Introduction to MATLAB

Instructor:

Azeem Iqbal

Lab Instructor

Centre for Experimental Physics Education (CEPE)

Lahore University of Management Sciences (LUMS)

Lahore, Pakistan

Contents

- Introduction to MATLAB
- Layout
- Basic arithmetic operations
- Creating vectors and matrices
- Matrix arithmetic
- Data manipulation
  - Introduction to “for” Loops
- Graphs and plotting
  - Introduction to plotting
  - Multiple plots
  - Resolution of graph
- Curve fitting
  - Least square curve fitting of linear data

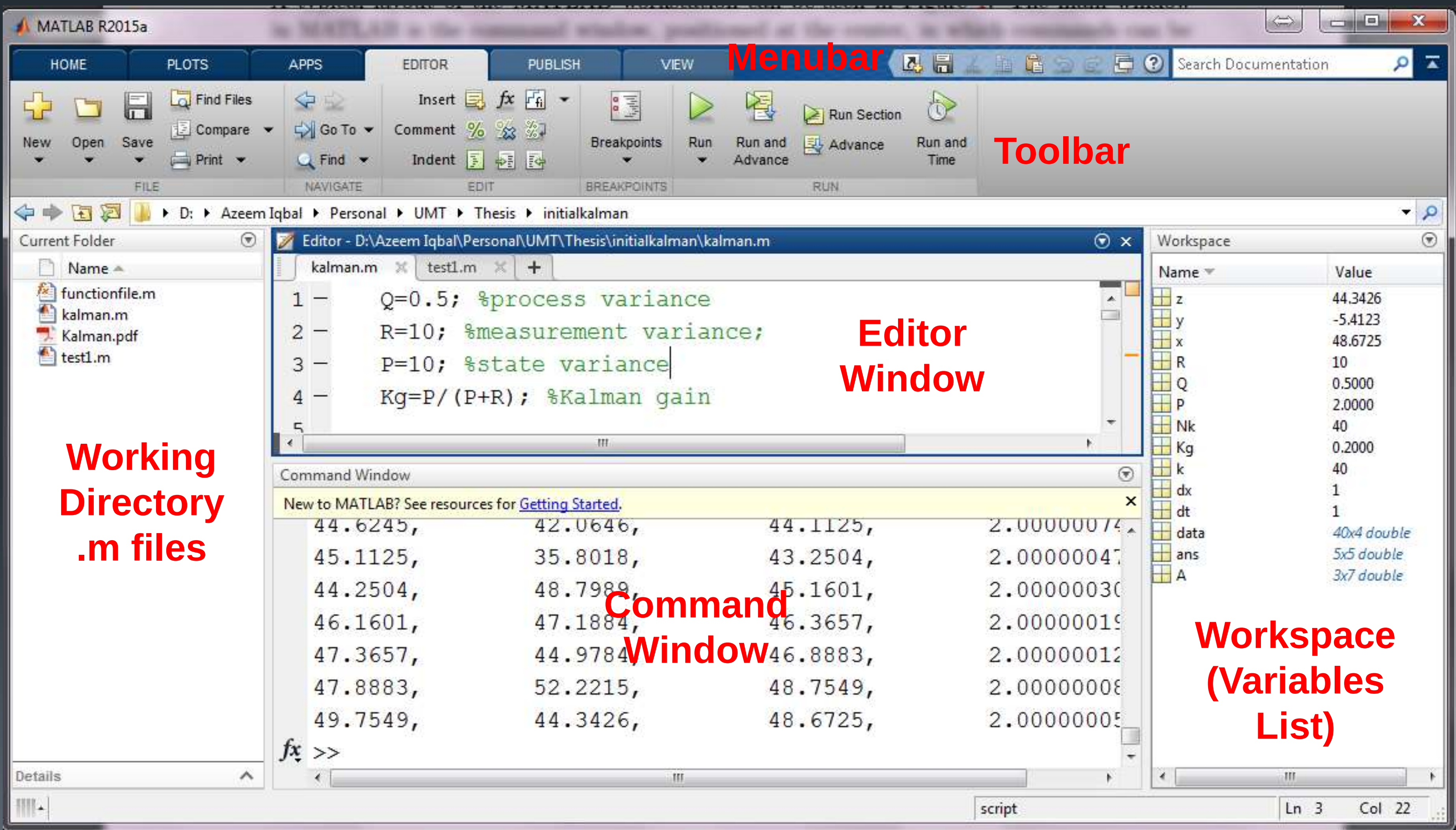
Introduction to MATLAB

- MATLAB stands for Matrix Laboratory.
- Developed by Cleve Moler from the University of New Mexico in the late 1970s.
- It is a high-performance language for technical computing and integrates computation, visualization, and programming environment.
- It has sophisticated data structures, contains built-in editing and debugging tools, and supports object-oriented programming.
- MATLAB was first adopted by researchers and practitioners in control engineering.
- It is now also used in education, in particular the teaching of linear algebra, numerical analysis, and is popular amongst scientists involved in image processing.

Introduction to MATLAB

Layout

Layout


MATLAB R2015a
Menubar
HOME PLOTS APPS EDITOR PUBLISH VIEW
Toolbar
Editor Window
Working Directory .m files
Command Window
Workspace (Variables List)
Current Folder
Workspace
Command Window
Editor - D:\Azeem Iqbal\Personal\UMT\Thesis\initialkalman\kalman.m
Q=0.5; %process variance
R=10; %measurement variance;
P=10; %state variance
Kg=P/(P+R); %Kalman gain

Introduction to MATLAB

Basic arithmetic operators

Basic Arithmetics

```
a = 5;
b = 4;
```

Summation	sum	=	a + b;
Difference	diff	=	a – b;
Product	prod	=	a * b;
Division	div	=	a / b;
Exponent	exp	=	a^2;
Square root	sqrt	=	sqrt(b);

Basic Arithmetics

Concept of precedence:

P E M D A S

			Order of precedence
P	=	Parentheses	1
E	=	Exponents	2
M	=	Multiplication	3
D	=	Division	4
A	=	Addition	5
S	=	Subtraction	6

Which ever comes first in left to right order of equation

Basic Arithmetics

Solve:

$$6 \div 2\,(2 + 1)$$

What is the answer
1 or 9?
The correct answer is “9”

Introduction to MATLAB

Creating vectors and matrices

Vector and Matrices

- Vector

```
x = [1 2 5 1]   Row or Column matrix
```

- Matrix

```
x = [1 2 3 ; 5 1 4 ; 3 2 -1]
```

$$x = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 1 & 4 \\ 3 & 2 & -1 \end{pmatrix}$$

- Transpose

```
y = x'
```

$$y = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

Vector and Matrices

- evenlist = [2 4 6 8 10 12 14 16 18];
- evenlist2 = 2:2:18;
- evenlist3 = [2; 4; 6; 8; 10; 12; 14; 16; 18];

$$\begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \\ 14 \\ 16 \\ 18 \end{pmatrix}$$

- evenlist = evenlist3';

Matrix arithmetic

a = [2 4 6; 1 3 5; 7 9 11];

$$\begin{pmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 7 & 9 & 11 \end{pmatrix}$$

Type size(a) to check the size of matrix, in this case it is 3x3 matrix

- You may want to extract a few values using:
  a(2,2)
  a(2,:)
  a(:,2)

Dot and scalar product

$$a = \begin{pmatrix} 5 & 8 & 9 \\ 2 & 4 & 6 \\ 1 & 3 & 5 \end{pmatrix} \qquad b = \begin{pmatrix} 4 & 6 & 7 \\ 2 & 1 & 3 \\ 5 & 3 & 8 \end{pmatrix}$$

a * b Matrix Multiplication

a .* b Element by Element Multiplication

Data manipulation using “for” Loops

Basic structure:

for (condition)
statements
end

Generate the first 15 Fibonacci numbers

```
f=[1 1];

for k=1:15
    f(k+2) = f(k+1) + f(k);
end
```

Let’s practice!

