Steering Paramagnetic Leidenfrost Drops in an Inhomogeneous Magnetic Field*

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The Leidenfrost Effect is a phenomenon in which a liquid drop levitates on a surface that is significantly hotter than its boiling point. When we create Leidenfrost drops using a paramagnetic liquid, such as oxygen, we end up with liquid drops that hover above a surface with negligible friction, and since oxygen is paramagnetic, can be controlled by a magnetic field.

KEYWORDS

Paramagnetism · Leidenfrost effect · Conservation of energy · Conservation of angular momentum · Equation of motion · Central force motion

1 Objectives

In this experiment, we will,

1. observe the Leidenfrost Effect using liquid oxygen,
2. steer oxygen drops inside an inhomogeneous magnetic field,
3. record the motion of these drops using a digital camera,
4. learn how to extract information from large data sets,

5. learn and implement techniques in numerical computing such as numerical differentiation and integration, and

6. compare experimental results with theoretical predictions.

2 Introduction

Many of us have inadvertently observed the Leidenfrost Effect in our kitchen. When we pour water onto a pre-heated pan which has a temperature far above the hundred degrees boiling point of water, water drops begin to fizz around in random directions for a few seconds before they eventually evaporate. Have you ever wondered why they don't evaporate immediately after falling onto the surface? It's because of the Leidenfrost Effect. A very thin layer of super hot vapour forms underneath the water drop. The water drop levitates on this insulating layer and does not come in direct contact with the surface hence it's lifetime is extended, and it jostles around since the friction between the drop and the surface is extremely small.

This phenomenon becomes more vivid with liquid oxygen since room temperature is already much higher than the boiling point of oxygen. Therefore, no additional heating of a surface is required. Furthermore, liquid oxygen is a paramagnetic substance, allowing us to observe it's dynamics inside a magnetic field.

From the standpoint of magnetism, materials are divided into five broad categories. They are ferromagnetic, paramagnetic, ferrimagnetic, antiferromagnetic and diamagnetic materials. Ferromagnetic materials may become permanently magnetised when placed in a magnetic field. A paramagnetic material remains magnetised only as long as it is inside the magnetic field. This means that **the dipoles of a paramagnetic material align in the direction of the applied magnetic field**, but dealign as soon as the applied field is removed. This is shown in the figure below.

The paramagnetism of oxygen can be described by the molecular orbital theory which explains that oxygen, O_2, has 2 unpaired electrons. These unpaired electrons act like magnetic dipoles imparting a paramagnetic character to oxygen [1].

The only challenge that remains is to form liquid oxygen drops. To achieve this, we take a hollow copper cone that is open from the top and place it in a clamp with it's pointed edge facing down. We pour in liquid nitrogen which quickly cools down the copper cone. Within a few seconds, the oxygen in the air starts to condense on the outer surface of the cone, drips down to the tip and falls off. This way, we get a constant supply of liquid oxygen drops that lasts a few minutes. Beneath the cone is an aluminium bar with a thin depression along it's axis. You can adjust the height of the cone, and the inclination of the bar in order to vary the initial velocity of the drop. Black plexiglass sheets are placed to prevent nitrogen fumes from making the video recording unclear.

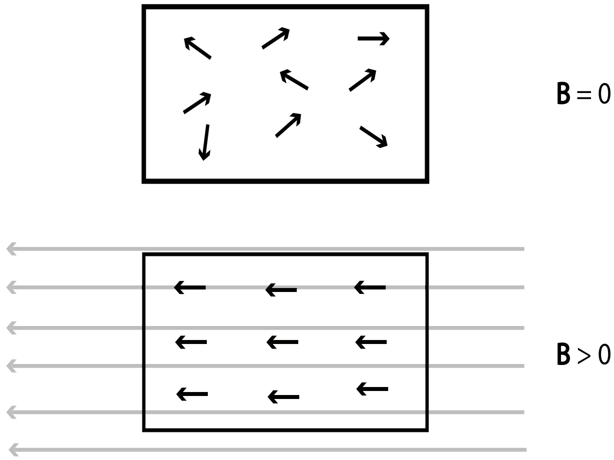

Figure 1: The small, black arrows inside the block represent the dipoles of the paramagnetic substance. The long, grey arrows represent the applied magnetic field.

At the other end of the aluminium bar is the horizontal surface underneath which the magnet is placed. The position of it's centre has been marked on the surface. Place a ruler on the surface to provide a reference scale when we analyse the recording.

