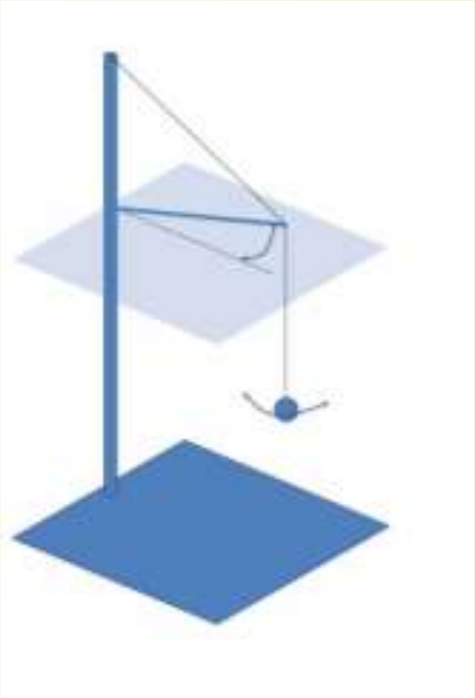
Problem 11. Azimuthal-Radial Pendulum

By: Aarij Atiq

Problem Statement

- Fix one end of a horizontal elastic rod to a rigid stand. Support the other end of the rod with a taut string to **avoid vertical deflection** and suspend a bob from it on another string. In the resulting pendulum the **radial oscillations** (parallel to the rod) can spontaneously **convert into azimuthal oscillations** (perpendicular to the rod) and vice versa. **Investigate the phenomenon.**



Outline of report

1
- Preliminary Observations

2
- Theory

3
- Experiments
- Experimental Set-up, and Results

- Conclusion

Construction of system

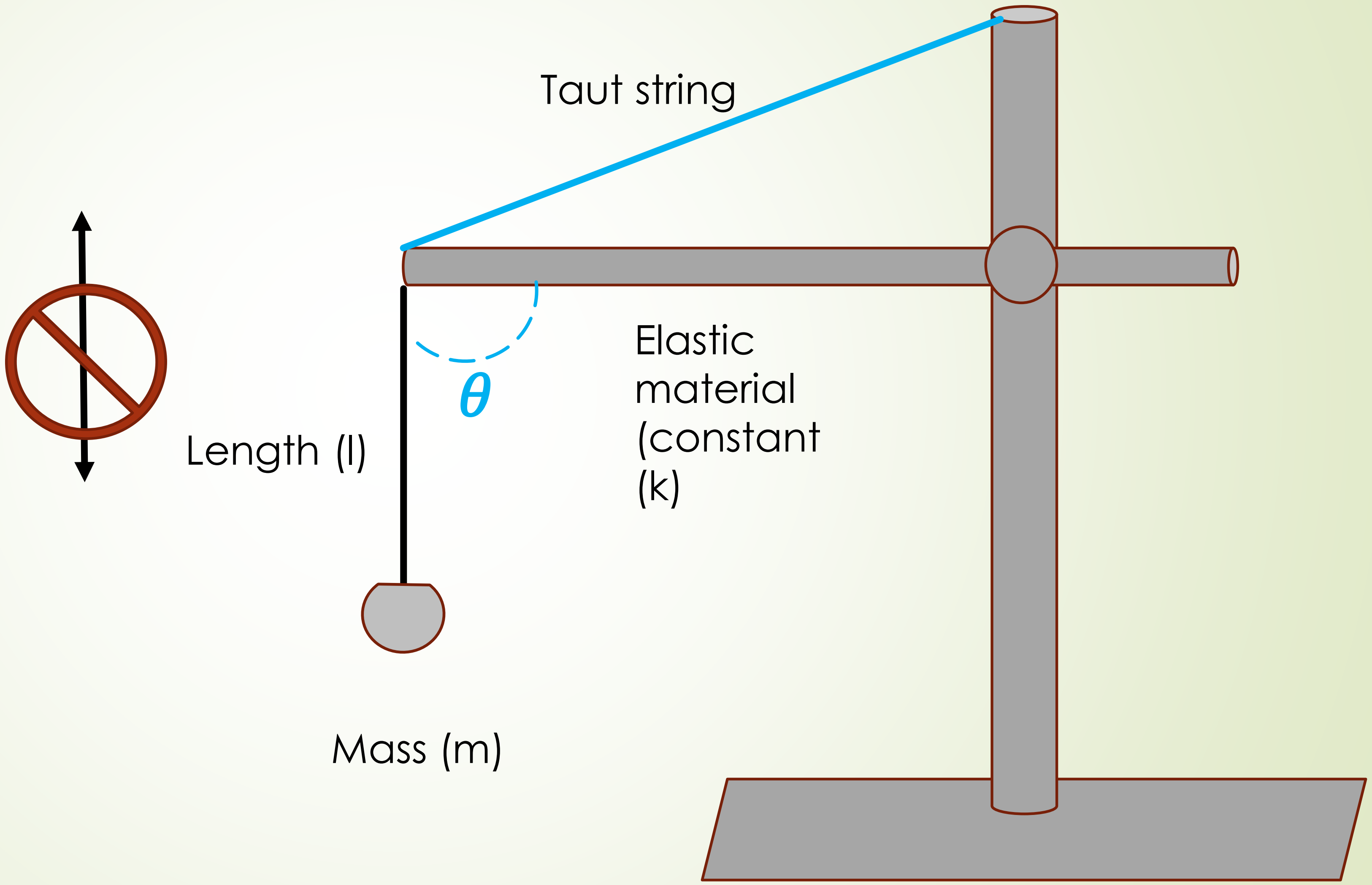


Diagram – Top View

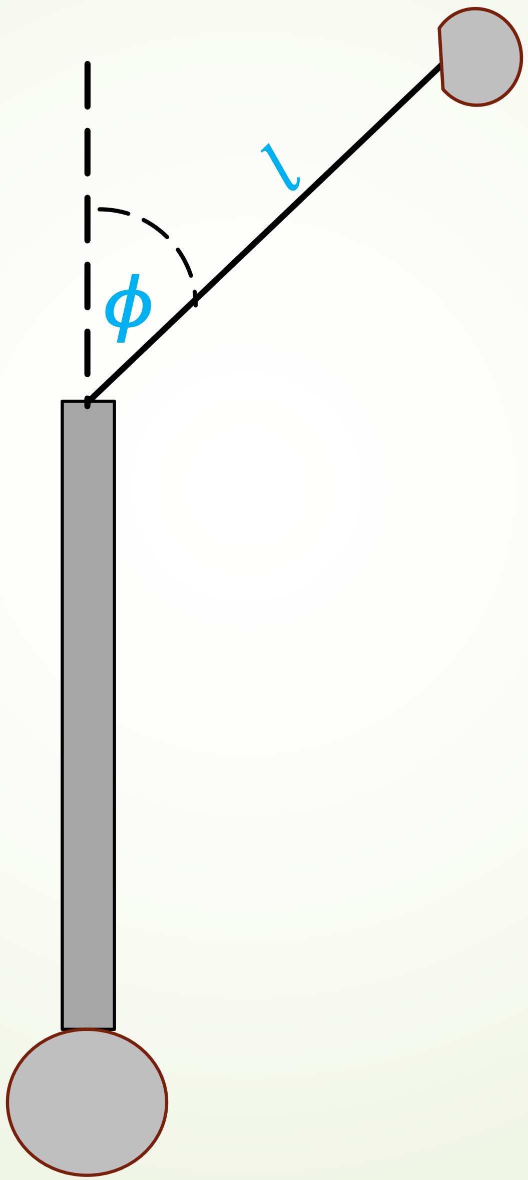
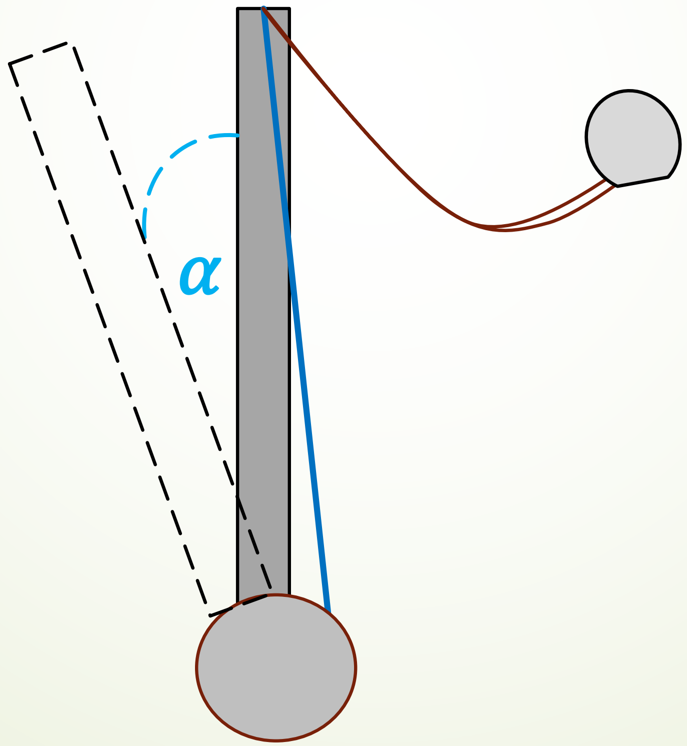


Diagram Top View

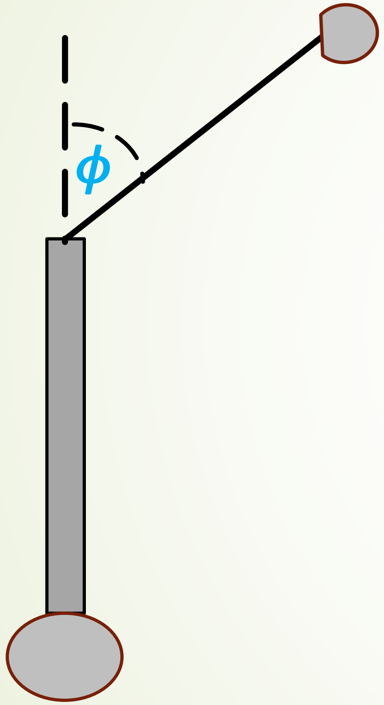


Observation Video

The pendulum behaves like a conical pendulum with an oscillating fulcrum

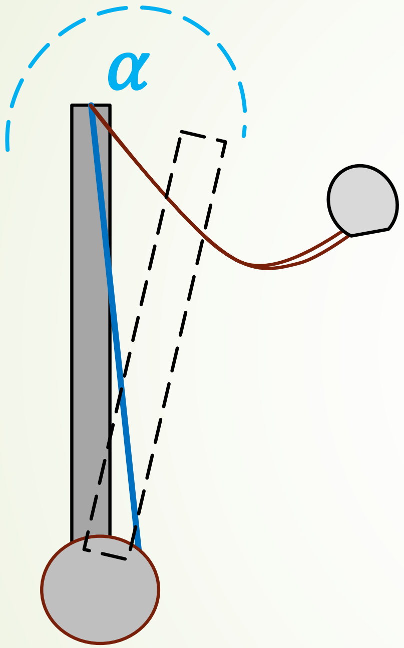
Radial oscillations → Azimuthal oscillations

Investigation- Conditions of the Experiment



The pattern isn't observed when disturbed in the phi direction

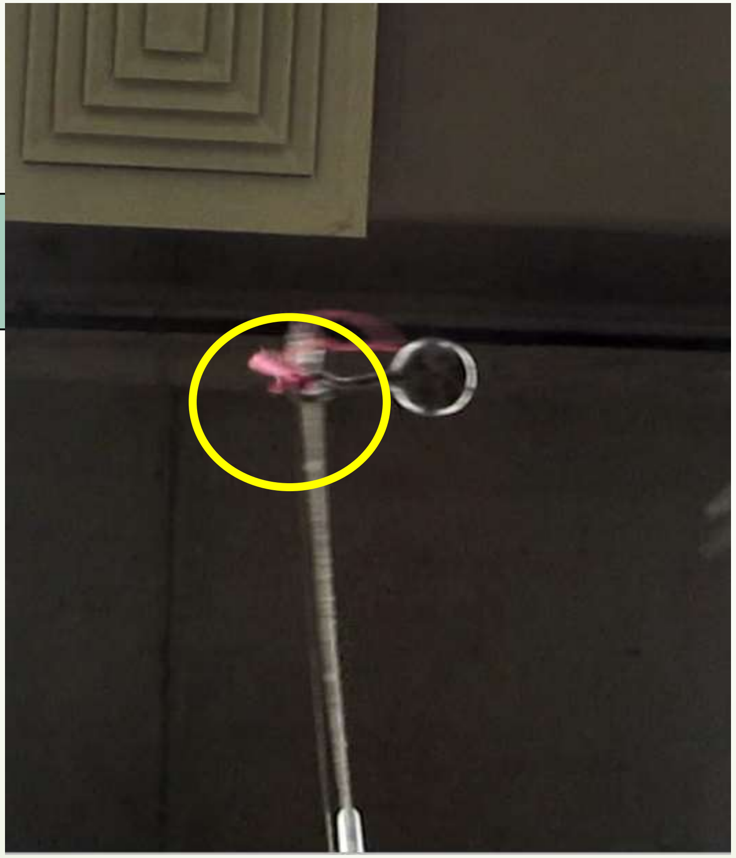
Investigation-Conditions

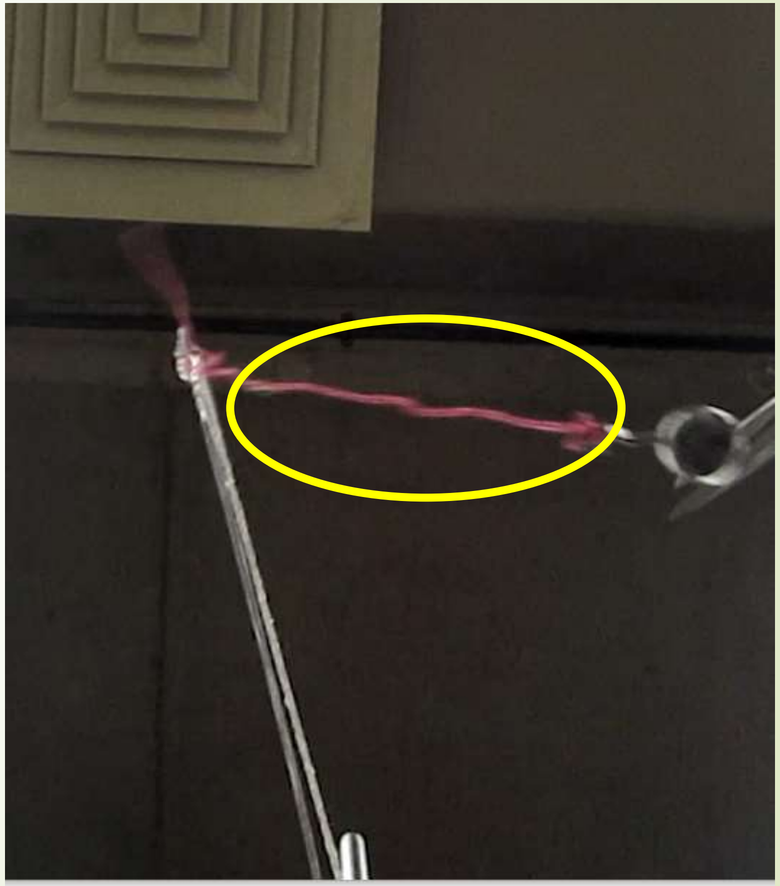


Behaves like a simple conical pendulum when disturbed in the α direction

Continued...