Solve the first exercise …

Contents

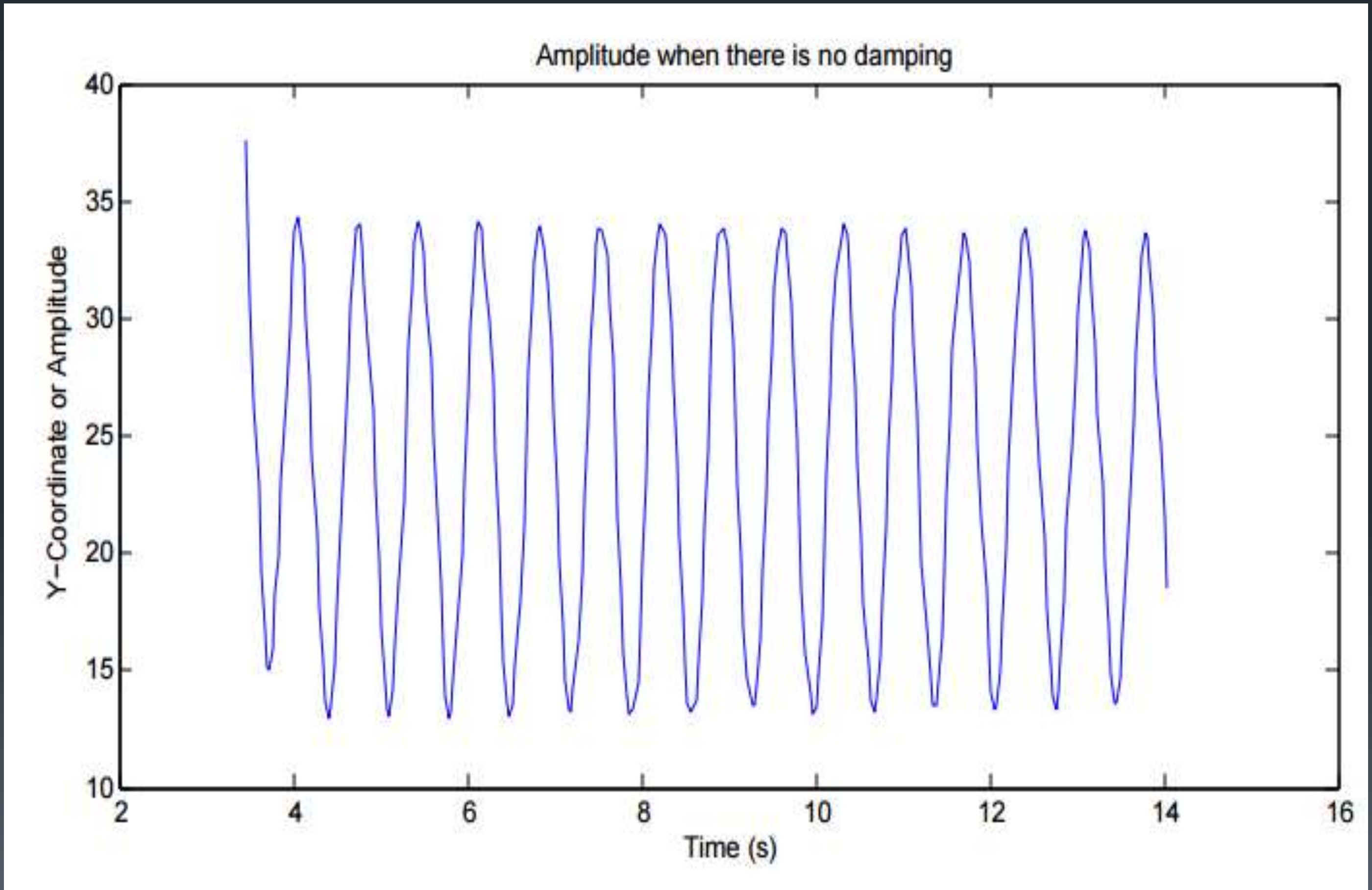
Introduction to MATLAB

Graphs and plotting

Graphs and plotting

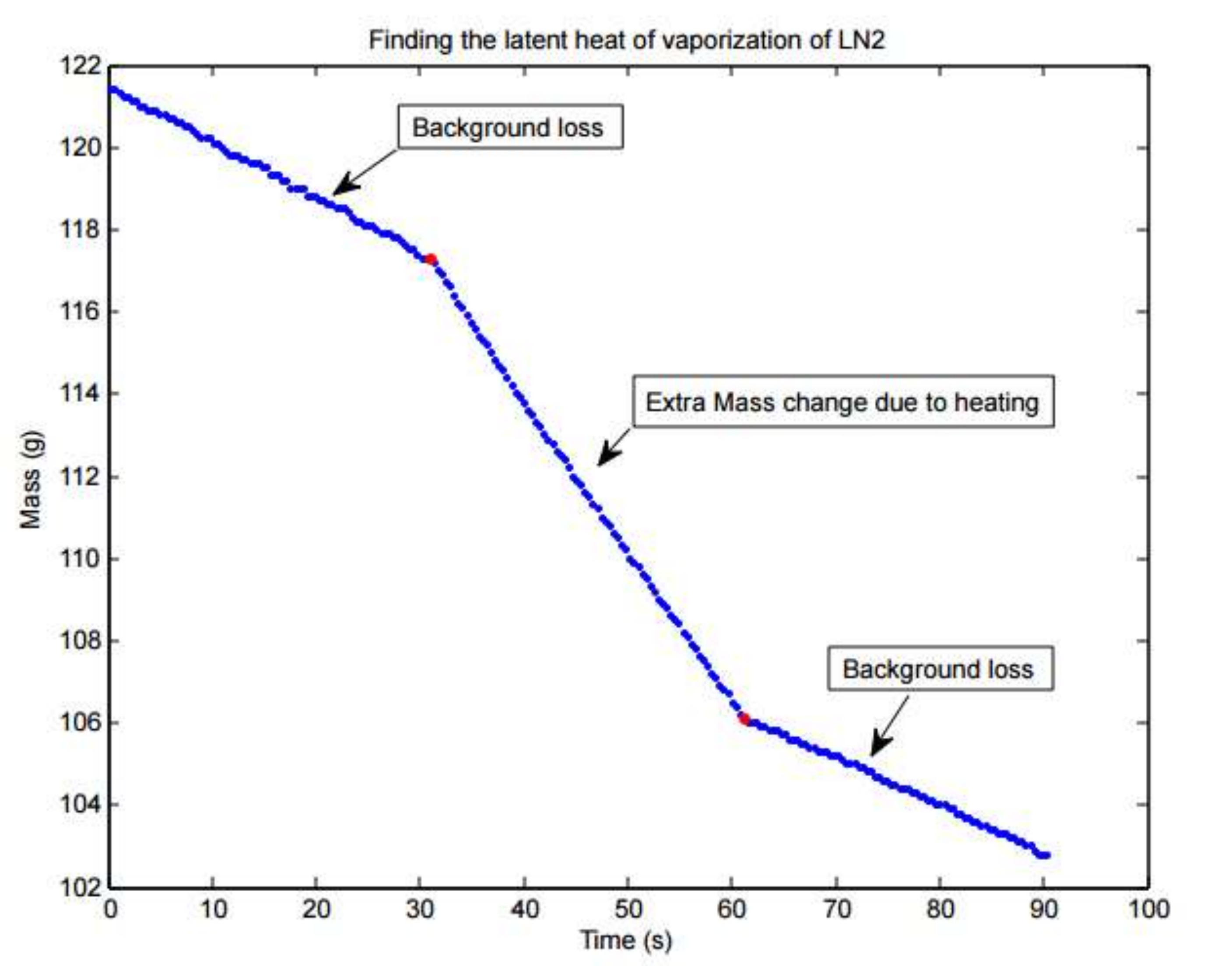
1. They act as visual aids indicating how one quantity varies when the other quantity is changed, often revealing subtle relationships.
2. Determine slopes and intercepts
3. Compare theoretical predictions and experimentally observed data.

Graphs and plotting



Sample Dataset

Sr#	Time (s)	Mass (g)
1	0.34	121.4
2	0.74	121.4
3	1.13	121.3
4	1.52	121.2
5	1.92	121.2
6	2.31	121.1
7	2.70	121.1
8	3.10	121.0
9	3.49	121.0
…	…	…
225	89.94	102.8
226	90.33	102.8


Finding the latent heat of vaporization of LN2
Background loss
Extra Mass change due to heating
Background loss
Mass (g)
Time (s)
122
120
118
116
114
112
110
108
106
104
102
0
10
20
30
40
50
60
70
80
90
100