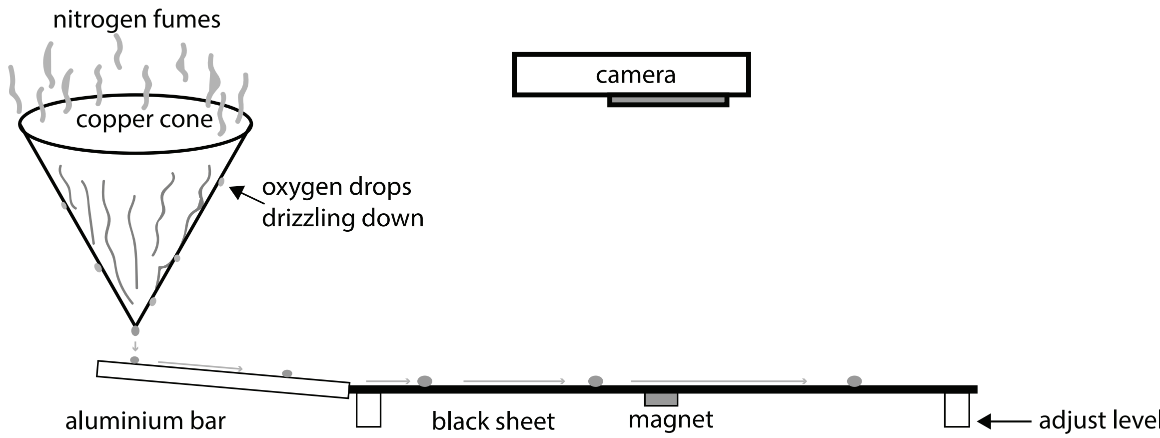

Figure 2: The experimental setup.

We direct these drops onto a horizontal sheet, underneath which we have placed a cylindrical magnet. The incoming drops have different velocities and initial displacements from the magnet's centre. We will refer to the initial velocity as v, the initial displacement from the magnet's centre as b, and the angle of deflection as α. These variables are shown in Figure (3).

Another important quantity that we will analyse is r_p, which is the pericenter of the trajectory. It

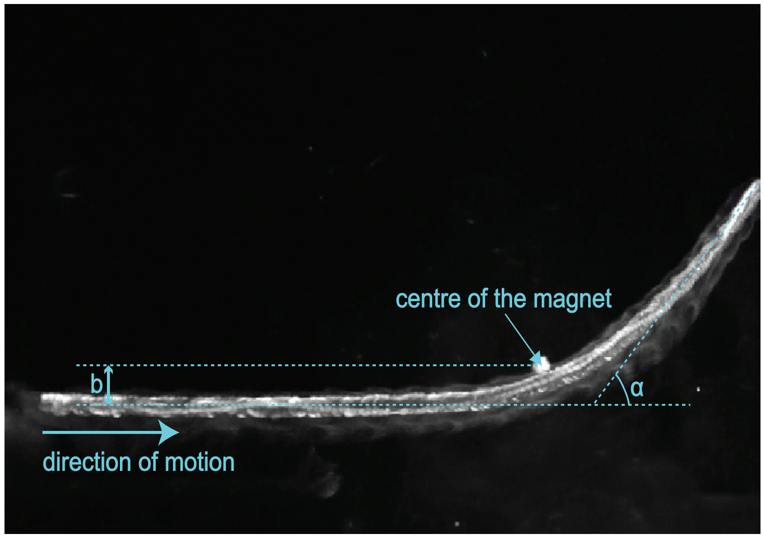

Figure 3: The trajectory of an oxygen drop projected towards a magnet. This is a view from the top. The figure is made of superimposed images of the drop at small time intervals. The fumes around the trajectory are due to evaporation of oxygen. The dotted line extends the initial line of motion of the drop. The frame rate of the camera used for this experiment is 240 fps. Note that b is the perpendicular distance between the centre of the magnet and the undeflected trajectory and α is the overall angle of deflection.

is the radial distance from the centre of the magnet to the point in the trajectory where the radial velocity of the oxygen drop is zero. Both r_p and α vary as functions of b and v, and analysing these relationships is the primary purpose of this experiment. This experiment closely follows the research work described in [2].