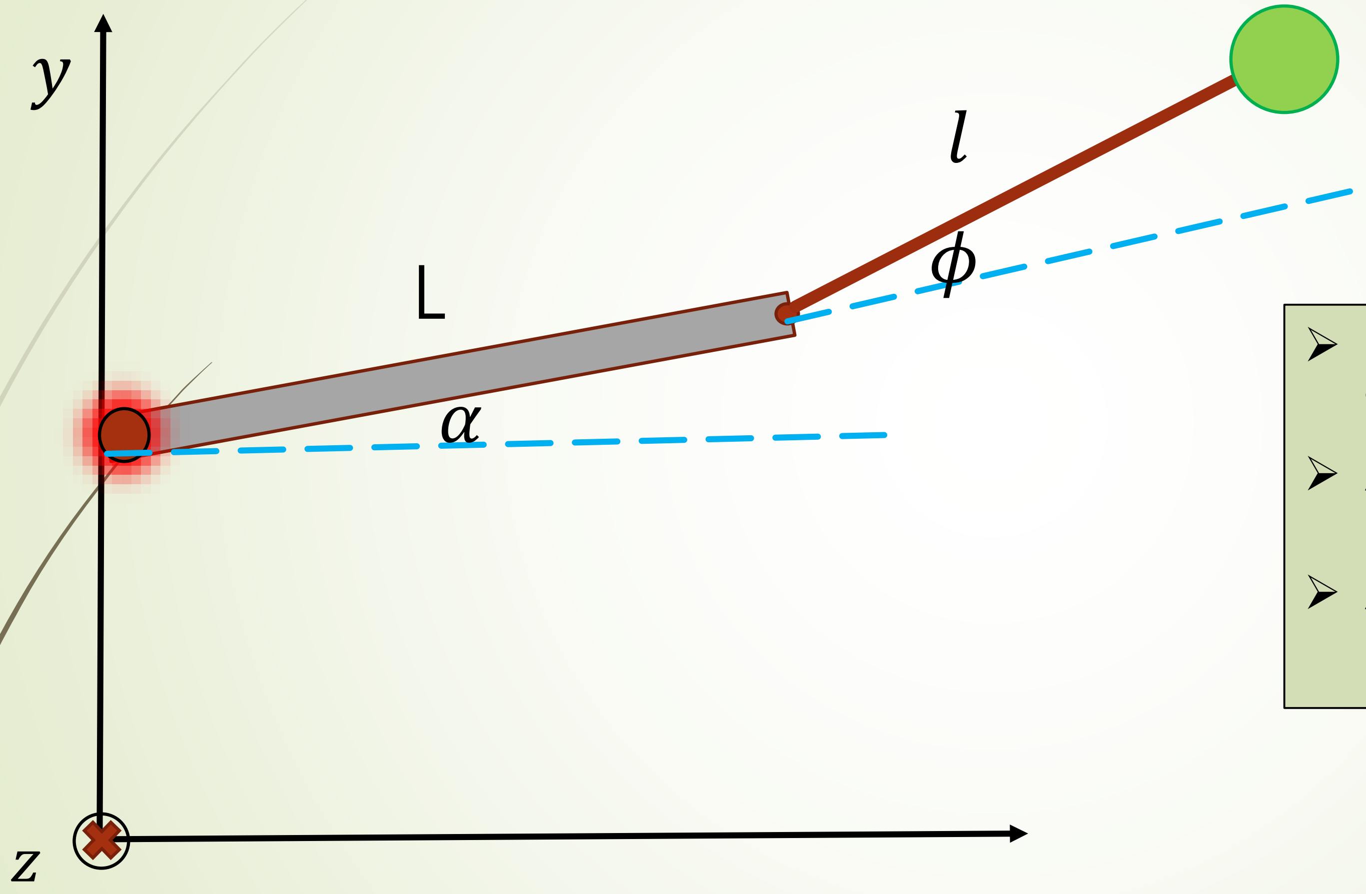
Effect achieved due to the contraction of the string





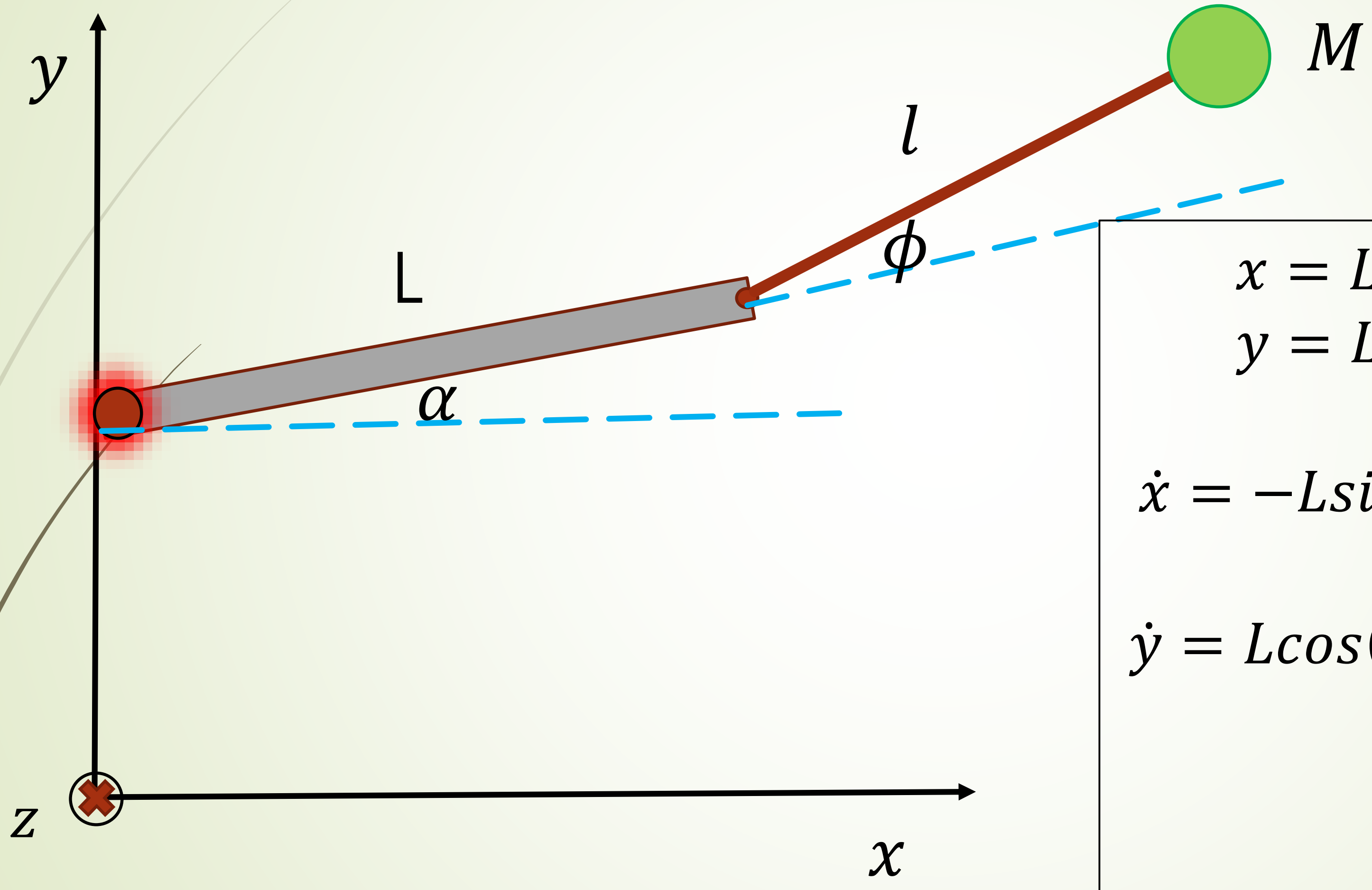
Behaves like a simple conical pendulum when disturbed in the α direction

Theory-Assumptions of the Model



- Ignoring the minimal deflection of the rod in the z-direction
- Assuming that l_2 remains constant (no contraction of the string)
- Assuming that the rod traces a circular motion α

Defining the generalized co-ordinates



$$x = Lcos(\alpha) + lsin(\theta)\cos(\theta)$$
$$y = Lcos(\alpha) + lsin(\theta)\sin(\phi)$$
$$z = -Lcos(\theta)$$
$$\dot{x} = -Lsin(\alpha)\dot{\alpha} + l(-\sin(\theta)\sin(\phi)\dot{\phi}$$
$$+\cos(\Phi)\cos(\Theta)\dot{\Theta}$$
$$\dot{y} = Lcos(\alpha)\dot{\alpha} + l\big(\sin(\Theta)\cos(\Theta)\,\dot{\Phi}\big) +$$
$$\sin(\Phi)\cos(\Theta)\dot{\Theta})$$

$$\dot{z} = lsin(\Theta)\dot{\Theta}$$

Lagrangian of the System

Kinetic Energy:

$$\frac{1}{2}\overbrace{M(\dot{x}^2+\dot{y}^2+\dot{z}^2)}^{\text{Velocity of the bob}}+\frac{1}{2}\overbrace{\left(\frac{1}{3}ML^2\right)}^{\text{Velocity of the rod}}\dot{\alpha}$$

Potential Energy:

$$\underbrace{-MgLcos(\theta)}_{\text{Potential Energy of bob}}+\underbrace{\frac{1}{2}k\alpha^2}_{\text{Elastic potential of the rod}}$$

Lagrangian= K.E-P.E

Euler-Lagrange Equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)=\frac{\partial L}{\partial \theta} \quad (1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right)=\frac{\partial L}{\partial \phi} \quad (2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right)=\frac{\partial L}{\partial \alpha} \quad (3)$$

$$ML(\ddot{y}\cos(\alpha) - \ddot{x}\sin(\alpha)) = -k\alpha + \frac{1}{3}ML^2\ddot{\alpha}$$

$$\ddot{y} = \ddot{x}\tan(\phi)$$

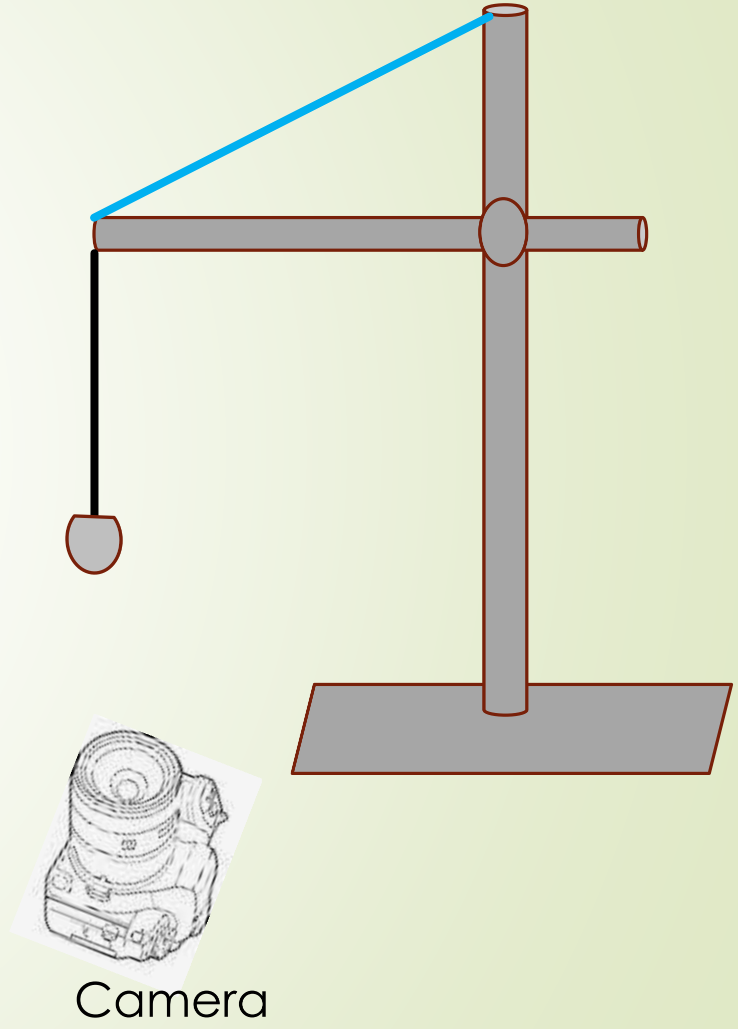
$$Ml\ddot{x}\cos(\phi)\cos(\Theta) + Ml\ddot{y}\sin(\phi)\cos(\theta) + \ddot{z}sinn(\theta) = -Mglsin(\theta)$$

Governing Equations

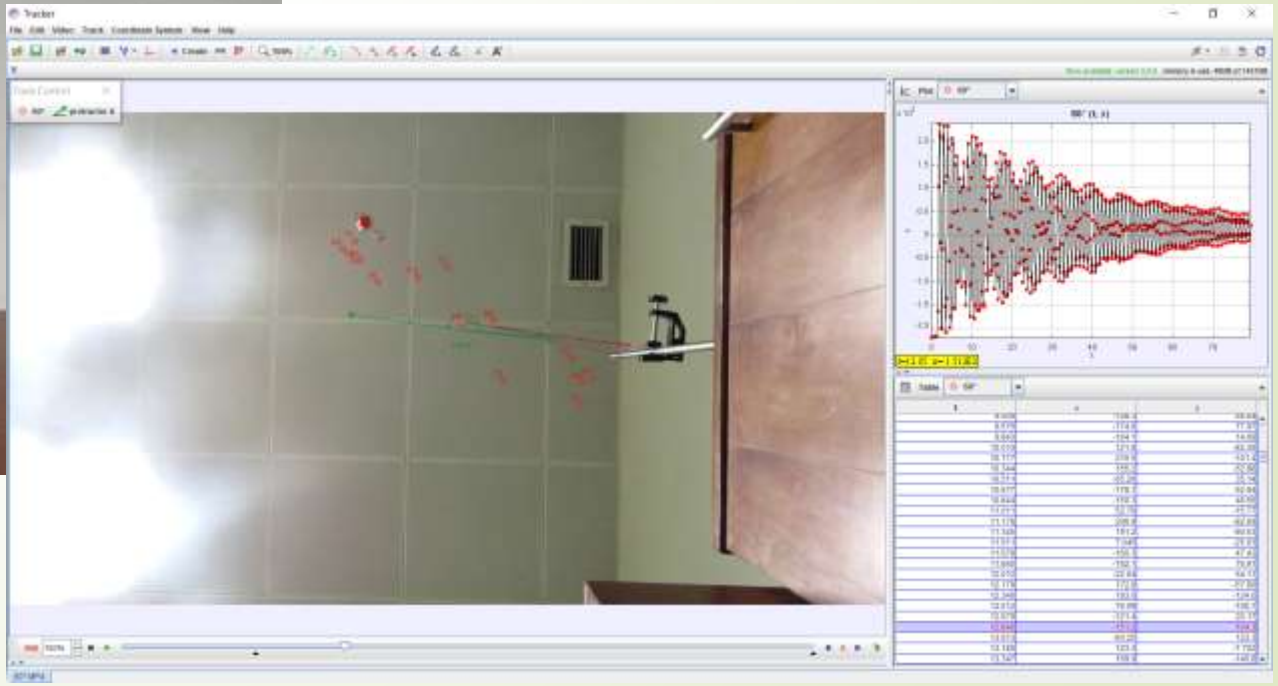
Can be written as 2nd order differential equations describing the acceleration in the (alpha),(theta) and (phi) directions