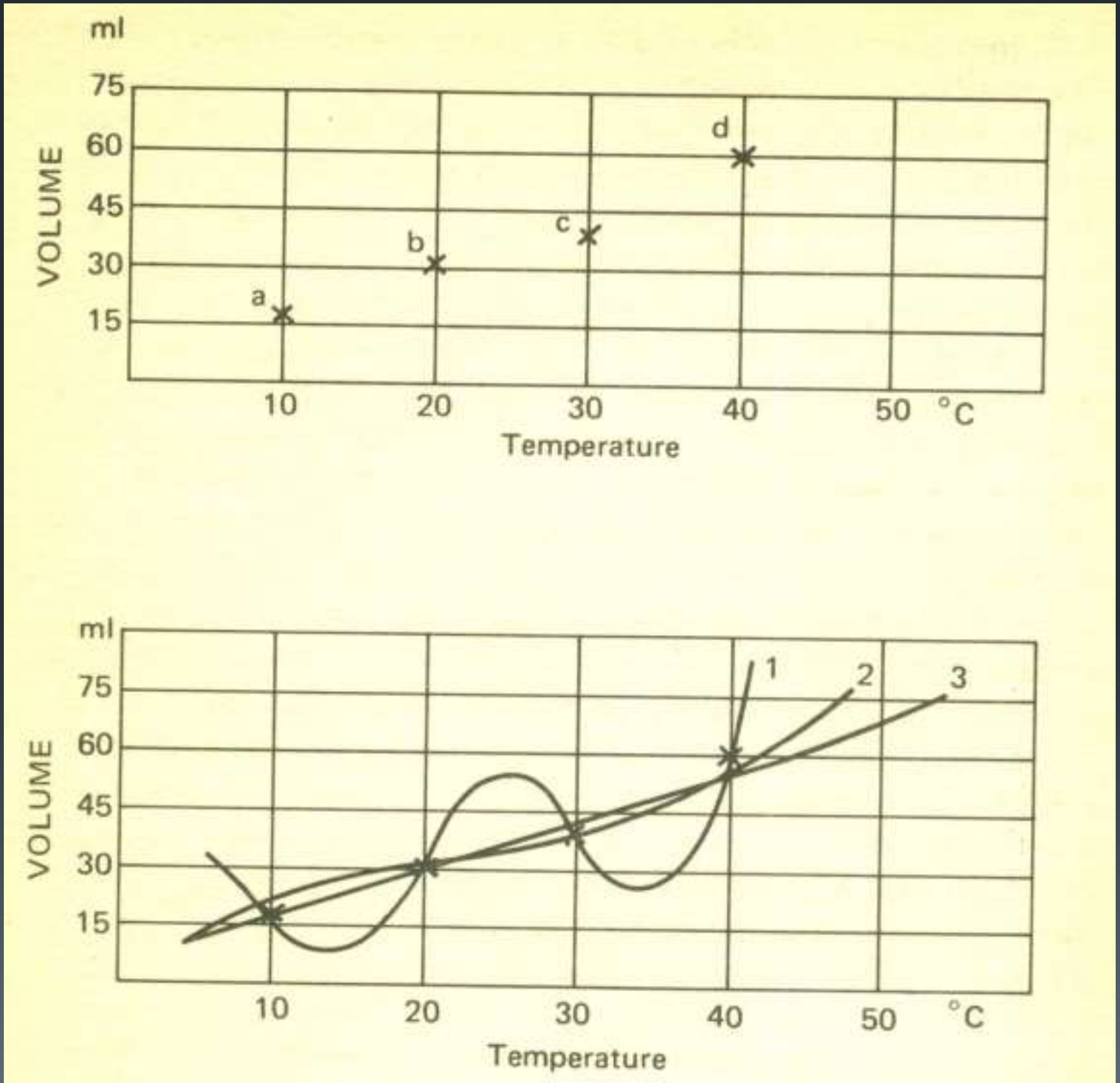
Introduction to plotting

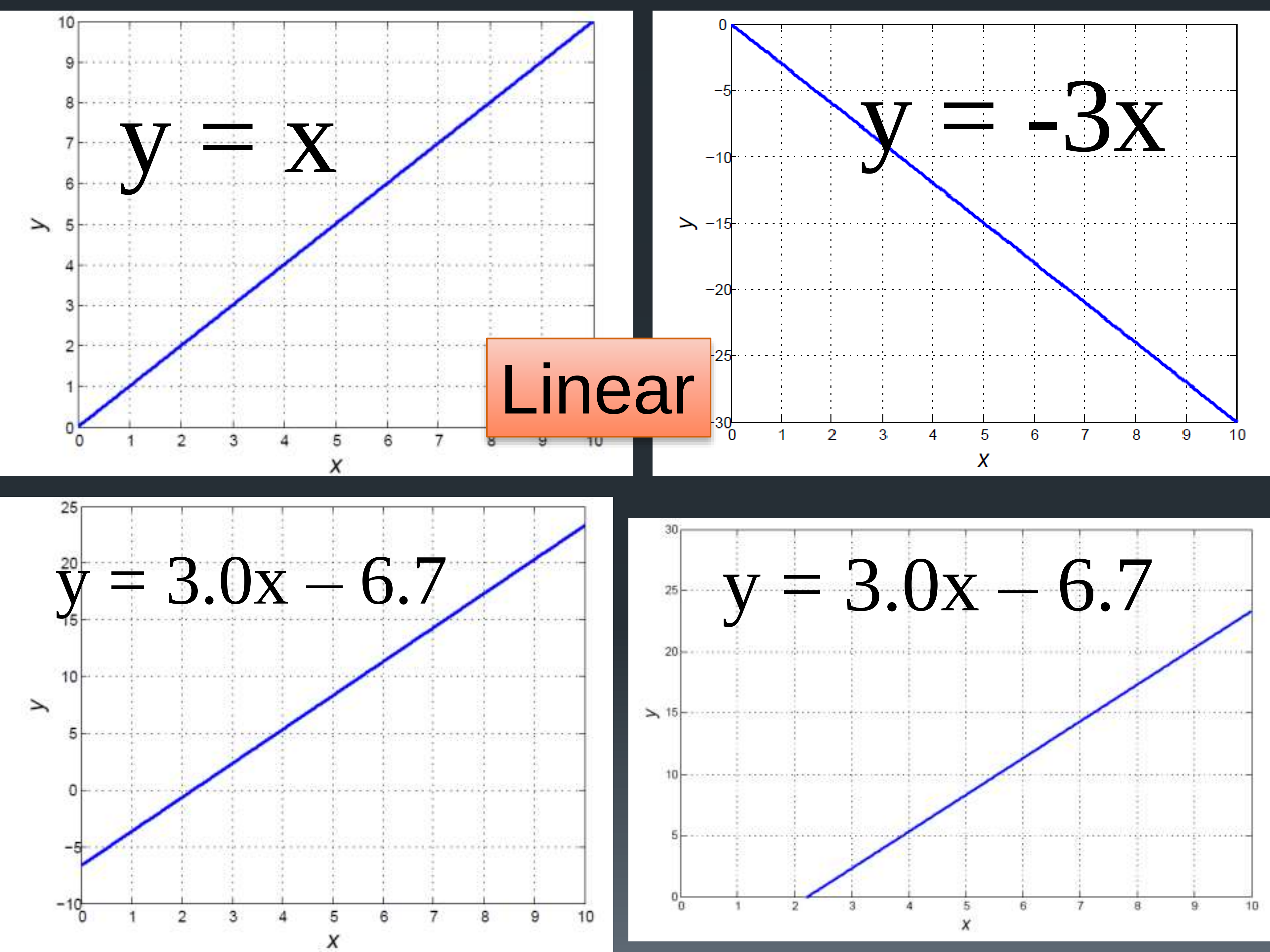
- Matlab can generate plots of a number of types
  - e.g.
    - Linear plots
    - Line plots
    - Logarithmic plots
    - Bar graphs
    - Three-dimensional plots

- In lab we will primarily work with two-dimensional plots by creating two "vectors" or an independent and dependent quantity.

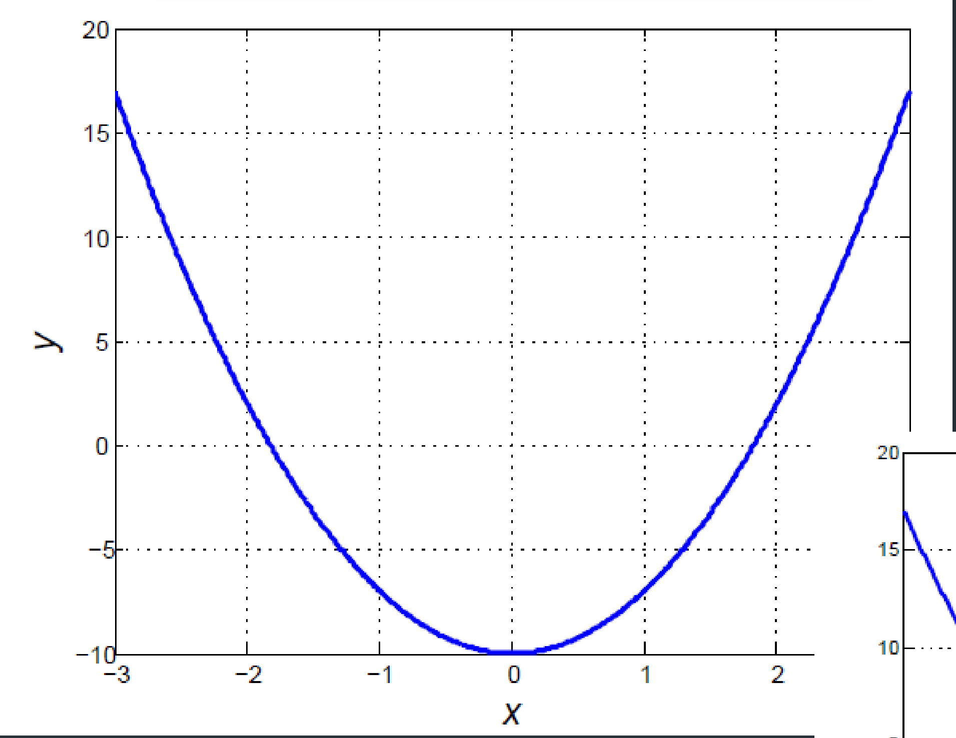
- It is customary to plot independent variable (the "cause") on horizontal axis and dependent variable (the "effect") on the vertical axis.

