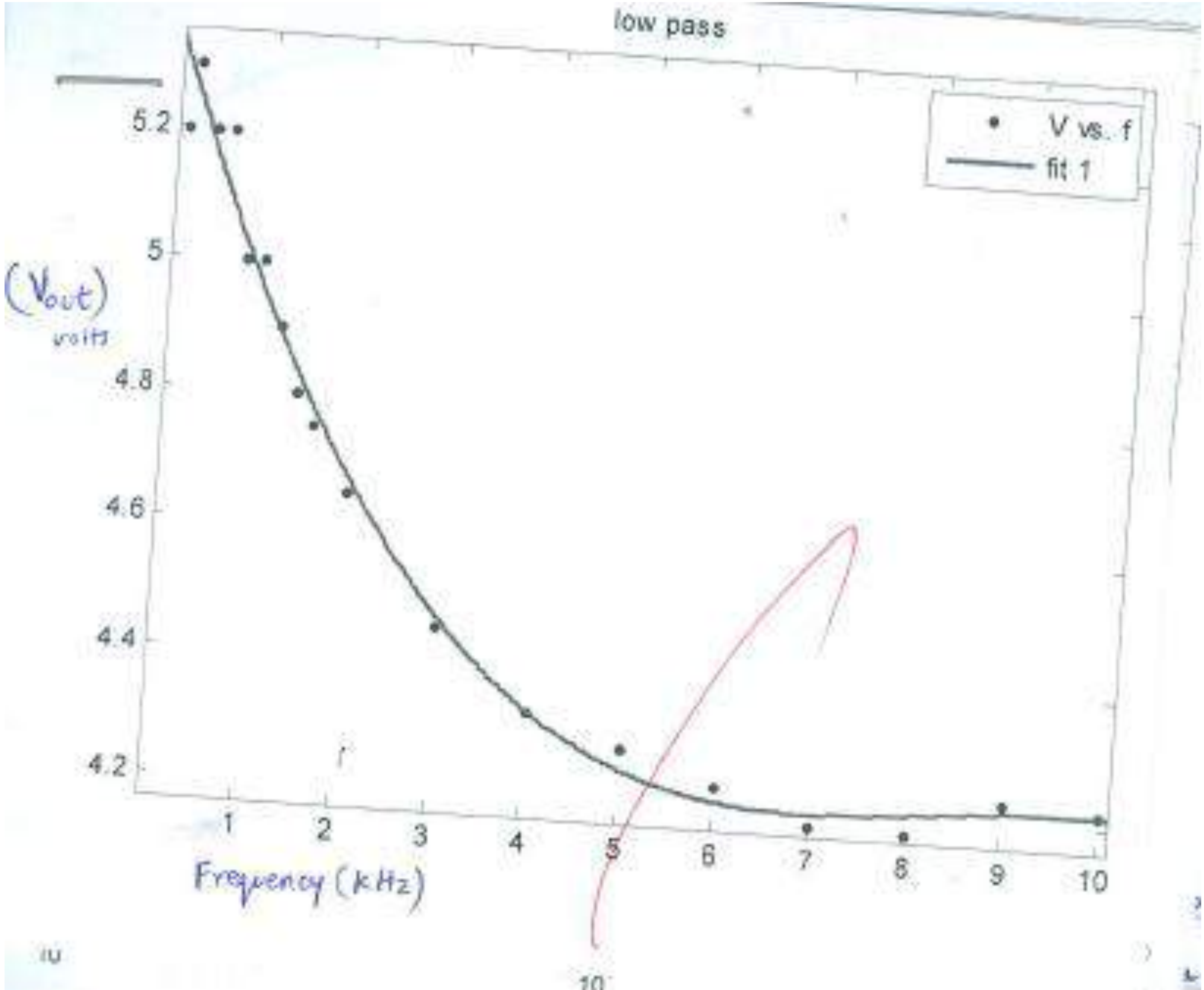
Uncertainty in radius= 0.005/(6)^0.5=0.002 (Analog instrument)