It is amazing to see how similar this scenario is to that of asteroids passing by a planet. In fact, while you are performing the experiment, you might be able to produce low velocity oxygen drops that fall into orbit around the centre of the magnet!

The next two subsections may not seem to have any direct connection to each other or this subject, but they are necessary in order to reach our first main aim, which is to derive the equation of motion of an oxygen drop in the magnetic field of a cylindrical magnet.

2.1 An Introduction to Polar Coordinates

For simplicity, we will work with a polar coordinate system in this experiment, with the centre of the magnet defined as the origin of our polar plane. However, you need to know how position and velocity are defined in a polar plane in order to be able to understand the mathematical description of this experiment. It will perhaps be easier if we introduce polar coordinates by comparing them directly with Cartesian coordinates. Every point in a cartesian plane can be defined by it's x and y coordinates. A point in polar coordinates is defined instead by r and θ, where r is the point's distance from the origin and θ is the angle between the line joining the point to the origin and the horizontal axis. The base vectors in a cartesian plane, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, point in the horizontal and vertical directions respectively, and have fixed directions. The base vectors in polar coordinates, $\hat{\mathbf{r}}$ and $\hat{\theta}$ however, are not as simple. Figure (4) shows how $\hat{\mathbf{r}}$ and $\hat{\theta}$ are related to the Cartesian unit vectors.

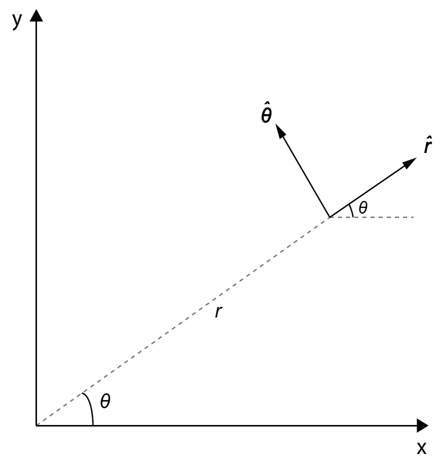

Figure 4: A figure showing how a point is defined using r and θ in a polar plane.

From this figure, we can quickly establish the following relationships.

$$\hat{\mathbf{r}} = \hat{\mathbf{i}}\, cos\theta + \hat{\mathbf{j}}\, sin\theta \tag{1}$$

$$\hat{\theta} = -\hat{\mathbf{i}}\, sin\theta + \hat{\mathbf{j}}\, cos\theta \tag{2}$$

Notice that $\hat{\mathbf{r}}$ and $\hat{\theta}$ do not have fixed directions, as both are functions of θ.

Now that we have defined and compared the unit vectors in each coordinate system, we can move on to define the position vector in each system. In Cartesian coordinates, we have

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \tag{3}$$

while in polar coordinates,

$$\mathbf{r} = r\hat{\mathbf{r}} \tag{4}$$

One could migrate from the polar to Cartesian coordinate system using the relations, $x = r\,cos\theta$ and $y = r\,sin\theta$, and conversely,

$$r = \sqrt{x^2 + y^2} \tag{5}$$

$$\theta = \tan^{-1}\frac{y}{x} \tag{6}$$

Q 1. Differentiate Equations 3 and 4 to obtain expressions for velocity in both coordinate systems.

You should obtain the following equations.

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\,\hat{\mathbf{i}} + \dot{y}\,\hat{\mathbf{j}} \tag{7}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\,\hat{\mathbf{r}} + r\dot{\theta}\,\hat{\theta} \tag{8}$$

The two terms in Equation 7, $\dot{x}\,\hat{\mathbf{i}}$ and $\dot{y}\,\hat{\mathbf{j}}$, represent velocity in the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ directions respectively. In the case of Equation 8, the term $\dot{r}\,\hat{\mathbf{r}}$ is for the velocity in the radial direction and $\dot{\theta}\,\hat{\theta}$ is for the velocity in the tangential direction.

This is all that we need to know about motion in polar coordinates for now. You will soon realise that working in polar coordinates will greatly simplify this problem.

2.2 The Magnetic Field Profile

In order to begin deriving the equation of motion, it is important to know the magnetic energy per unit volume at each point in the plane. This is given by the following equation,

$$E_{\text{mag}} = -\frac{\chi}{2\mu_0}|\mathbf{B}|^2 \tag{9}$$

where $\chi = 0.0035$ at $-183°$C is the magnetic susceptibility of liquid oxygen, $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of free space and $\mathbf{B}$ is the magnetic field. Refer to Appendix A for a derivation of this equation.