Typical models that fit typical experimental data

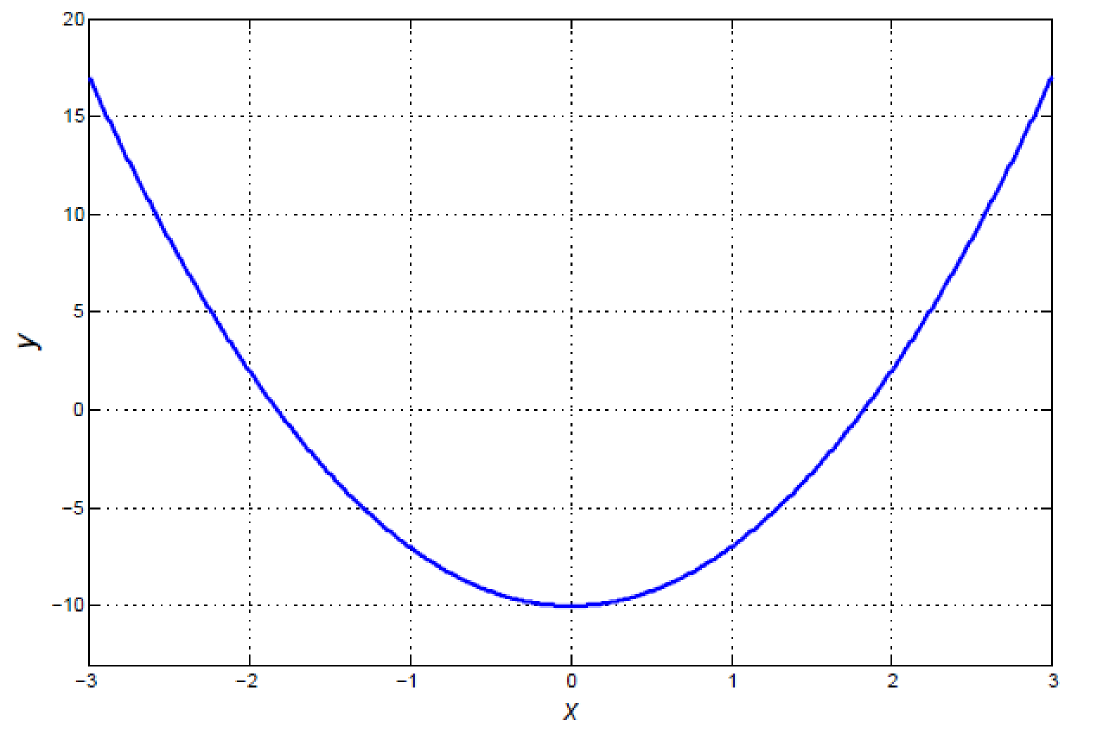


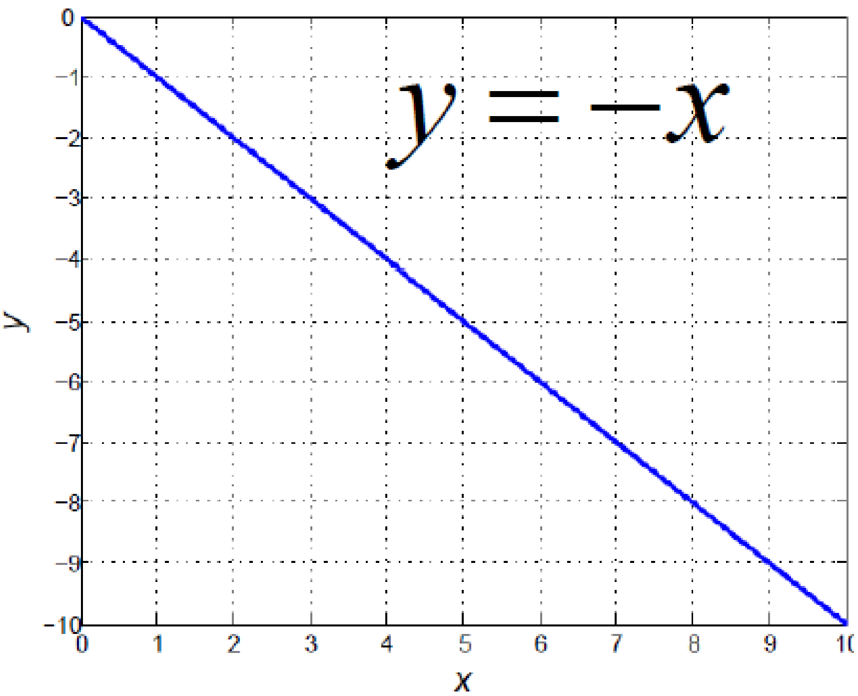

y = x
Linear
y = -3x
y = 3.0x – 6.7
y = 3.0x – 6.7
x
y

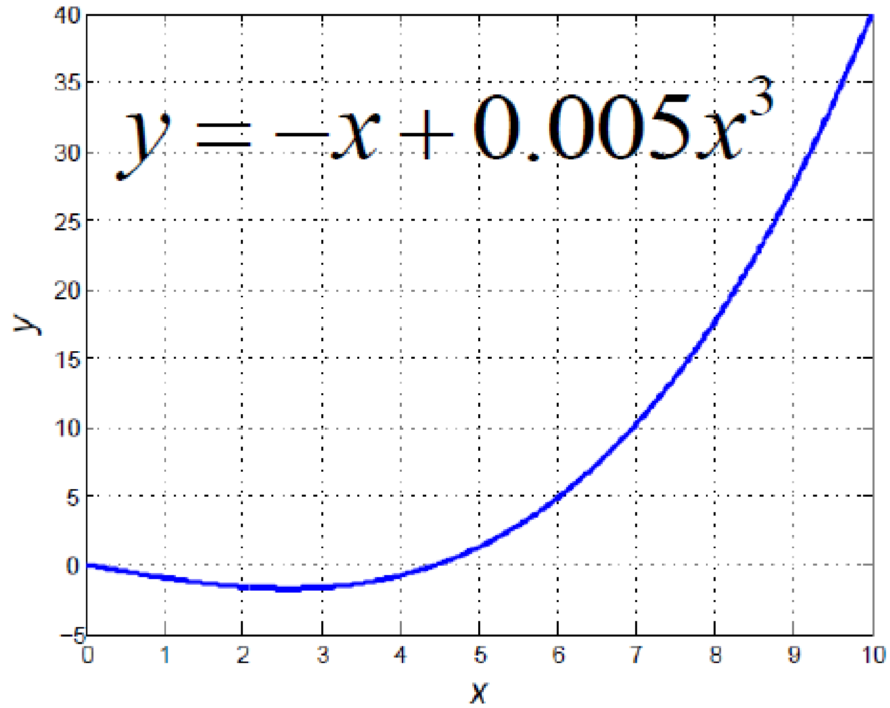
$Y = 3x^2 - 10$

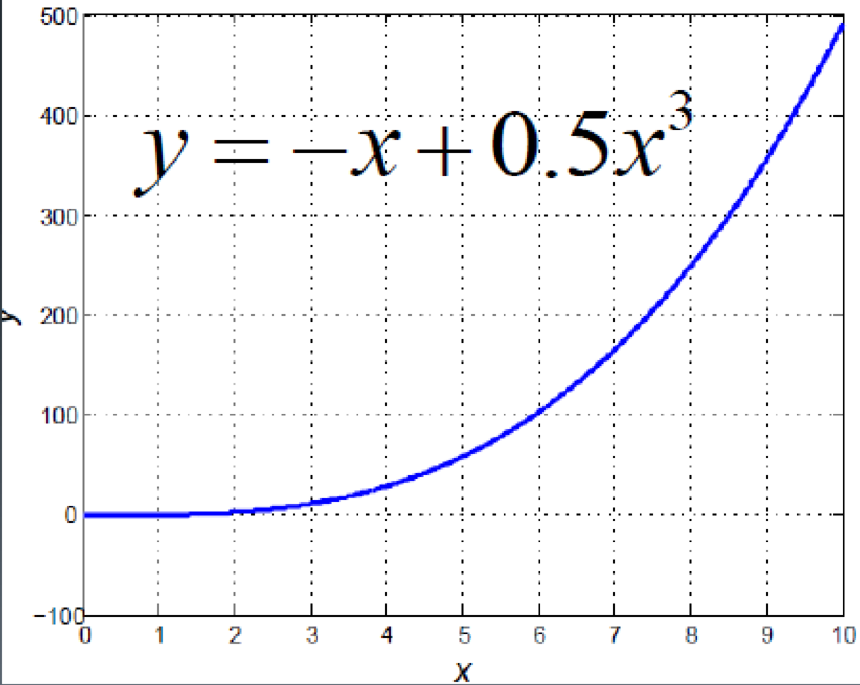


Quadratic

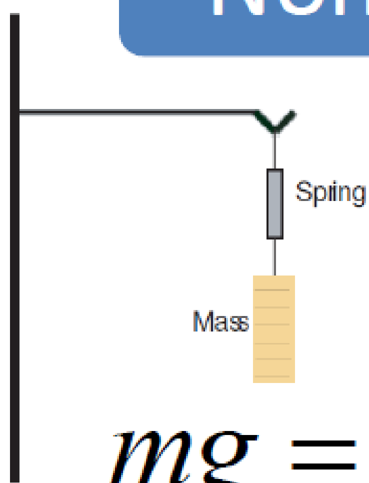






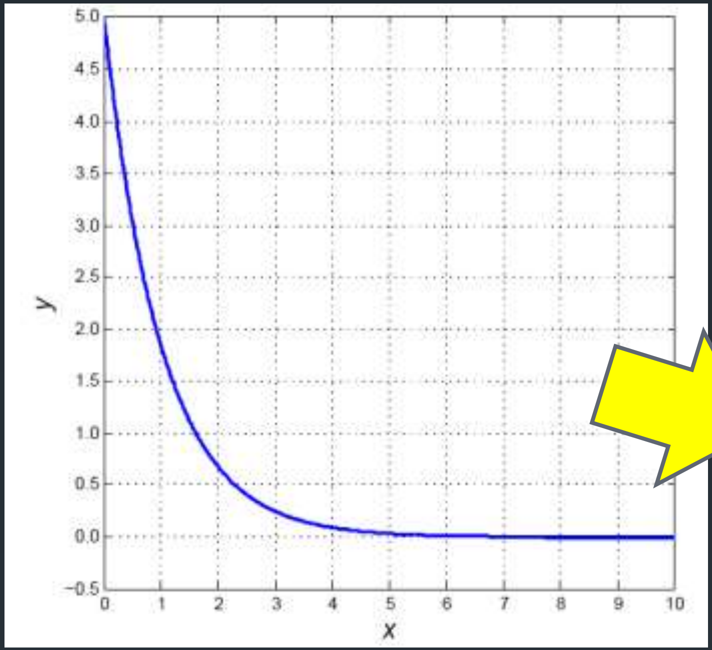


Nonlinearity



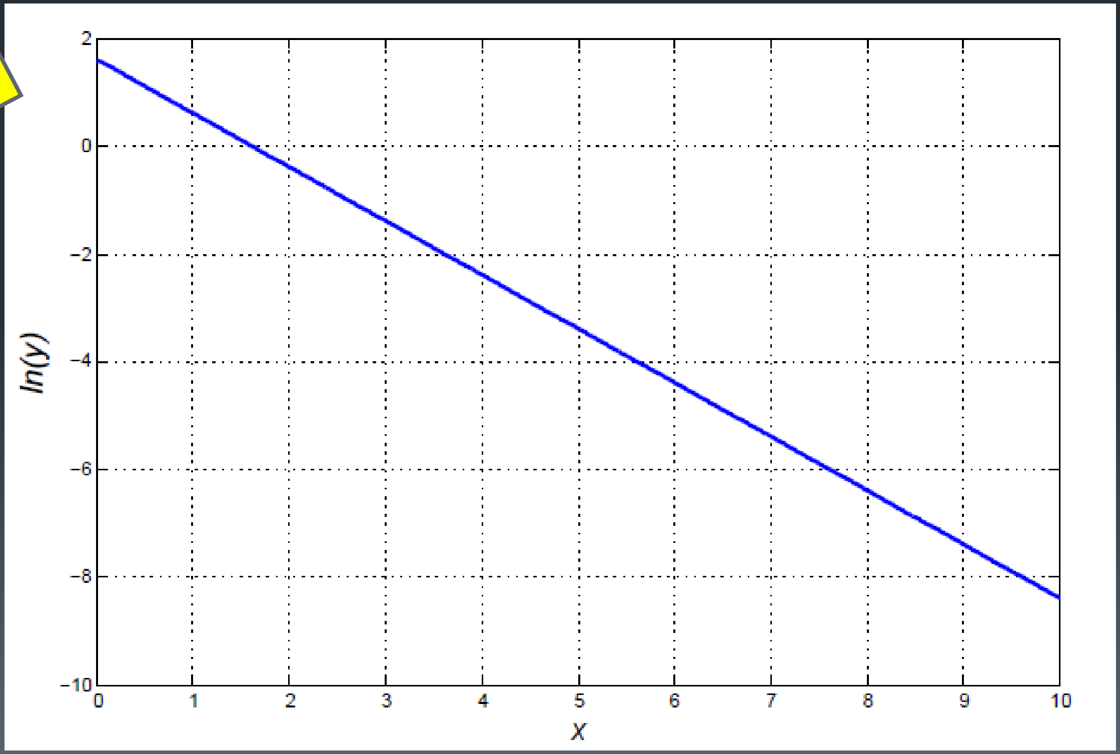
$$mg = -kx + \beta x^3$$