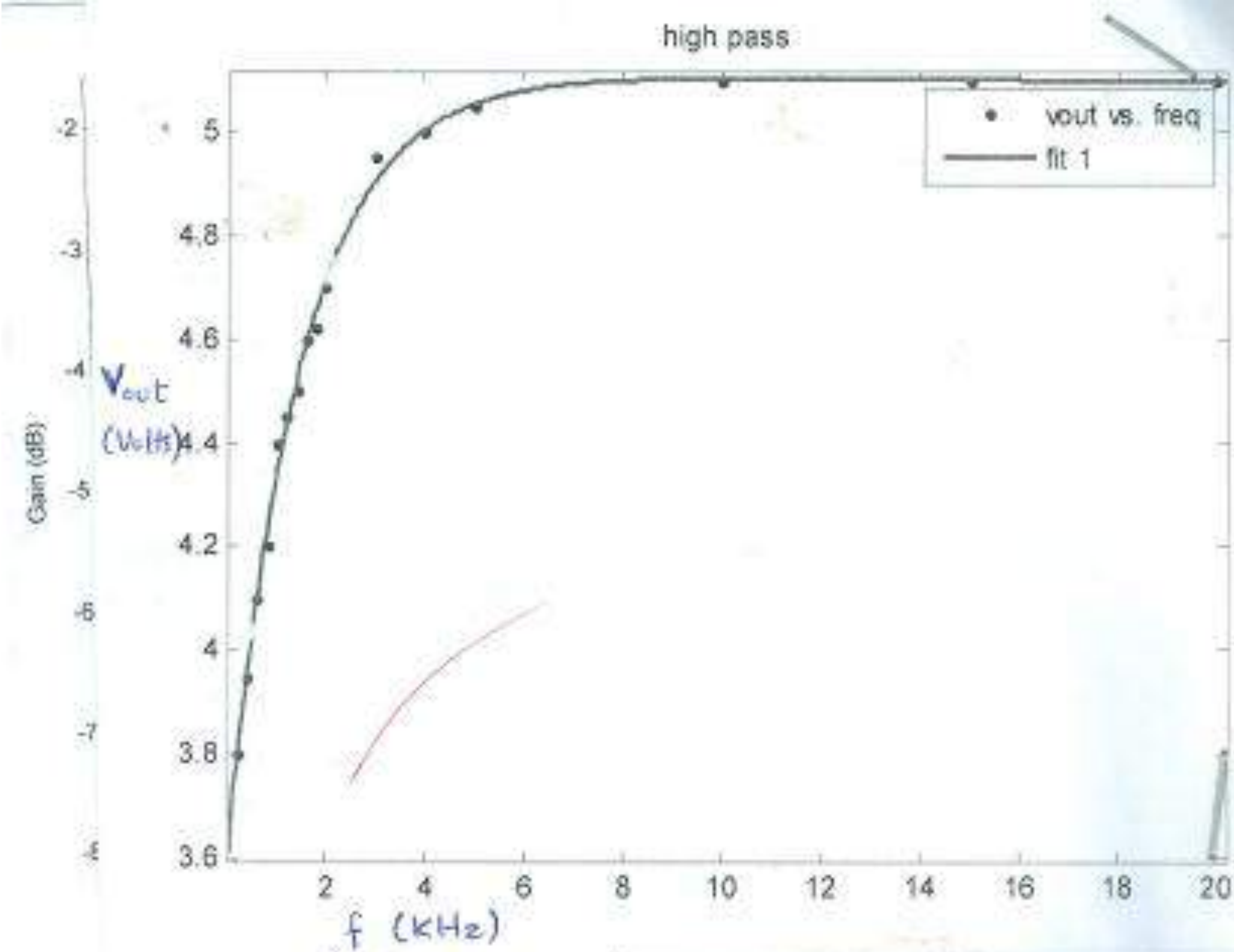

height loss of the mass hanger with time
0.8
0.7
0.6
0.5
0.4
0.3
0.2
height(m)