Experimental Set-up

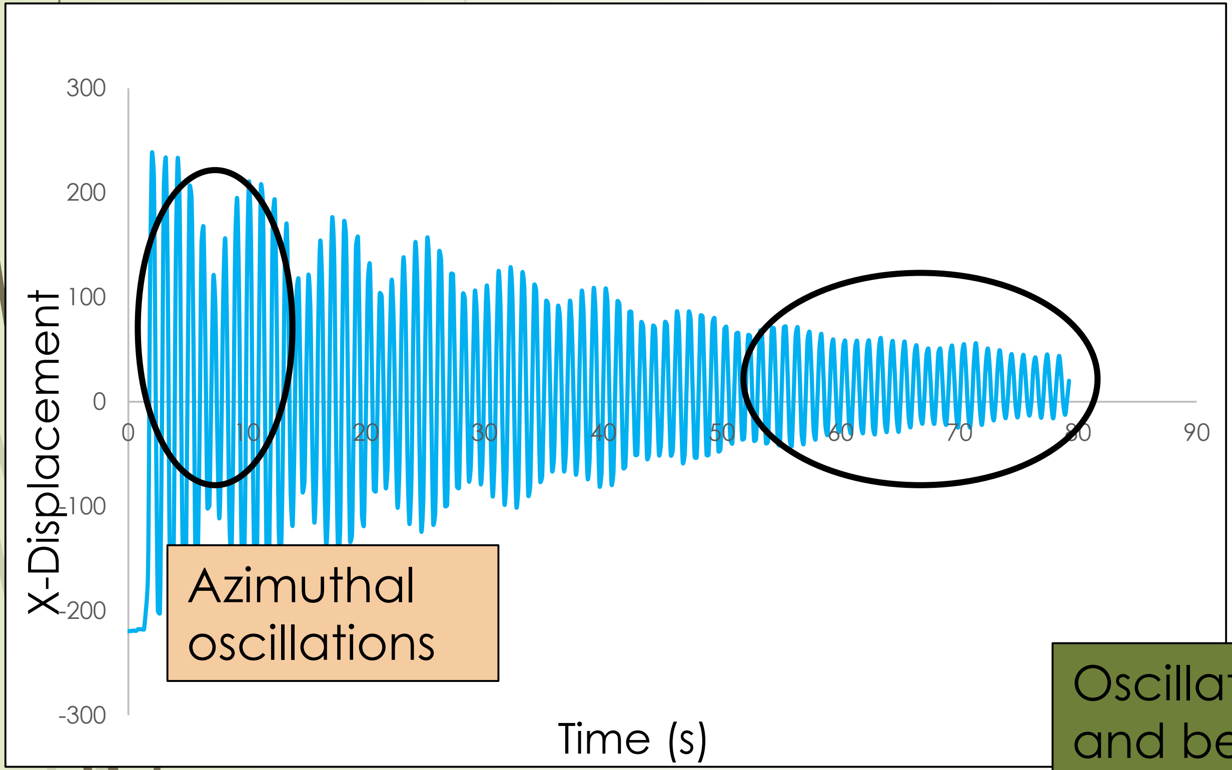
- Rigid stand
- Strings
- Bobs of variable masses
- Camera
- Release mechanism for bob