Linearization

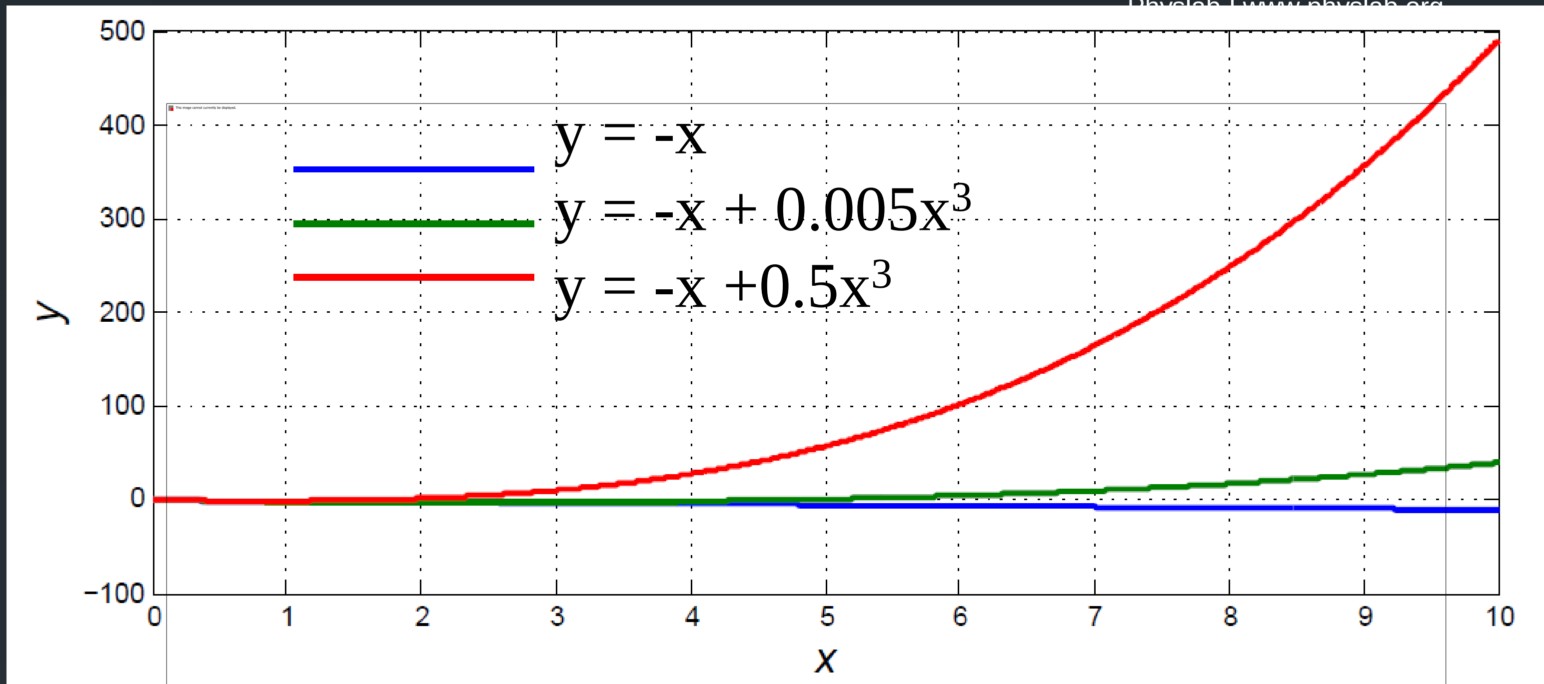


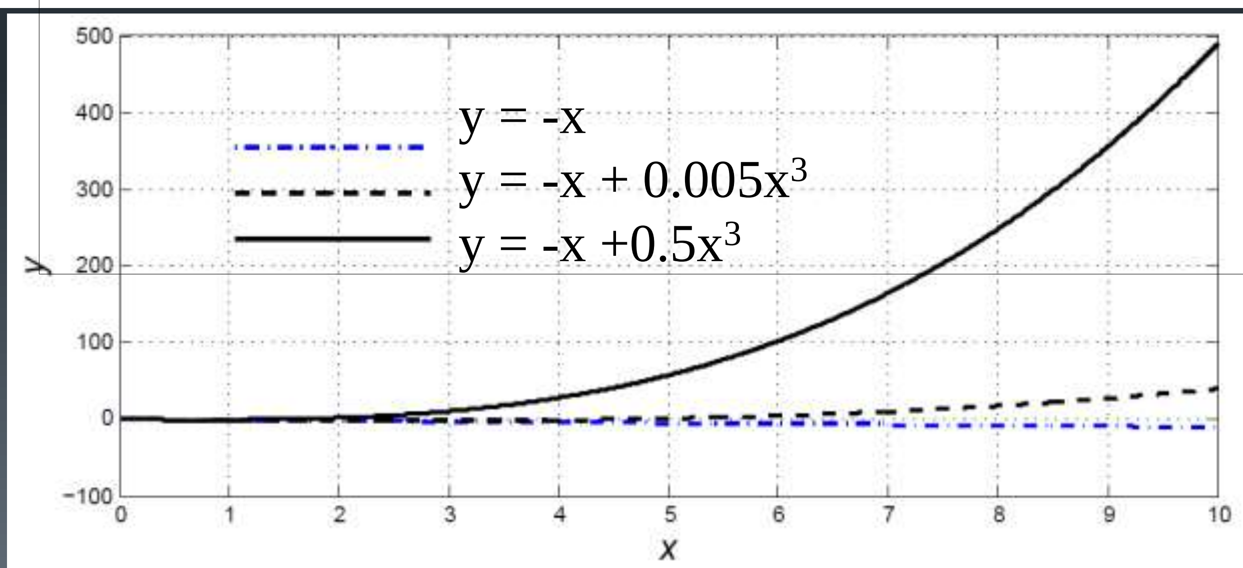
$$y = 5e^{-x}$$

$$\ln(y) = \ln(5) - x$$



Superposing Graphs





Example

- Let's consider an example of a stretched string fixed at one end to a rigid support, is strung over a pulley and a weight of 1.2 kg is attached at the other end. The string can be set under vibrations using a mechanical oscillator (woofer) connected to the signal generator.