Q 2. Measure the horizontal component of the magnetic field $\mathbf{B}$ using the provided Gaussmeter and plot it as a function of the distance from centre of the magnet r.

Q 3. Use the following equation to fit the data and find the values of the constants, E_0, q and r_0. What are their units?

$$E_{\text{mag}} = \frac{-E_0}{q + (r/r_0)^6} \tag{10}$$

Far from the magnet, the magnetic field varies as $1/r^3$, and since E_{mag} is proportional to $|\mathbf{B}|^2$, the magnetic energy varies as $1/r^6$.

2.3 Deriving the Equation of Motion

We will use the principles of conservation of energy and angular momentum to derive the equation of motion of an oxygen drop passing through the magnetic field produced by a single cylindrical magnet. This scenario involves a central force acting on the oxygen drop, which is the magnetic force. Therefore, we consider it to be a *central force motion* problem [3]. The initial kinetic energy of the drop is it's total energy. **As the drop gets closer to the magnet, it's kinetic energy changes and it begins to gain magnetic energy.** Since it's motion under the influence of the magnetic field is no longer linear, we define it's kinetic energy to be the sum of it's radial and tangential kinetic energies. For this, we define the centre of the magnet as the origin of our polar plane. The following is the expression for the initial kinetic energy per unit volume,

$$E_{\text{initial}} = \frac{1}{2}\rho v^2 \tag{11}$$

where ρ is the density of liquid oxygen and v is it's initial velocity.

Q 4. Use your expressions for radial and tangential velocity in a polar plane, Eq. (8), to write the expressions for the radial and tangential kinetic energy per unit volume, E_{radial} and $E_{\text{tangential}}$.

Q 5. Use the principle of energy conservation to write down the equation of motion of an oxygen drop.

Due to the axial symmetry of this problem, we can use the conservation of angular momentum to further simplify the equation of motion.

Q 6. Show that the conservation of angular momentum implies,

$$\dot{\theta} = \frac{bv}{r^2}. \tag{12}$$

Q 7. Use Eq. (12) to eliminate $\dot{\theta}$ from the equation derived in Question 5.

Eventually your derivation should lead to the following equation of motion.

$$\rho\, v^2 = \rho\, \dot{r}^2 + \rho \frac{b^2\, v^2}{r^2} - \frac{2E_0}{q + (r/r_0)^6}. \tag{13}$$

3 Tracking the Motion of Liquid Oxygen Drops

Our goal is to vary the distance b, and observe the effect it has on the distance to the pericenter of the trajectory, r_p, and the angle of deflection α.

3.1 Procedure

Caution: Wear gloves while using liquid nitrogen. Once you have poured liquid nitrogen into the cone, handle the apparatus very carefully to prevent it from spilling. Do not touch anything close to the cone with bare hands. The metallic parts of the clamp and stand, and the aluminium bar become extremely cold during the experiment.

Pour liquid nitrogen into the cone and wait for it to start producing oxygen drops. Vary the distance b and observe the angle of deflection α. For example find a velocity that results in a large maximum value of α (greater than 45°), by varying the height of the cone and the inclination of the aluminium bar. Once you have selected the velocity, adjust the camera on top of the magnet in a way that the oxygen drop is in the field of view of the camera before it is under the influence of the magnetic field and stays in it a few seconds after leaving it's influence again. The ruler should also be in the field of view, as it will help in calibrating the reference scale. Note that it will take multiple trials to obtain an interesting set of trajectories.

Begin the experiment by pouring liquid nitrogen into the cone. Once the oxygen drops begin to fall, set b to zero and begin recording at a high frame rate (preferably 240 fps). After every 2 to 3 drops, move the rod by pushing it from both ends in order to slightly increase b. You will also have to simultaneously rotate the clamp so that the cone remains on top of the rod and liquid oxygen keeps falling onto it. Continue to do so until the angle of deflection becomes extremely small. Stop the recording and transfer the file to the computer.

Try to make sure that the recording does not contain more than 10,000 frames. Otherwise the software will take a long time to load all frames for analysis.

3.2 Video Tracking

Start the application **Tracker** on your computer [4].

1. Go to `File` → `Open File` and select the saved video file.
2. On the top menu, select clip setting, and set the correct fps for your video. You can also exclude unnecessary frames by changing the start and end frames.