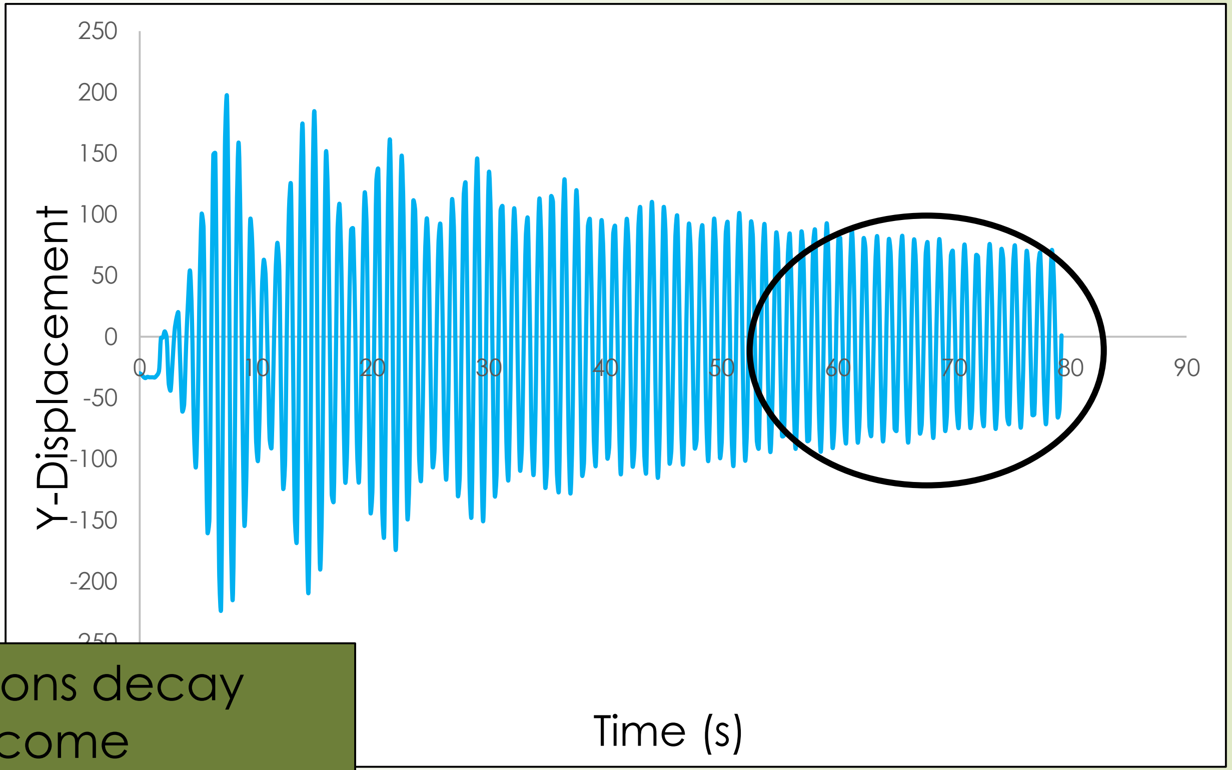
Experimental Set-up



Displacement-Time Graphs

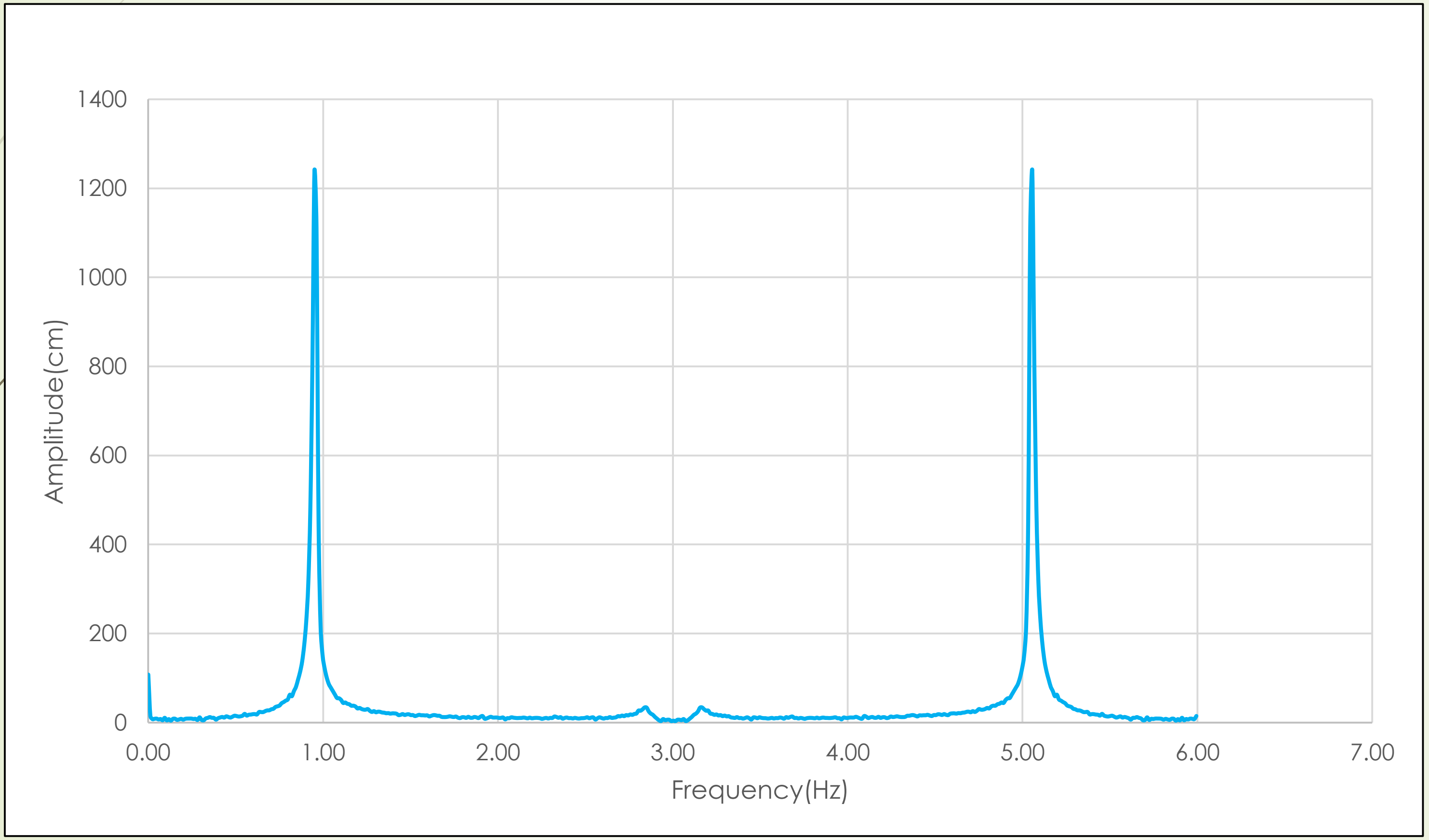


Azimuthal oscillations



Oscillations decay and become indistinguishable

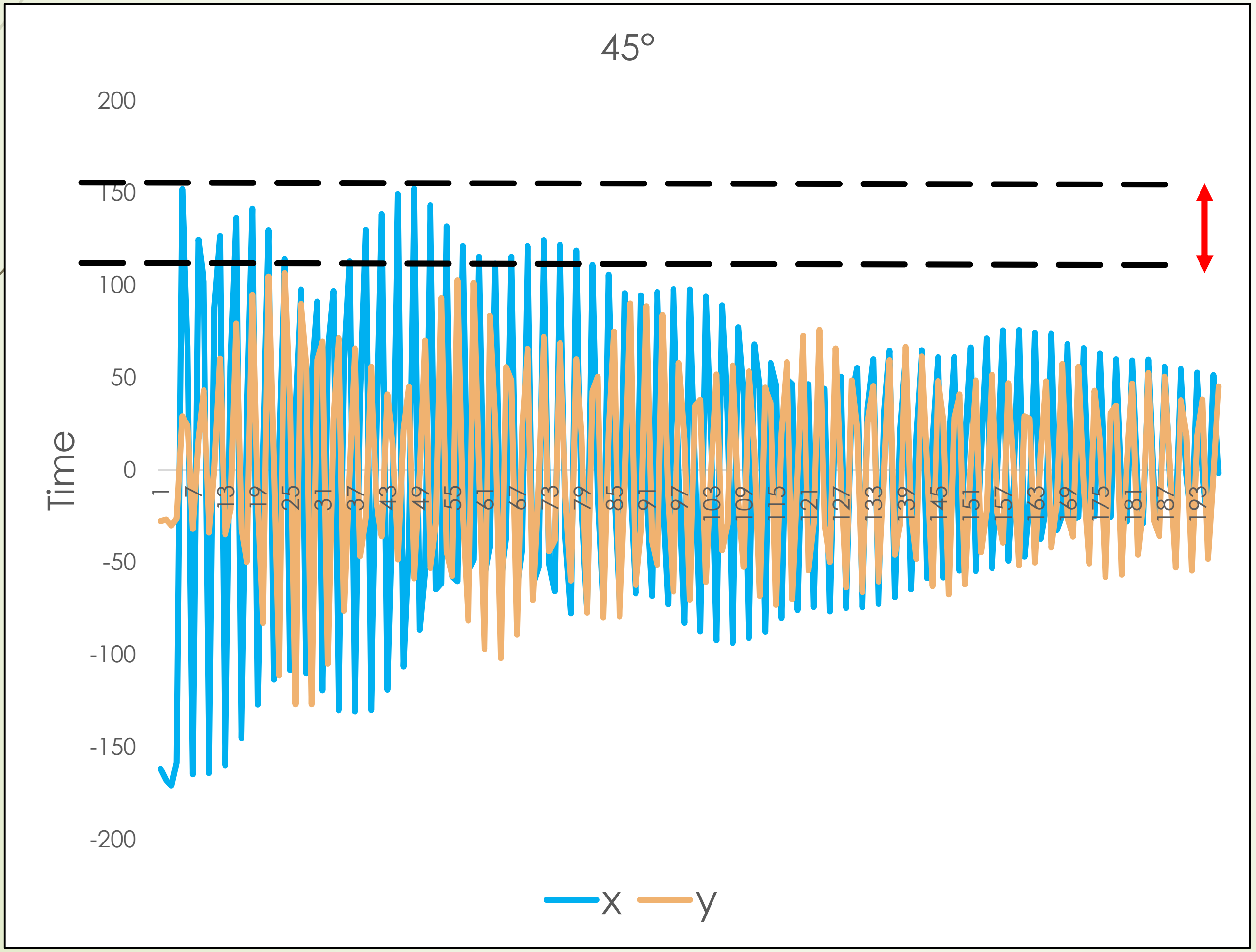
Fourier Analysis



Two bands corresponding to the Azimuthal and Radial Oscillations

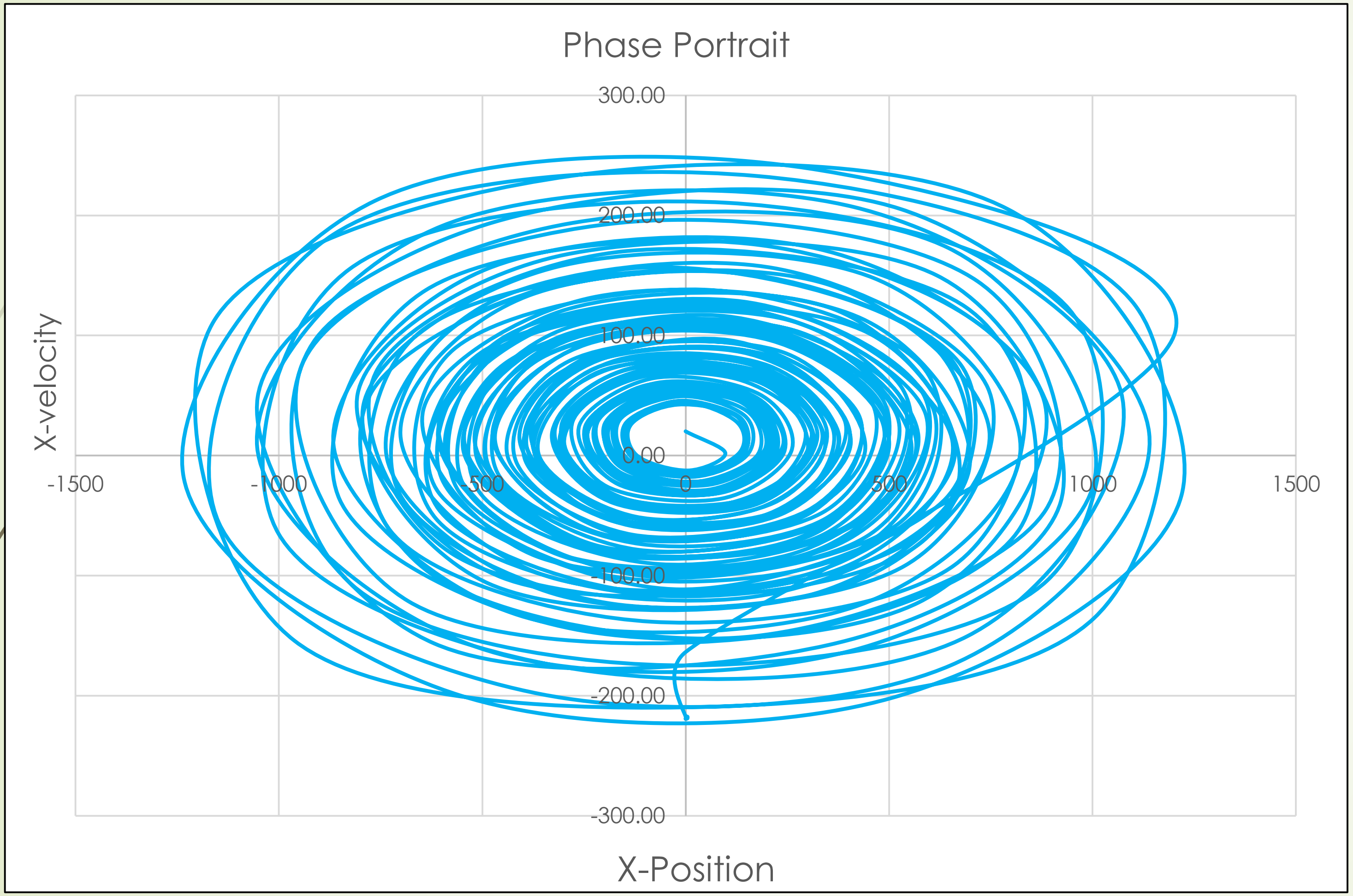
Comparing the Amplitudes

Amplitude Ratio= $\frac{Max\ ampltude\ in\ X\ direction}{Max\ ampltude\ in\ the\ Y\ direction}$



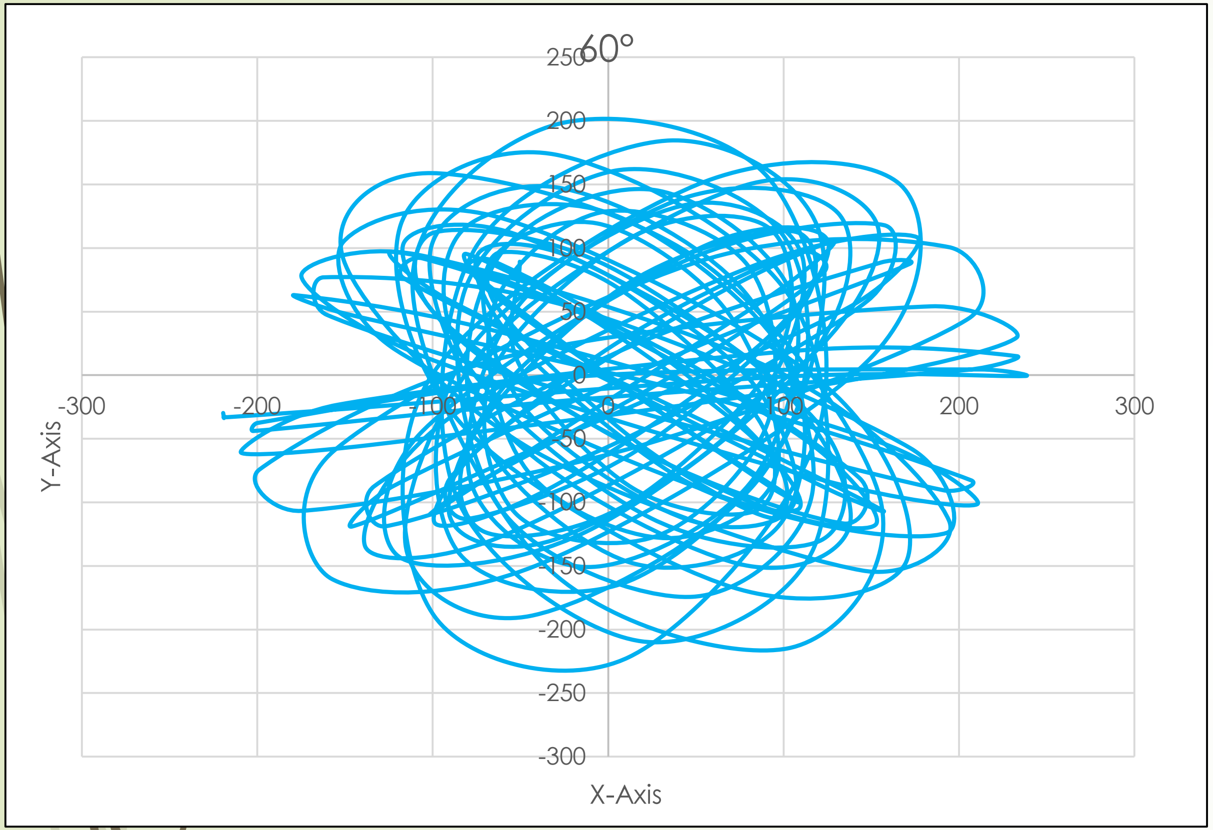
Obtained using the Max function in Excel