- The relation of angular velocity (w) with the wave vector (k) is called the dispersion relation and given by,

$$w(k) = n\sqrt{\frac{T}{\mu}} \times \frac{\pi}{L}$$

$$w(k) = n\sqrt{\frac{T}{\mu}} \times \frac{\pi}{L}$$

$$2\cancel{\pi}f = n\sqrt{\frac{T}{\mu}} \times \frac{\cancel{\pi}}{L}$$

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \times n$$

$$y = \underline{m}\,x + c$$

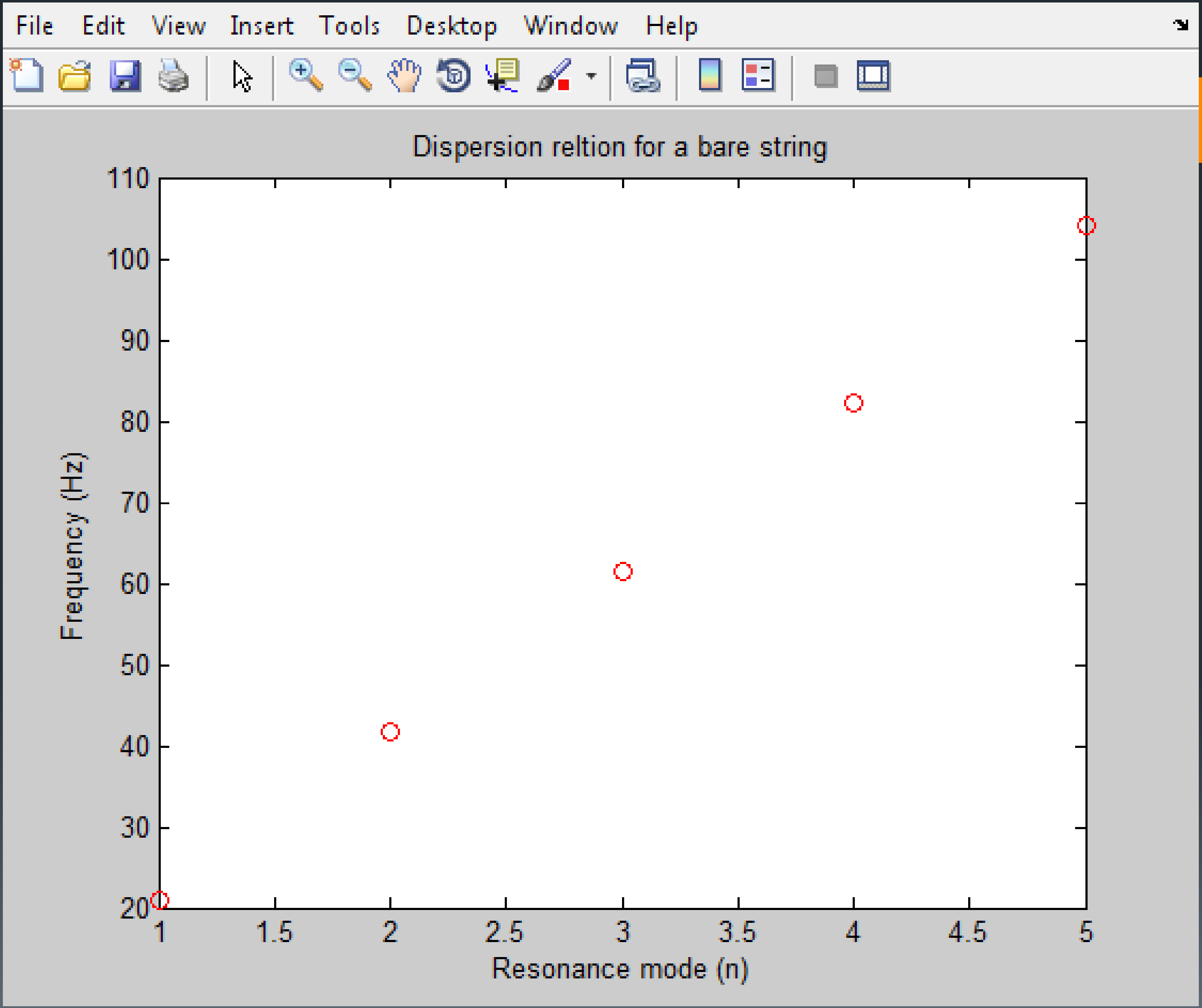
Example

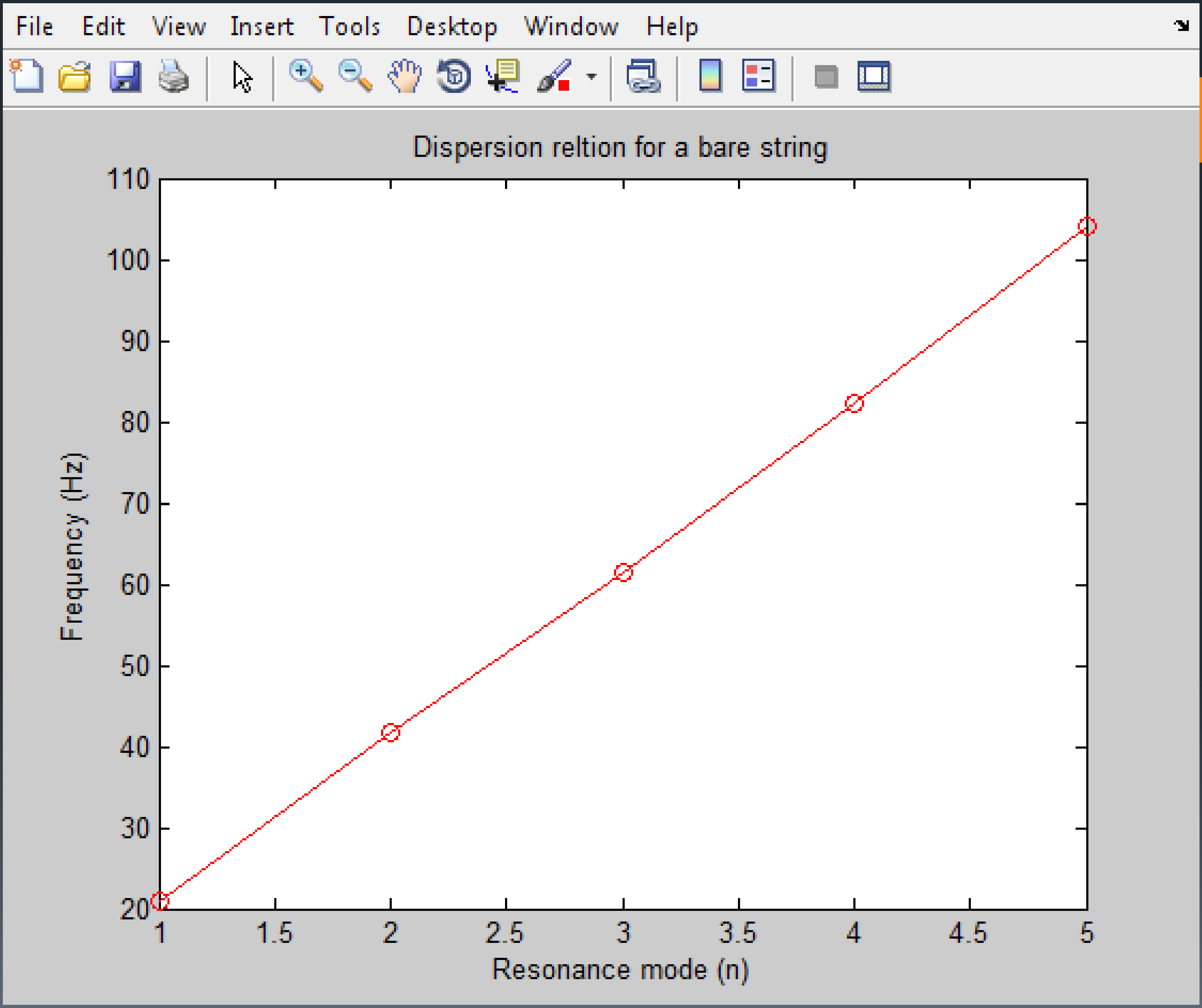
Plot the following experimental data:

Resonance mode (n)	**1**	**2**	**3**	**4**	**5**
Frequency (Hz)	20.82	41.82	61.32	82.32	104.1

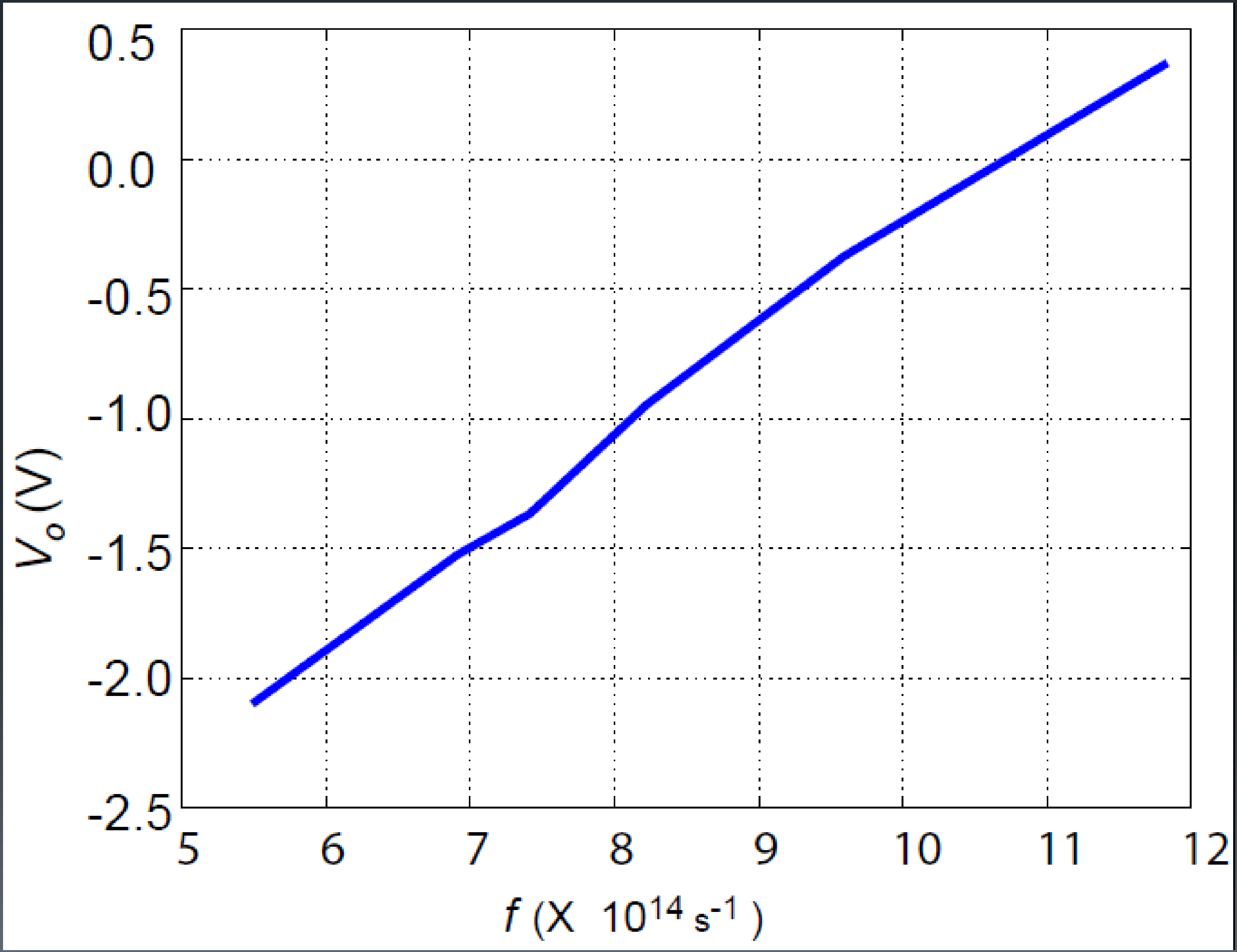
Commands:

```
n=[1 2 3 4 5];
f=[20.82 41.82 61.32 82.32 104.1];
plot(n,f)
xlabel(`Resonance mode (n)')
ylabel(`Frequency (Hz))')
title('Dispersion relation for a bare string')
```

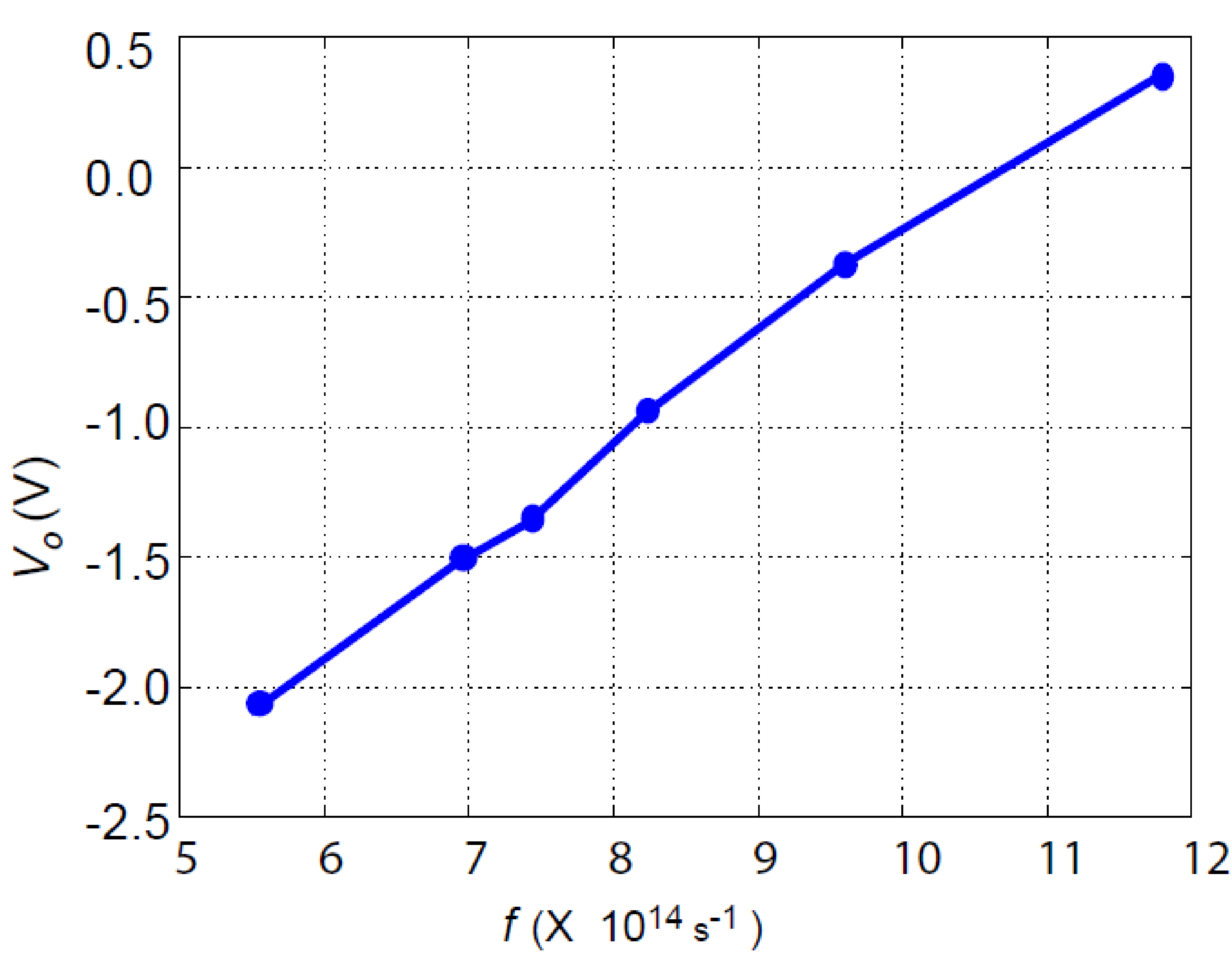

File Edit View Insert Tools Desktop Window Help
Dispersion reltion for a bare string
Frequency (Hz)
110
100
90
80
70
60
50
40
30
20
1
1.5
2
2.5
3
3.5
4
4.5
5
Resonance mode (n)