3. On the top menu, select the drop down menu from `Show, hide or create calibration tools`, go to `New` → `Calibration Stick`

4. Zoom into the ruler. Bring the cursor to a mark, hold shift and click to place one end of the calibration stick. Repeat the same for the other end, and then enter the actual distance to set a reference scale.

5. On the top menu, click `Show or hide the coordinate axis`. Place the origin of the axis on the centre of the magnet.

6. On the top menu, select `Create a new track`. Hold shift, and click the oxygen drop that you wish to track. You will notice that a new mass, named 'mass A' will appear. Make sure you identify the drop before it gets under the influence of the magnetic field.

7. Now go to `mass A` → `Autotracker`. To create a key frame that the auto tracker will use to keep track of the drop, hold shift and control, and click the drop. Try to select the entire drop and not just a certain part of it so that the auto tracker does not lose track of it.

8. Click `Search` to run the auto tracker. If you notice that it has lost track of the drop, you can either delete all frames by going to `Delete` → `Clear All`, rewind to certain frames to delete them by going to `Delete` → `This Point`, or rewind back and delete all later points by going to `Delete` → `Later Points`.

9. If the auto tracker does not work at certain points, you can manually select the drop by holding shift and clicking. You can also change the key frame by holding shift and control and clicking.

10. Notice the data being collected on the right side as the auto tracker continues to search. Once the drop is out of the magnet's influence, press the stop button in the auto tracker window.

11. On the graph visible on the right side of the window, set the vertical axis to y, and the horizontal axis to x. To make the analysis easier, you need to make sure that the y coordinate of the drop is constant before it comes under the influence of the magnetic field. Rotate the coordinate axis and observe the graph of y against x. Set it at an angle at which the initial path of the drop is parallel to the x axis.

12. Double click the graph to open the `Data Tool`. Go to `File` → `Export Data` and save it with a `.txt` extension.

13. This way, you have a text file that contains columns labelled x, y, and t, that describe the entire trajectory of the drop. Delete the first 2 header lines so that it becomes easier to import the data into MATLAB.

14. Repeat this procedure for at least 10 drops in the video. Make sure that the next drop has a greater value of b.

If you are not clear about any of the processes, go to `Help` → `Tracker Help`.

3.3 Analysis

At this point, we should be clear that our goals are to

1. find values for r_p (the distance to the pericenter of the trajectory),
2. find values for α (the angle of deflection) and
3. compare them to the theoretical values obtained from the equation of motion.

The first step is to understand how we can extract these values for each drop from the equation of motion. This is explained in Section 3.4. The next step is to extract these values from our experimental data, and compare them to the theoretical values obtained from the equation of motion, Eq. (13). Section 3.5 explains this task.

3.4 Predicting the Peripheral Radius r_p

Recall Eq. (13) for the ensuing analysis. The first value we need to obtain from this equation is r_p. **Recall that r_p is the distance between the center of the magnet and the drop at which the radial velocity is equal to zero.** Therefore, in order to find r_p, we must set $\dot{r}$ in the equation of motion to zero. Once we do this, we obtain the following equation.

$$\rho\, v^2 = \rho \frac{b^2\, v^2}{r_p^2} - \frac{2E_0}{q + (r_p/r_0)^6} \tag{14}$$

This is an algebraic equation that cannot be solved analytically. An appropriate method is to graphically solve such an equation. This method involves plotting the two sides of the equation on the same graph. The point of intersection represents the solution. In this case, the two sides are $\rho\, v^2$ and $\rho\frac{b^2\, v^2}{r^2} + \frac{2E_0}{q+(r/r_0)^6}$. The first term is a constant since it is the initial energy of the drop. The second term is a function of r that will intersect the initial energy at r_p.

Q 8. You have already found an equation for E_{mag} by fitting it to the data points. Select any suitable values for v and b. Write a MATLAB program that finds the value of r_p for these values. Typical initial velocities for this experiment vary from 20 cm/s to 50 cm/s.

3.5 Experimental Results

Your task is to write a computer program (e.g in MATLAB) that loads the text file for each drop, calculates the data that is required (v, r_p, α) and also computes the theoretical values. Later, experimental results will be compared with the theoretical predictions of r_p and (optionally) α.

It is entirely up to you to decide the structure of your program. The following points may be useful and can be considered as an 'appetizer' for the problem.