Phase Portrait

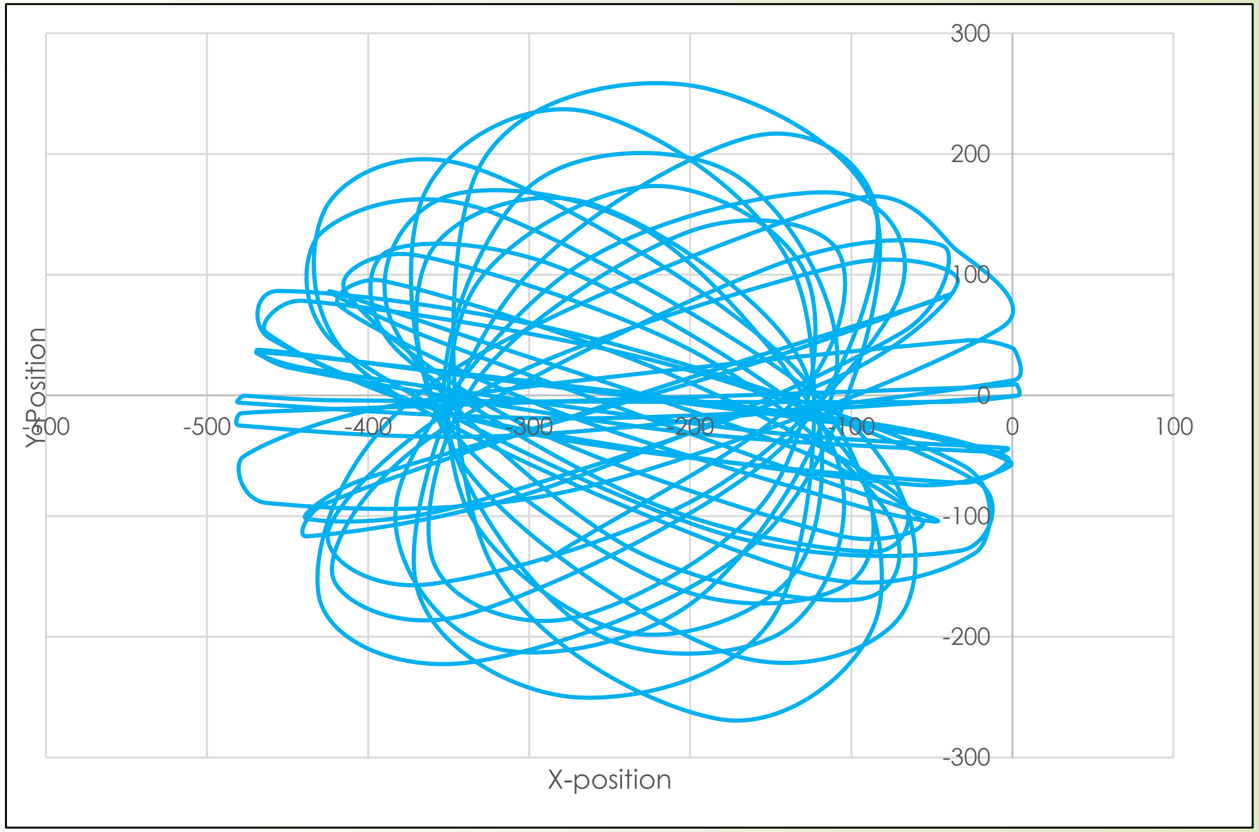


Shows non-chaotic, periodic motion for the general case

Experiment- release angle



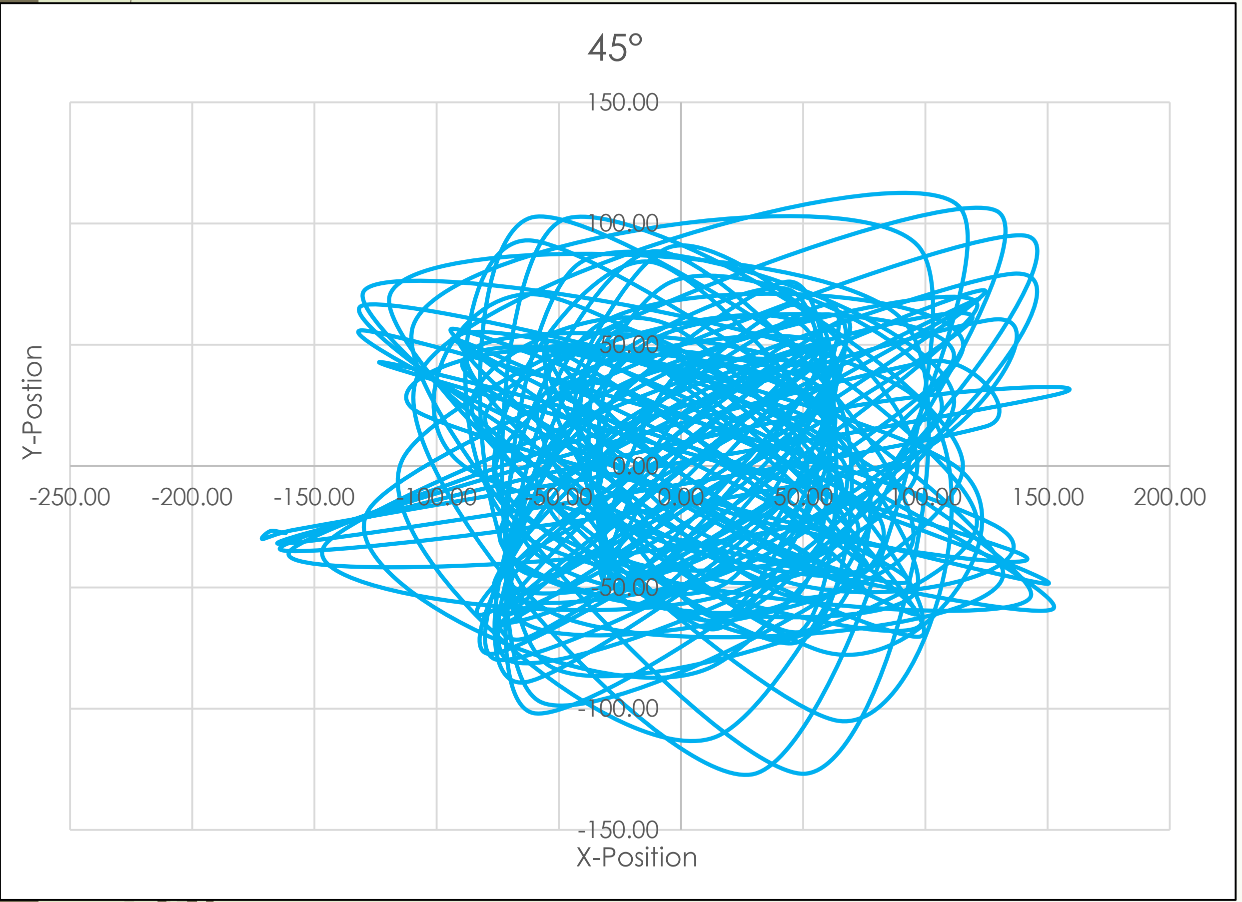
60°

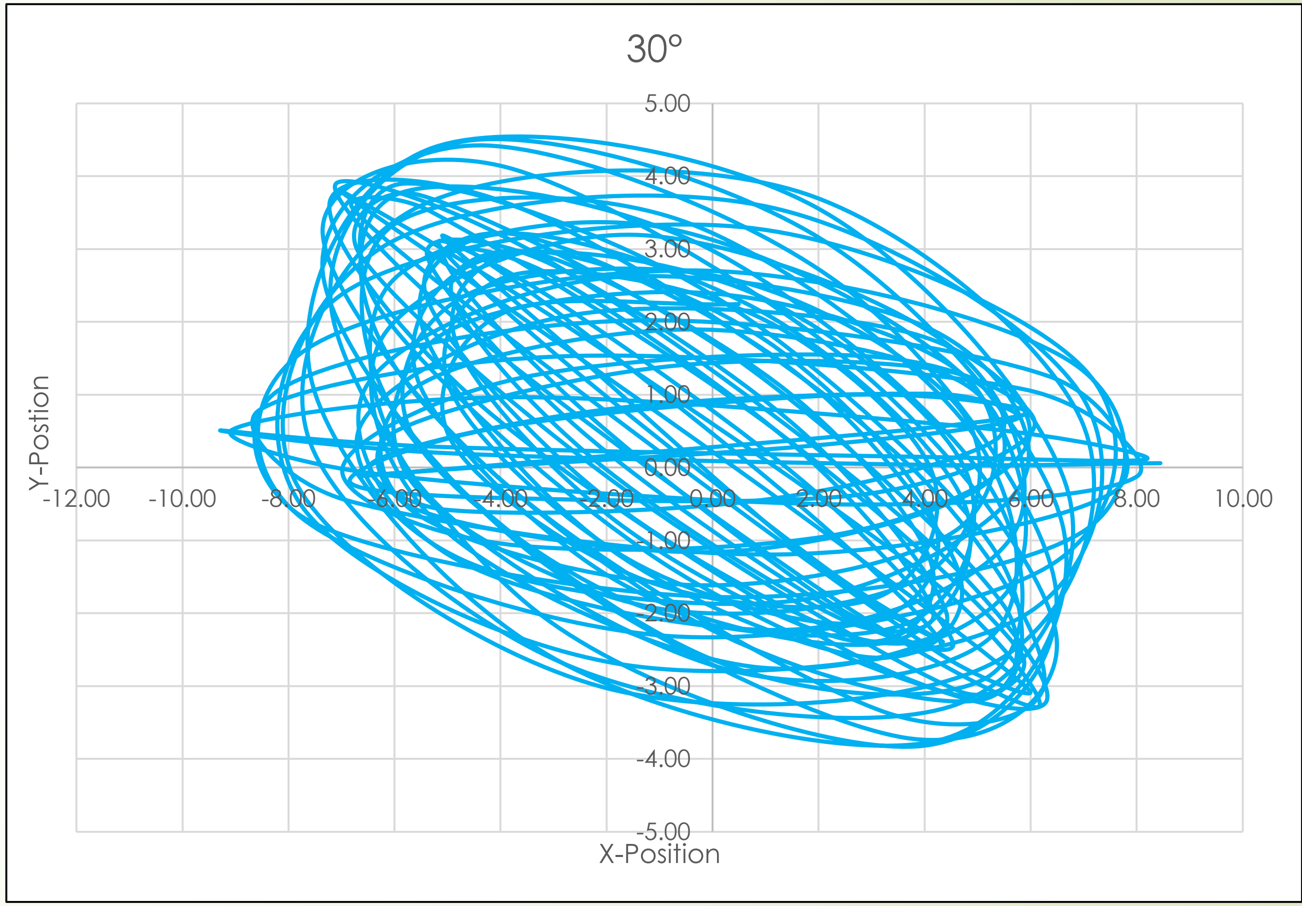


90°

Bigger angles => higher transitions rate

Initial Angles



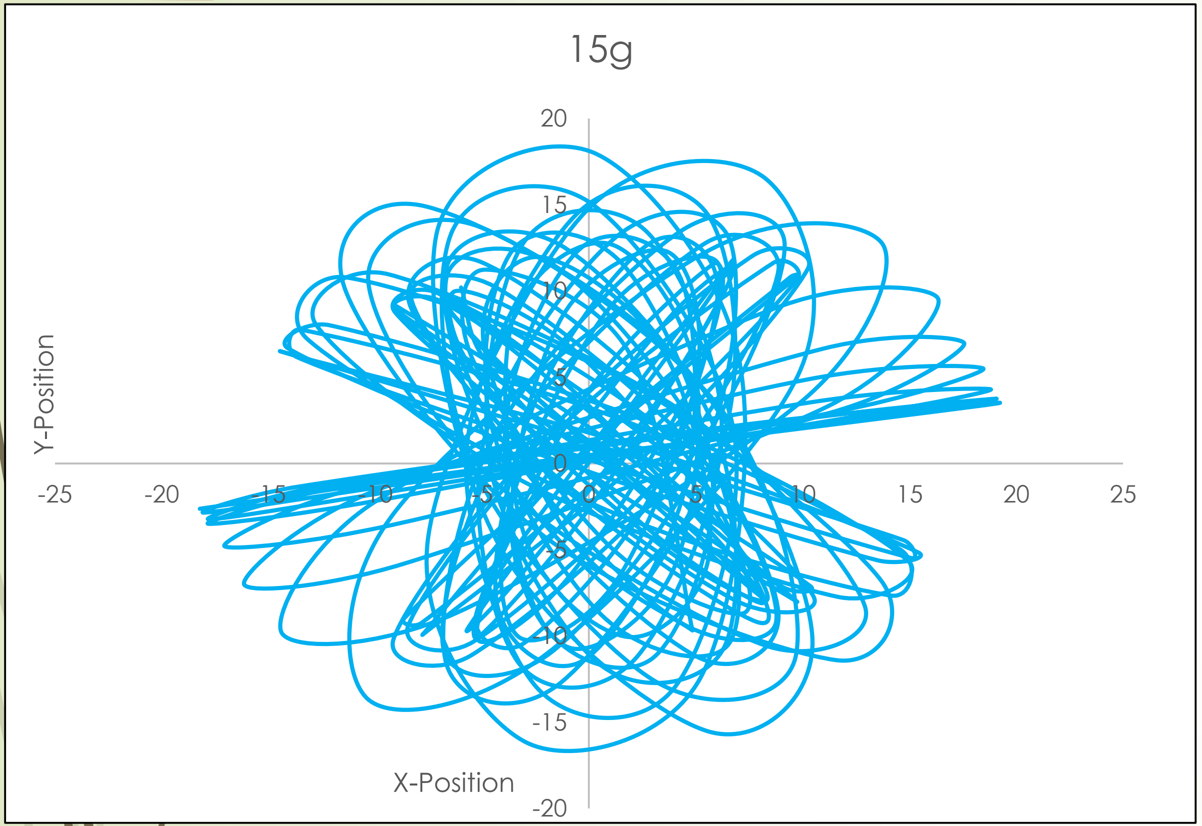


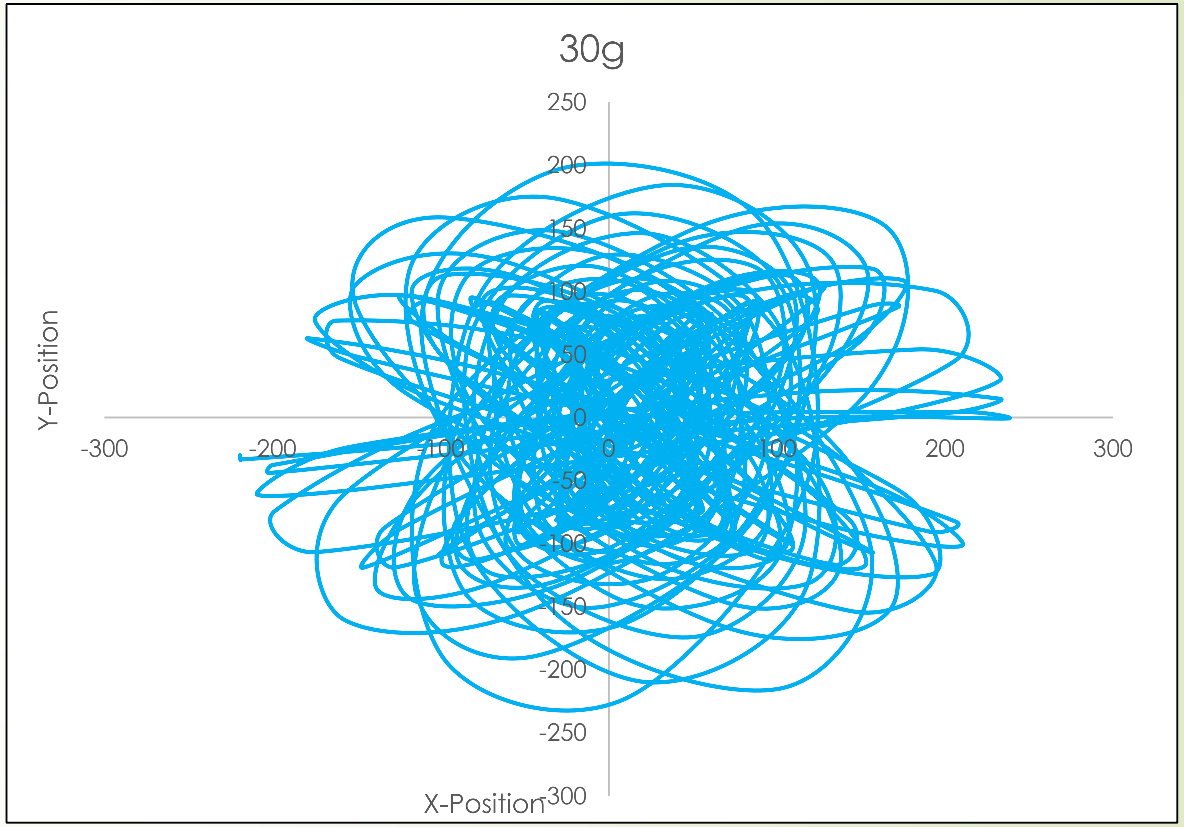
45°

30°

Smaller angles => Lower transitions rate

Change in Mass (light)