Plot of Intensity Vs Angles with errorbars
Intensity (V)
Angles(rad)
140
120
100
80
60
40
20
0
-20
-1
0
1
2
3
4
5
6
7


hall voltage vs current
y = 1.1*x - 0.035
data 1
linear
Hall Voltage, V (V)
Current, I (mA)
12
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9
10


low pass
V vs. f
fit 1
5.2
5
4.8
4.6
4.4
4.2
1
2
3
4
5
6
7
8
9
10
(Vout)
volts
Frequency (kHz)


high pass
vout vs. freq
fit 1
Gain (dB)
Vout (Volt)
f (KHz)
-2
-3
-4
-5
-6
-7
5
4.8
4.6
4.4
4.2
4
3.8
3.6
2
4
6
8
10
12
14
16
18
20

- 5.75

4.2

Monday 10th Nov. 2014

$\boxed{\text{Experiment \# 1.2A}}$

Q1:- Moment of Inertia of tennis ball.

$$I = \frac{2}{5} M\left(\frac{R_e^5 - R_i^5}{R_e^3 - R_i^3}\right)$$

First we find respective quantities needed to measure the moment of inertia of a hollow sphere and a composite ~~disk~~ cylinder

For composite ~~disk~~ cylinder

$R_1 = 4.34\ mm = 0.434\ cm$
$R_2 = 14.96\ mm = 1.496\ cm$
$h_1 = 5.78\ mm = 0.578\ cm$
$h_2 = 16.90\ mm = 1.690\ cm$
$T = 26.10/20 = 1.305\ secs$

For tennis ball (hollow sphere)

$R_e = 32.07\ mm = 3.207\ cm$
$R_i = 26.38\ mm = 2.638\ cm$
$M = 62.95g$
$T_{tennis} = \frac{38.71}{10} = 3.871\ secs$

I for tail section

$$I_{tail} = \frac{1}{2} \pi \rho h_1 R_1^4$$

$$I_{tail} = \frac{1}{2}(\pi)(2.7\times 10^{-3}\ g/mm^3)(5.78)(4.34)^4$$

$I_{tail} = ~~8.697\ g\ mm^2~~\ 0.087\ g\ cm^2$

I for short section

$$I_{shr} = \frac{1}{2} \pi \rho h_2 (R_2^4 - R_1^4)$$

$$I_{shr} = 0.5 \times \pi \times 2.7 \times \cancel{16.4}\ 1.69 (1.496^4 - 0.934^4)$$

$$I_{short} = 35.646 \text{ g cm}^2$$

$$I_{total} = I_{tall} + I_{short} = 35.733 \text{ g cm}^2$$

$$\boxed{I_{cylinder} = 35.733 \text{ g cm}^2}$$

We know that

$$T = 2\pi \sqrt{I/K}$$

or

$$K = \frac{4\pi^2 I}{T^2}$$

$$\boxed{K = 828.339} \text{ g cm}^2/\text{s}^2$$

Now

$$I_{tennis} = \frac{K T_{tennis}^2}{4\pi^2}$$

$$I_{tennis} = \frac{(828.339)(3.871)^2}{4\pi^2}$$

$$\boxed{I_{tennis} = 314.409\ g\,cm^2}$$

Theoretical value of moment of inertia of tennis ball

$$I_{tennis.\ theoretical} = \frac{2}{5}(62.95)\left(\frac{32.07^5 - 26.20^5}{32.07^3 - 26.20^3}\right)$$

$$\boxed{I_{tennis\ theoretical} = 364.087\ g\,cm^2}$$

Uncertainities

* Uncertainities in measurements done with vernier calliper. (Δr)

$$\Delta r = \frac{0.02}{2\times\sqrt{6}} = 4.08 \times 10^{-3}\ mm = 4.08 \times 10^{-4}\ cm$$

* Uncertainities in measurements done with stopwatch (ΔT)

$$\Delta T = \frac{0.01}{2\times\sqrt{3}} = 2.887 \times 10^{-3}\ secs.$$

* Uncertainities in quantities measured with weighing balance (ΔM)

$$\Delta M = \frac{0.01}{2\times\sqrt{3}} = 2.887 \times 10^{-3}\ grams.$$

Rating of digital devices not included in uncertainties.