1. Write a *for loop* that loads one text file at a time, extracts all useful values and adds them to an array.
2. Since the text file contains the position of the drop at each time in cartesian coordinates you may first need to convert these to polar coordinates.
3. To find the velocity, you need to apply an algorithm that numerically differentiates the data. One such algorithm is given below.

$$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} \tag{15}$$

   This is an approximation of the first derivative of a function $f(x)$ using a five-point stencil. It assumes that the spacing between two successive points is h.

Q 9. Plot the experimental r_p against b. On the same figure, plot the value of r_p predicted by the equation of motion and see if your experimental results agree with theoretical predicitons.

Q 10. Repeat the same process and plot α against b. (**Optional:** Also find the predicted values of α using the equation of motion.)

Now that you have completed this experiment, you must be able to appreciate how a combination of different concepts can give us a platform to study kinematics of a particle in almost zero friction. Not only does this help us understand the motion of a paramagnetic object, it also helps us simulate and observe events that are analogous to astronomical phenomena such as asteroids passing by a planet, or a satellite orbiting the earth. In fact, if we move the object at a certain velocity towards an incoming drop, we can recreate a magnetic version of the gravitational slingshot effect, making the oxygen drop accelerate into another direction. Consider exciting variants of this experiment.

Appendices

A Magnetic Energy

The magnetic energy per unit volume in a region is defined by the following equation,

$$E_{\text{mag}} = \frac{|\mathbf{B}|^2}{2\mu} \tag{16}$$

where $\mathbf{B}$ is the magnetic field and μ is the magnetic permeability of the medium. We will use this fundamental equation to derive the magnetic energy per unit volume for a paramagnetic substance. Let us first state all the relationships that are essential to the derivation of the magnetic energy per unit volume.

The permeability μ of a medium is defined as the product of the relative permeability μ_r and the permeability of free space μ_0.

$$\mu = \mu_r \mu_0 \tag{17}$$

The magnetic susceptibility is defined by the following relationship,

$$\mathbf{M} = \chi_m \mathbf{H} \tag{18}$$

where χ_m is the magnetic susceptibility, $\mathbf{M}$ is the magnetisation, and $\mathbf{B}$ is the magnetic flux density.

The magnetic flux density $\mathbf{M}$ is related to the magnetic field intensity $\mathbf{H}$ by the following relationship.

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu \mathbf{H} \tag{19}$$

By substituting the value of $\mathbf{M}$ from Eq.(18), we get

$$\mathbf{B} = \mu_0(H + \chi \mathbf{H}) = \mu \mathbf{H} \tag{20}$$

$$\mathbf{B} = \mu_0(1 + \chi)\mathbf{H} = \mu \mathbf{H} \tag{21}$$

$$\mu_0(1 + \chi) = \mu \tag{22}$$

$$\tag{23}$$

Using this final expression and Eq.(17), we can establish the following.

$$\mu_r = 1 + \chi \tag{24}$$

We can rewrite Eq.(16) as

$$E_{\text{mag}} = \frac{|\mathbf{B}|^2}{2\mu_r \mu_0} \tag{25}$$

By substituting the value of μ_r from Eq.(24), we get

$$E_{\text{mag}} = \frac{|\mathbf{B}|^2}{2\mu_0\,(1+\chi)} \tag{26}$$

We can now use the Binomial Series to approximate this expression, since $\chi << 1$.

$$E_{\text{mag}} = \frac{|\mathbf{B}|^2}{2\mu_0\,(1+\chi)} = \frac{|\mathbf{B}|^2}{2\mu_0}(1+\chi)^{-1} \approx \frac{|\mathbf{B}|^2}{2\mu_0}(1-\chi) \tag{27}$$

Further expansion leads to

$$E_{\text{mag}} = \frac{|\mathbf{B}|^2}{2\mu_0} - \chi\frac{|\mathbf{B}|^2}{2\mu_0} \tag{28}$$

The first term on the right hand side of Eq.(28) can be ignored. It is the magnetic energy in vacuum and can be considered as the baseline magnetic energy. The other term is the magnetic energy per unit volume for a paramagnetic substance with magnetic susceptibility χ.

Therefore, the magnetic energy per unit volume is defined the following equation.

$$E_{\text{mag}} = -\chi\frac{|\mathbf{B}|^2}{2\mu_0} \tag{29}$$

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[4] Tracker can be downloaded from https://physlets.org/tracker/.