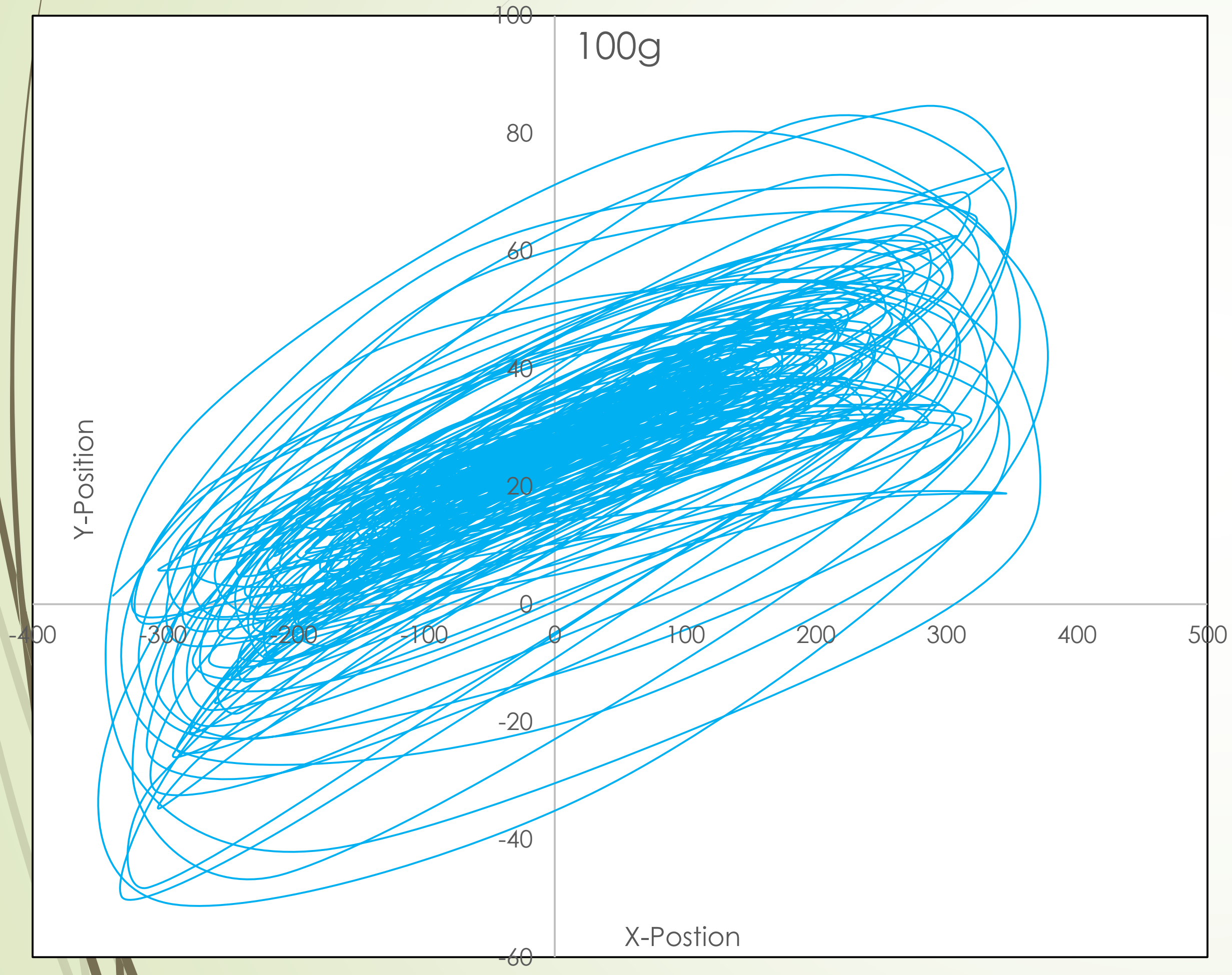




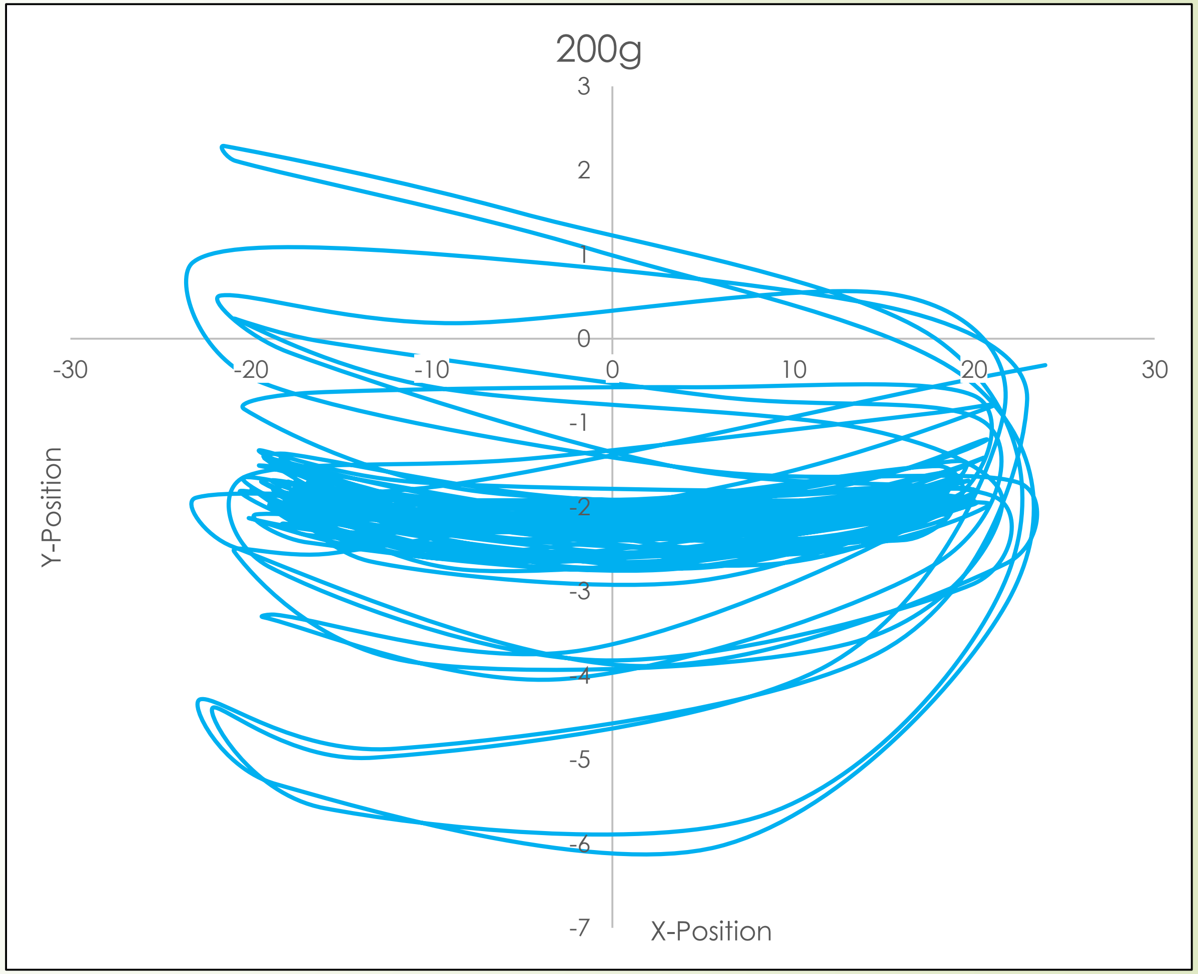
15g

30g

Change in Mass (heavy)

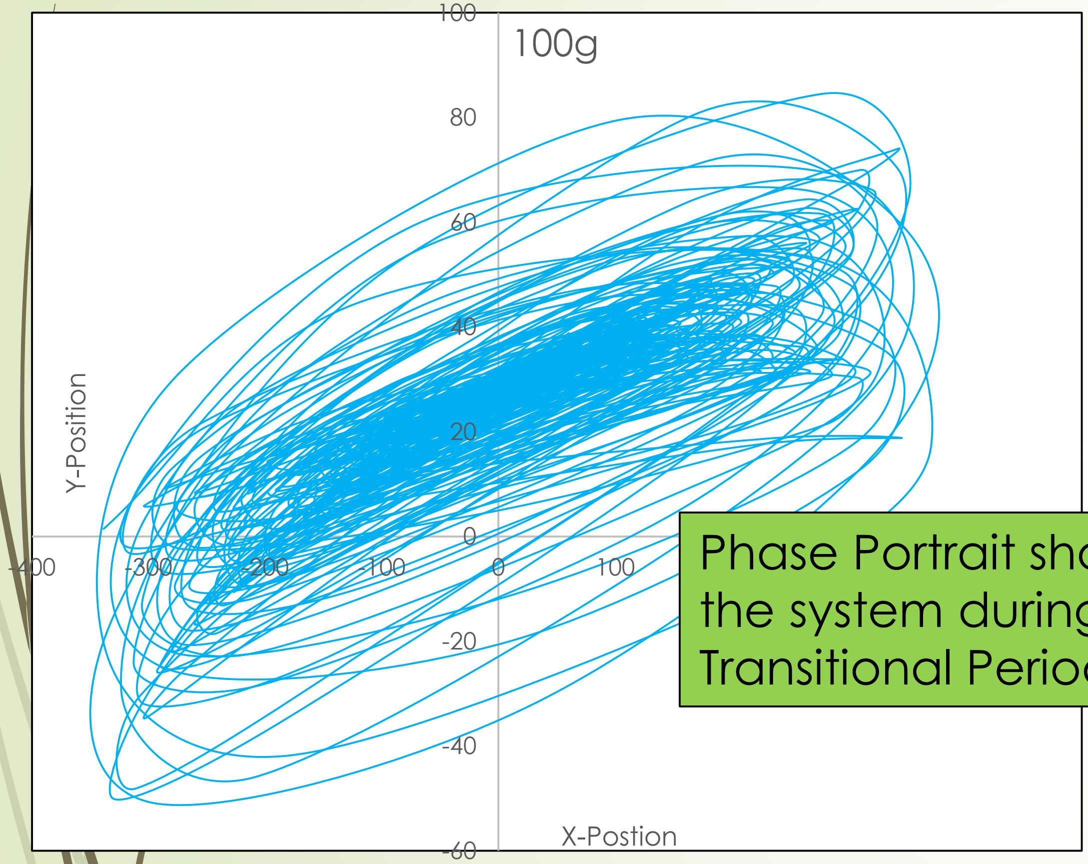


100g

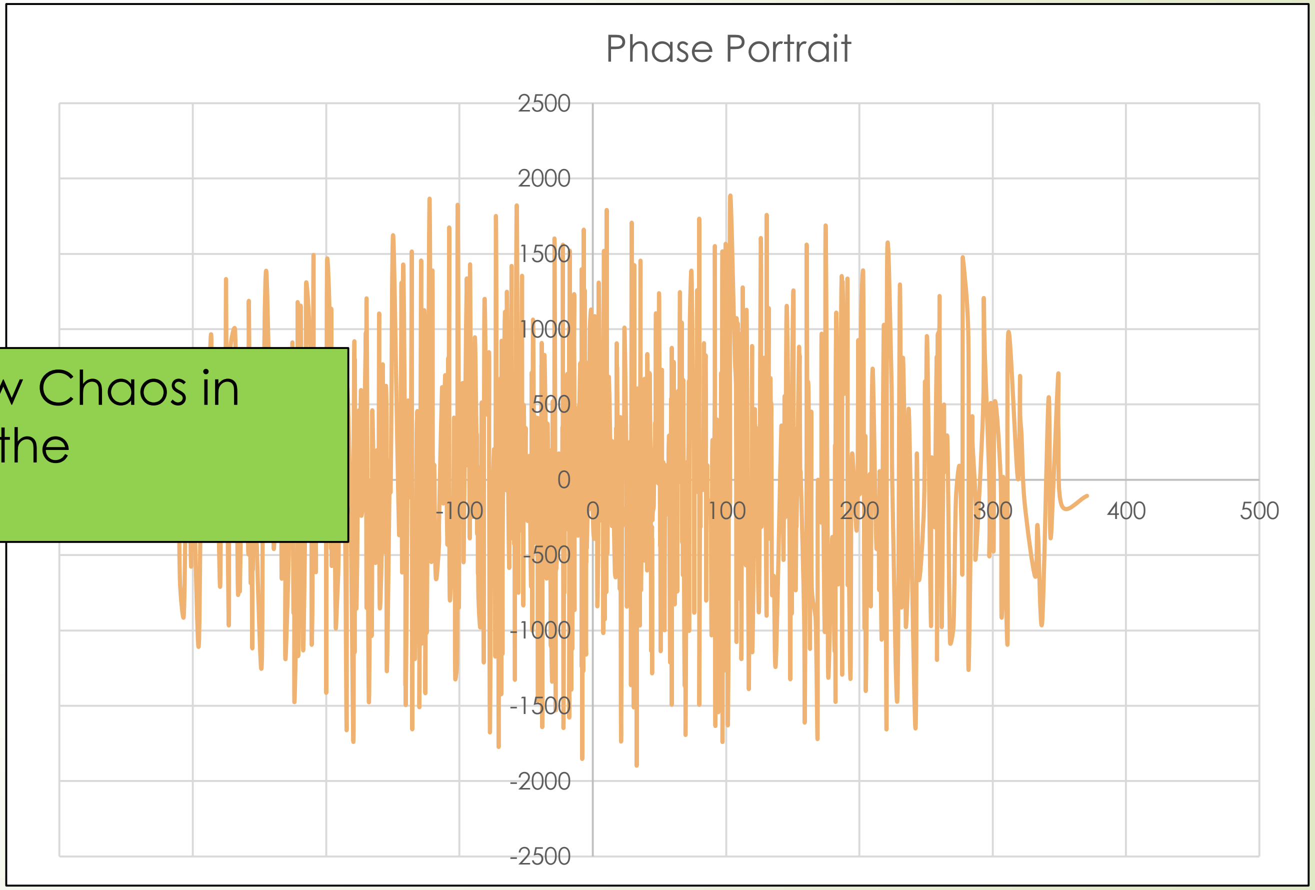


200g

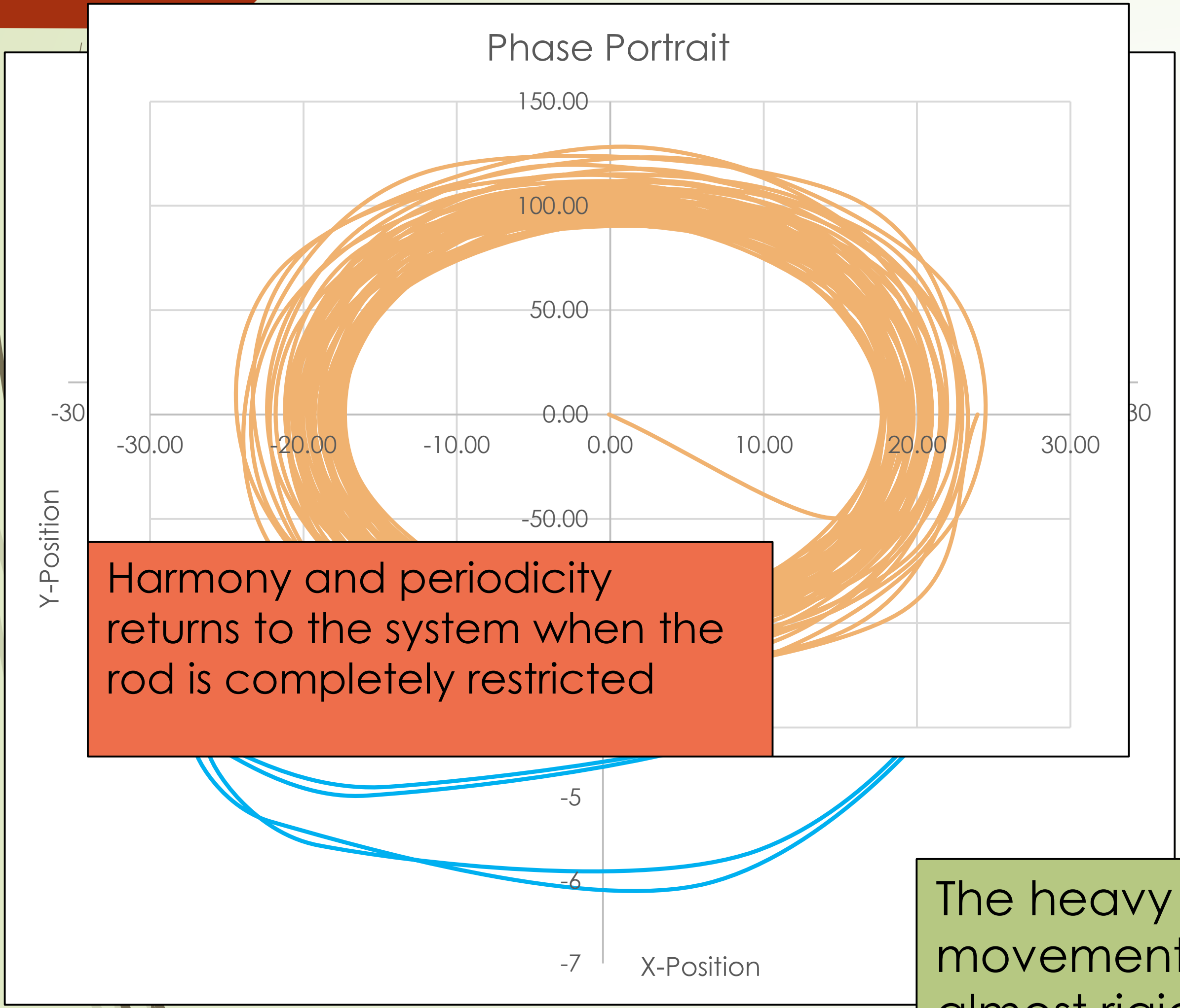
Effect of Mass on the Elasticity of rod



Phase Portrait show Chaos in the system during the Transitional Period

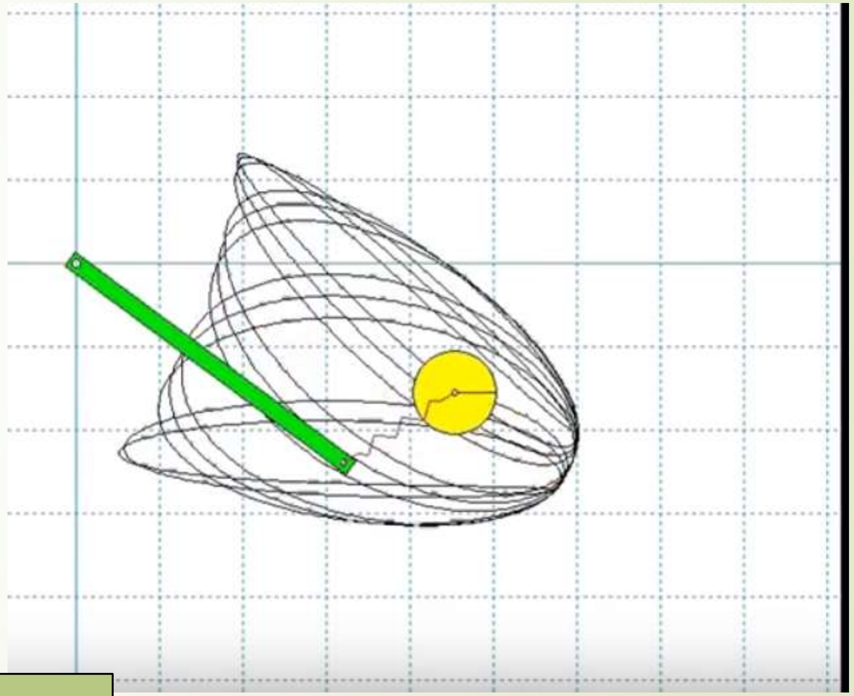


Effect of mass on the elasticity of rod



Harmony and periodicity returns to the system when the rod is completely restricted