Uncertainity in I_{tall}

$$\Delta I_{tall} = \sqrt{\left(\frac{\delta I_{tall}}{\delta h_1}\Delta h_1\right)^2 + \left(\frac{\delta I_{tall}}{\delta R_1} \times \Delta R_1\right)^2}$$

$$\Delta I_{tall} = \sqrt{\left(\frac{1}{2}\pi \rho R_1^4 \times \Delta h_1\right)^2 + \left(4 \times \frac{1}{2}\pi \rho h_1 R_1^3 \times \Delta R_1\right)^2}$$

$$\Delta I_{tall} = \sqrt{\left(0.5(\pi)(2.7)(0.481)^4(1.03\times10^{-4})\right)^2 + \left(2\times(\pi)(2.7)(0.578)(0.481)^3(9.08\times10^{-4})\right)^2}$$

$$\Delta I_{tall} = \sqrt{3.77\times10^{-9} + 1.07\times10^{-7}}$$

$\Delta I_{tall} = 3.33 \times 10^{-4}$ g cm^2

Uncertainity in I_{short}

$$\Delta I_{short} = \sqrt{\left(\frac{1}{2}\pi\rho(R_2^4 - R_1^4)\times\Delta h_1\right)^2 + \left(\frac{1}{2}\pi\rho h_2 R_2^3 \times 4 \times \Delta R_2\right)^2 + \left(-\frac{1}{2}\pi\rho h_2 R_1^3 \times 4 \times \Delta R_1\right)^2}$$

$$\Delta I_{short} = \sqrt{7.4\times10^{-5} + 1.5\times10^{-3} + 9.1\times10^{-7}}$$

$\Delta I_{short} = 0.04$ g cm^2

$\Delta I_{cylinder} = (3.33 \times 10^{-4} + 0.04)\ gcm^2$

$\boxed{\Delta I_{cylinder} = 0.04\ gcm^2}$

Uncertainity in $I_{tennis\ theoretical}$

$$\frac{\delta I_{tt}}{\delta R_e} = \frac{2}{5} M \left[\frac{-5R_e^7 + 5R_e^4 R_i^3 - 3R_e^7 + 3R_e^2 R_i^5}{(R_e^3 - R_i^3)^2} \right]$$

$$= -2075.99$$

$$\frac{\delta I_{tt}}{\delta R_i} = \frac{2}{5} M \left[\frac{(R_e^3 - R_i^3)(-5R_i^4) - (R_e^5 - R_i^5)(-3R_i^2)}{(R_e^3 - R_i^3)^2} \right]$$

$$= 102.83$$

$$\frac{\delta I_{tt}}{\delta M} = \frac{2}{5} \left[\frac{R_e^5 - R_i^5}{R_e^3 - R_i^3} \right] = 5.78$$

$$\Delta I_{tt} = \sqrt{\left(\frac{\delta I_{tt}}{\delta R_e} \Delta R_e \right)^2 + \left(\frac{\delta I_{tt}}{\delta R_i} \times \Delta R_i \right)^2 + \left(\frac{\delta I_{tt}}{\delta M} \times \Delta M \right)^2}$$

Ans 1:- $\boxed{I_{\text{tennis theoretical}} = (364.09 \pm 0.85)\ g\,cm^2}$

Now

$$\Delta K = \sqrt{\left(\frac{4\pi^2}{T^2} \times \Delta I_{cylinder}\right)^2 + \left(4\pi^2 I_{cylinder} T^{-1} \times -2 \times \Delta T\right)^2}$$

$$\boxed{\Delta K = 6.5\ g\,cm^2/s^2}$$

Ans 2:- $\boxed{\text{Torsional constant } (K) = (828 \pm 6)\ g\,cm^2/s^2}$

Now

$$I = \frac{KT^2}{4\pi^2}$$

10.11.2013

$$\Delta I_{\text{tennis experimental}} = \sqrt{\left(\frac{T^2}{4\pi^2} \times \Delta K\right)^2 + \left(\frac{2TK}{4\pi^2} \times \Delta T\right)^2}$$

$$\Delta I_{\text{tennis experimental}} = 2.48\ g\,cm^2$$

Ans 3:- $\boxed{I_{\text{tennis experimental}} = (314.4 \pm 2.5)\ g\,cm^2}$