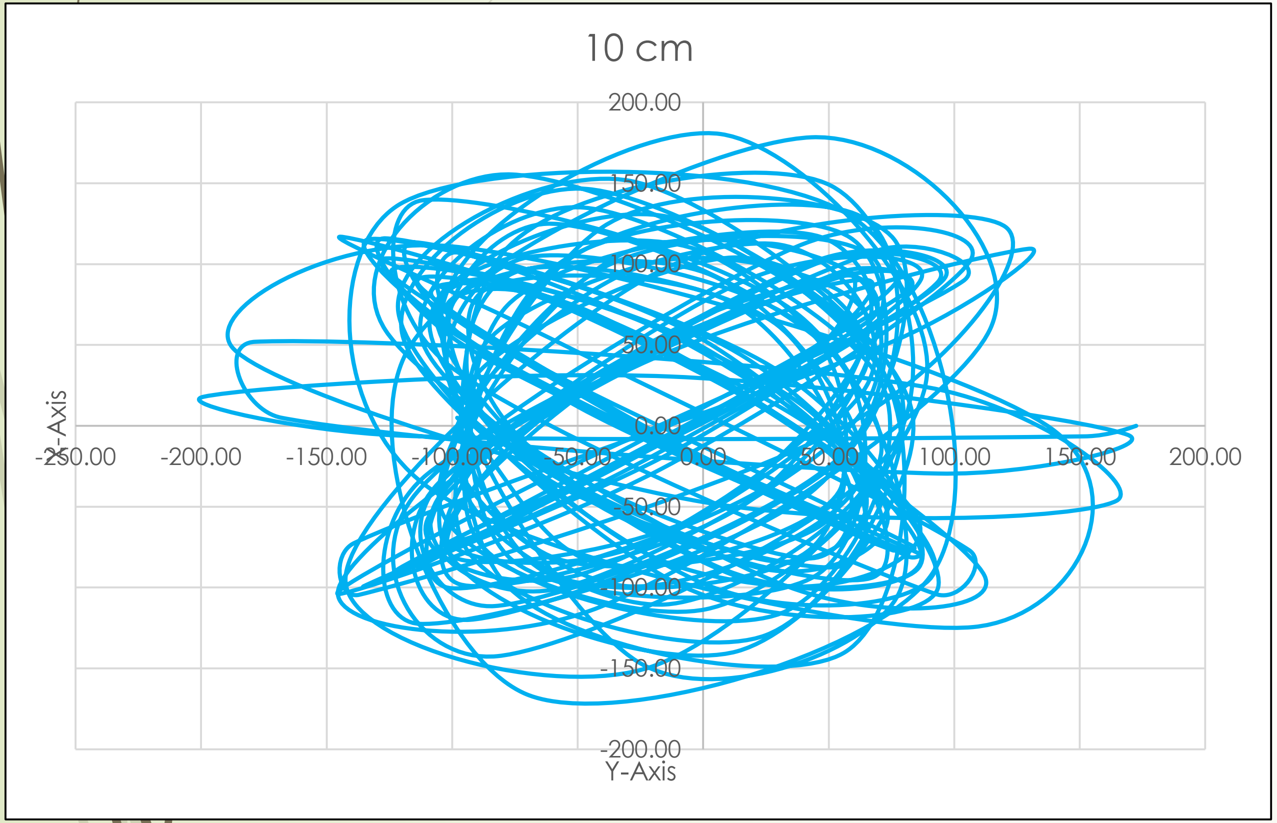
*

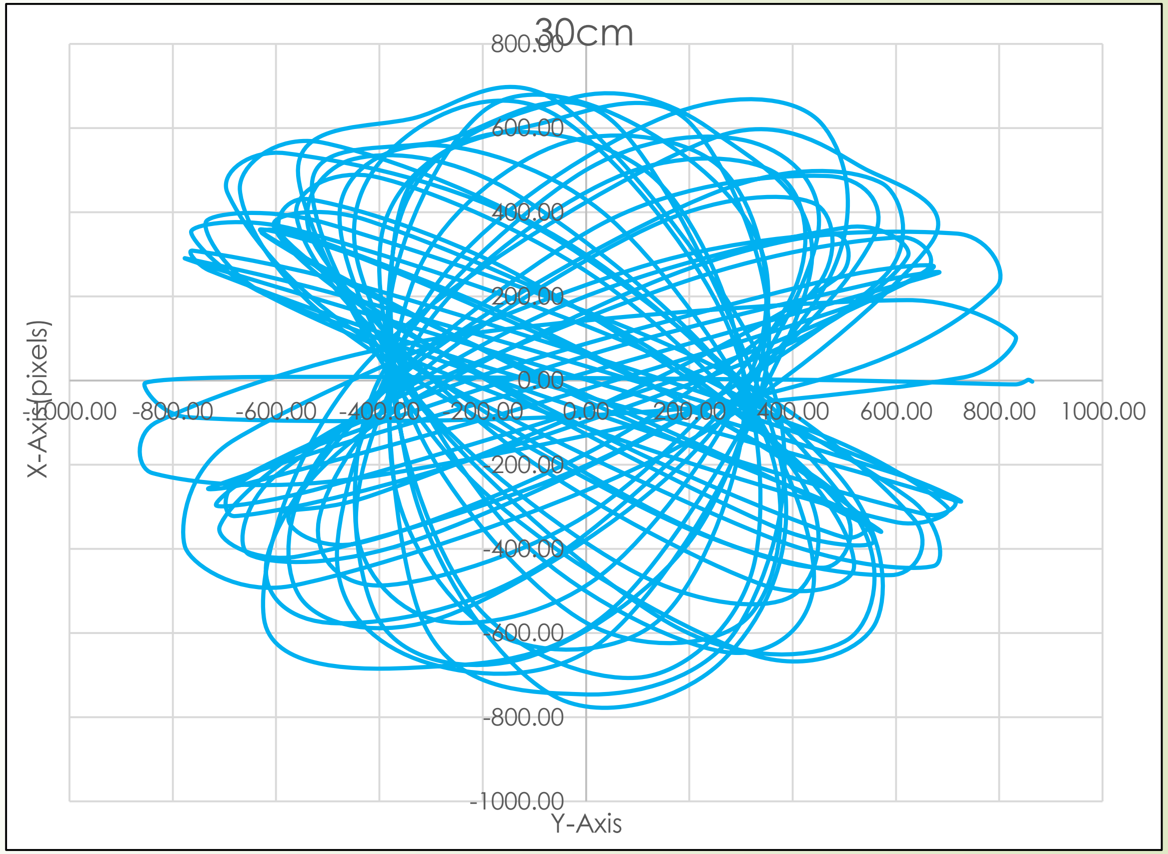


The heavy mass limits the movement of rod, leaving an almost rigid rod that can rotate(but not bend)

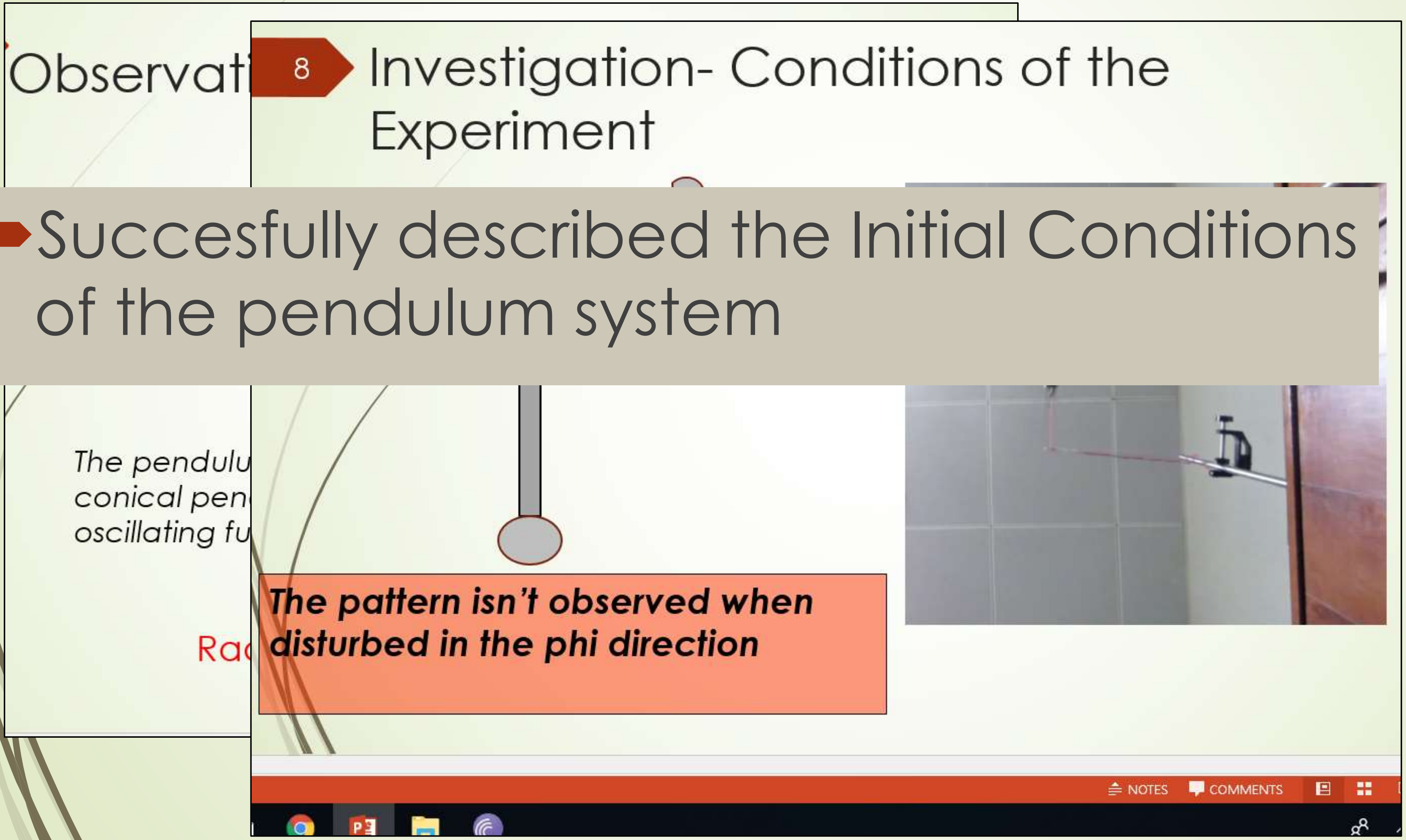
https://www.youtube.com/watch?v=1BhCXEd1zM4

Length of the String





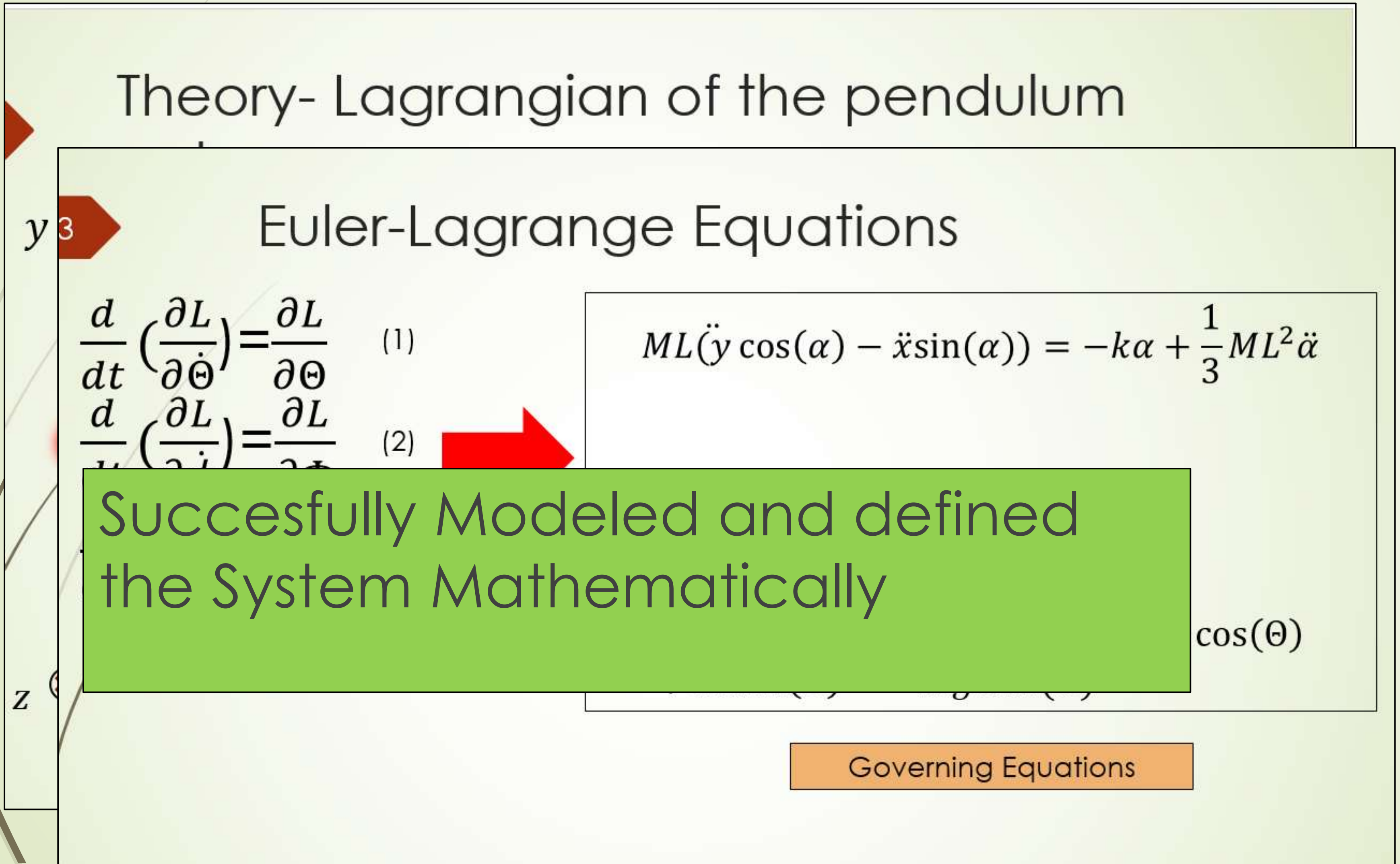
Conclusion-Preliminary



- Succesfully described the Initial Conditions of the pendulum system

Pendulum only works when disturbed in the Radial Direction

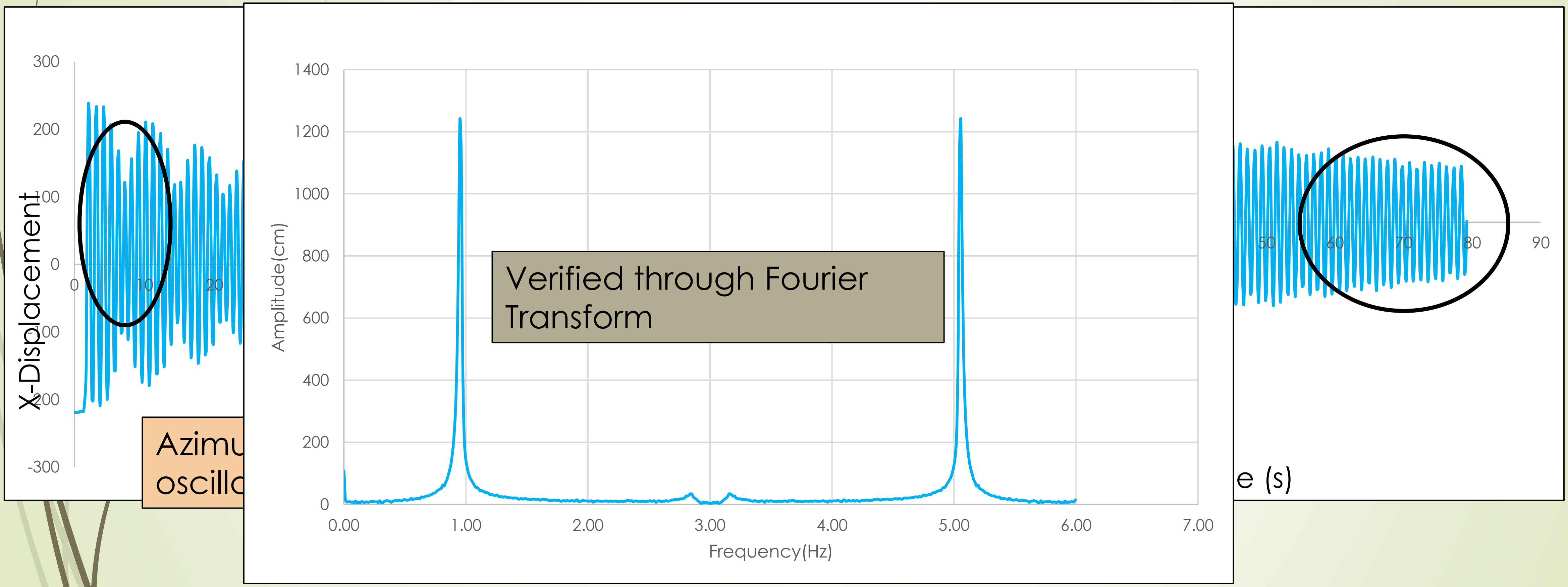
Conclusion-Theory



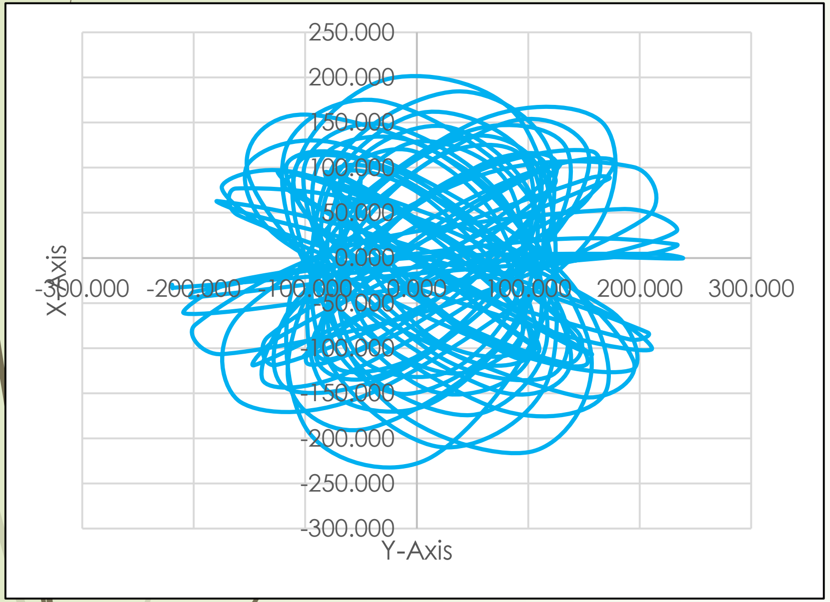
Succesfully Modeled and defined the System Mathematically

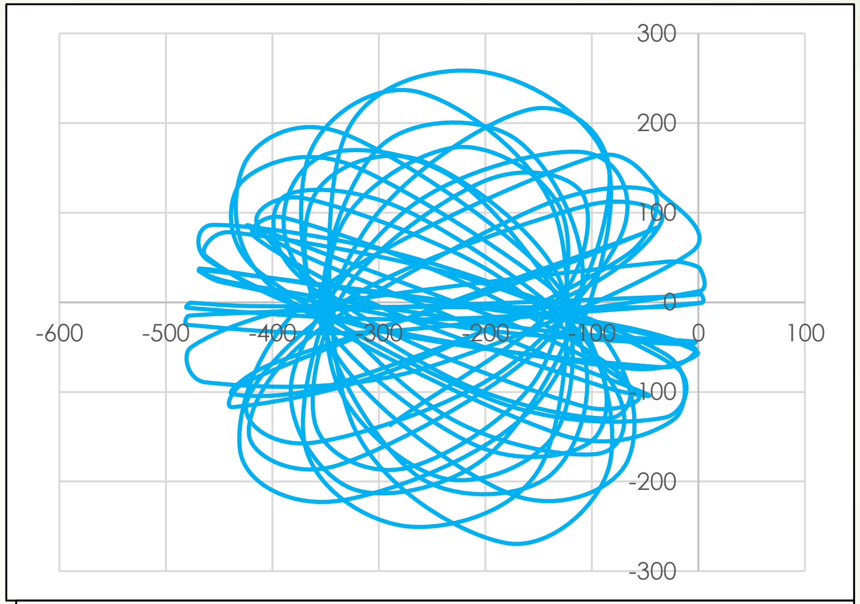
The bob has 3 degrees of Freedom which complicate the model

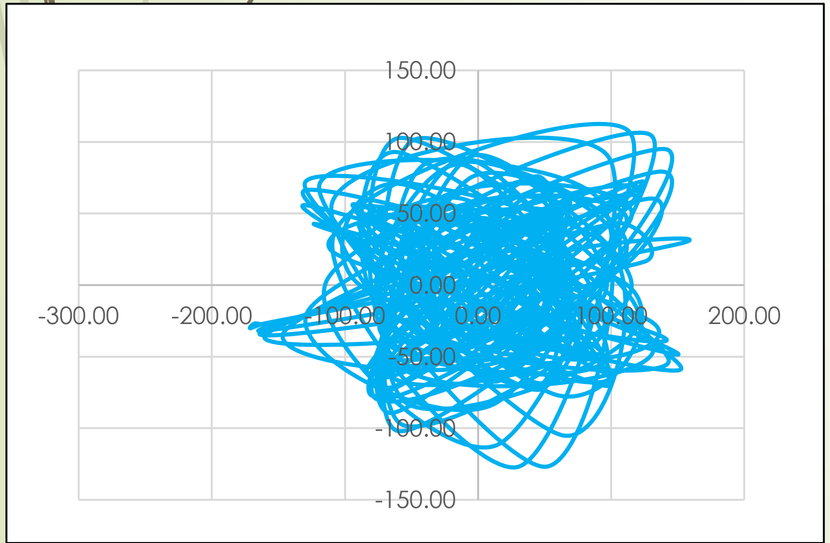
Conclusion-Experiments(Describing Frequencies)

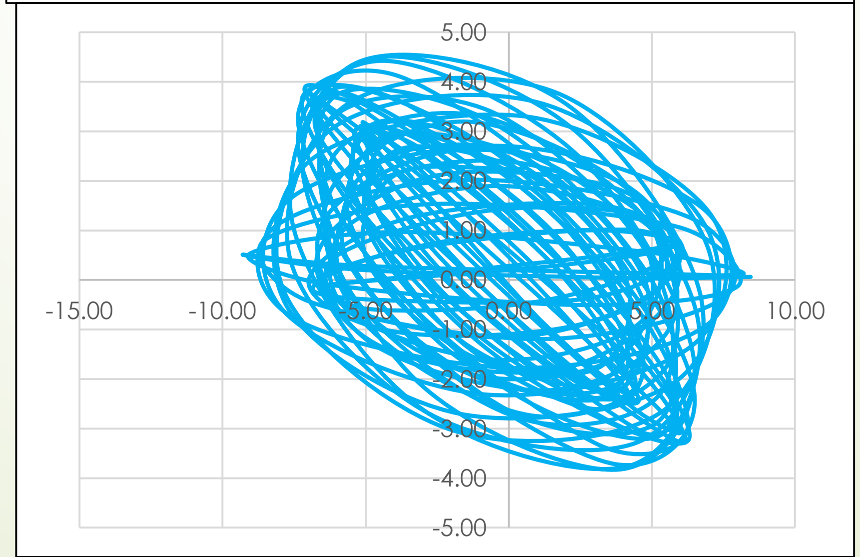


Conclusion-Experiments









Investigated and Explained relations of the system to 3 critical parameters

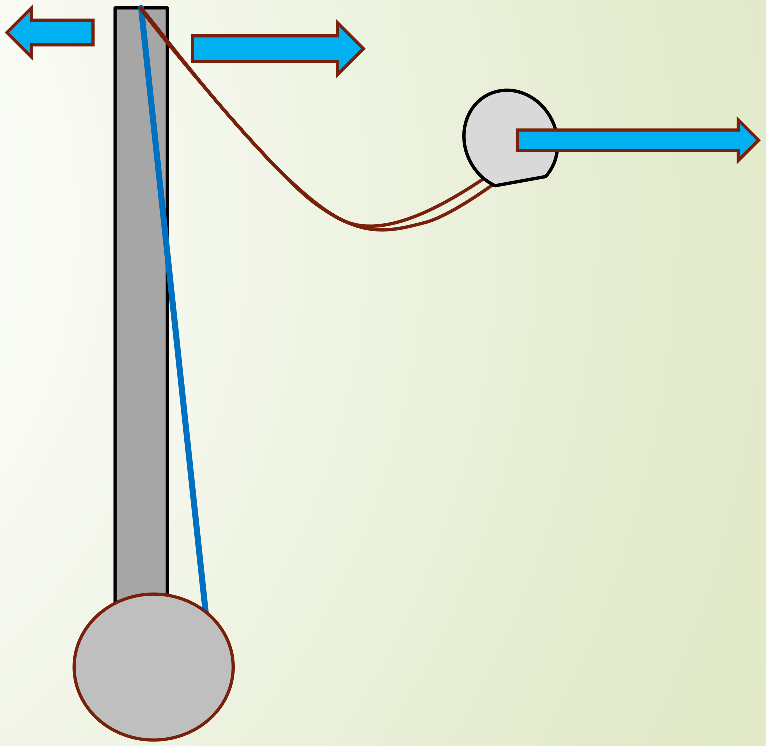
Thankyou!

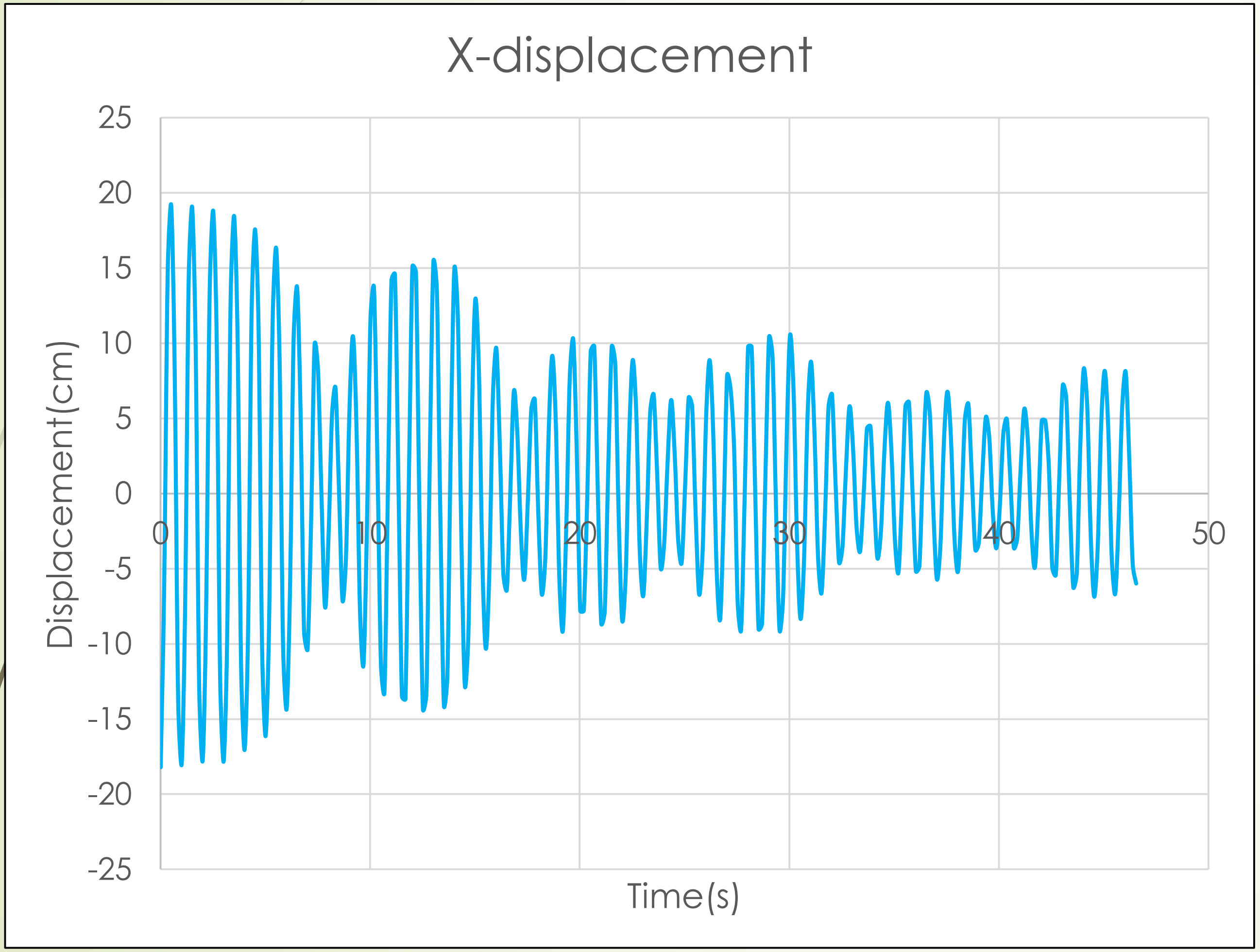
Relevant Parameters

- Length of the string
- Elasticity of the rod
- Weight of the bob
- Initial Angles

Theory- Conservation of energy

- The radial oscillations of the bob disturb the equilibrium of the elastic rod causing to move in the direction of the bob's motion.
- Since the rod is fixed on one end, it traces a circular path (azimuthal oscillations)
- The restoring force of the rod acts against the motion of the bob.
- This sets up a chain of energy transfer where the motion of the bob is transferred into the azimuthal perturbations of the rod




X-displacement
Displacement(cm)
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
10
20
30
40
50
Time(s)