File Edit View Insert Tools Desktop Window Help
Dispersion reltion for a bare string
Frequency (Hz)
110
100
90
80
70
60
50
40
30
20
1
1.5
2
2.5
3
3.5
4
4.5
5
Resonance mode (n)

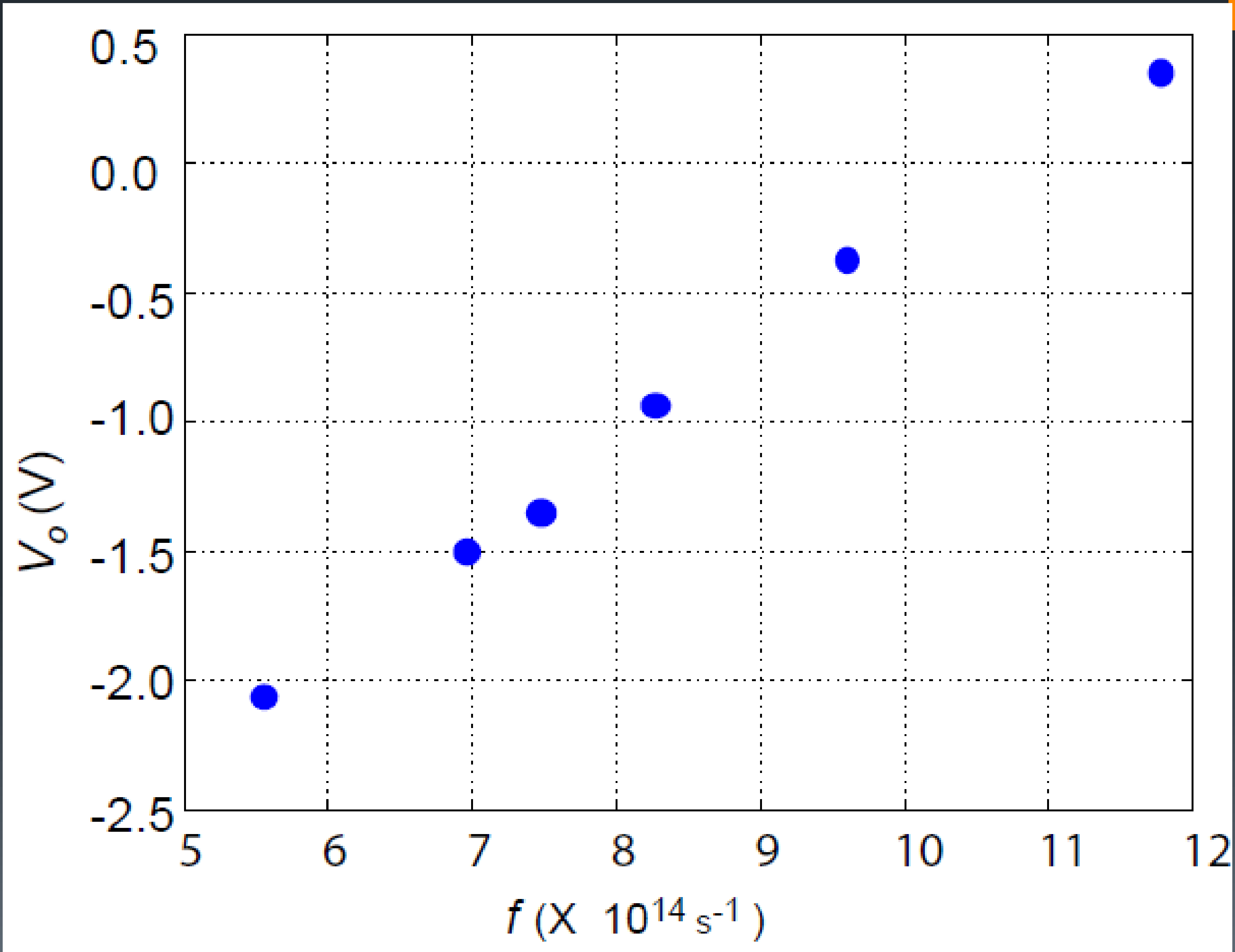
Possible Ways of Plotting
(a) Not acceptable



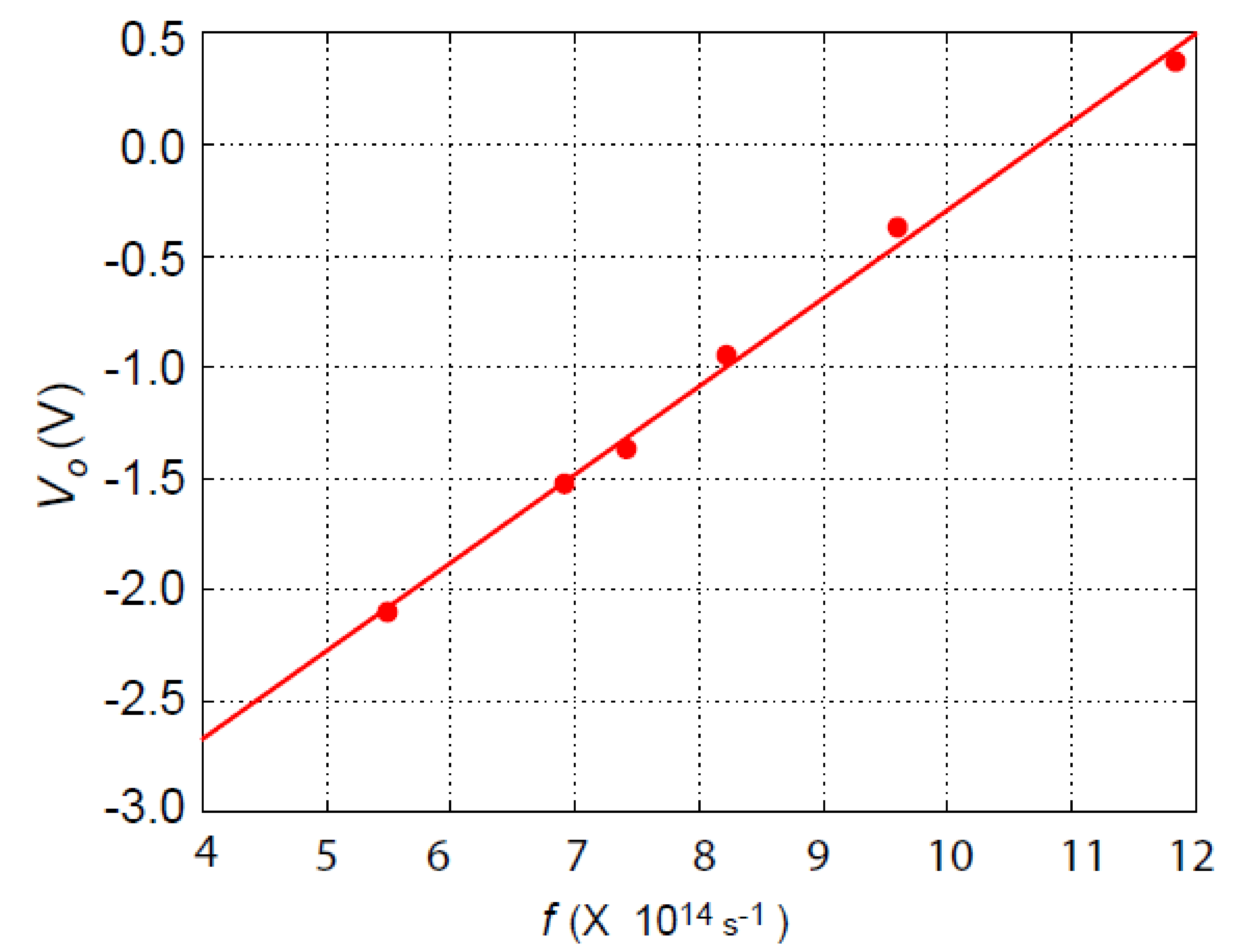
(b) Barely acceptable



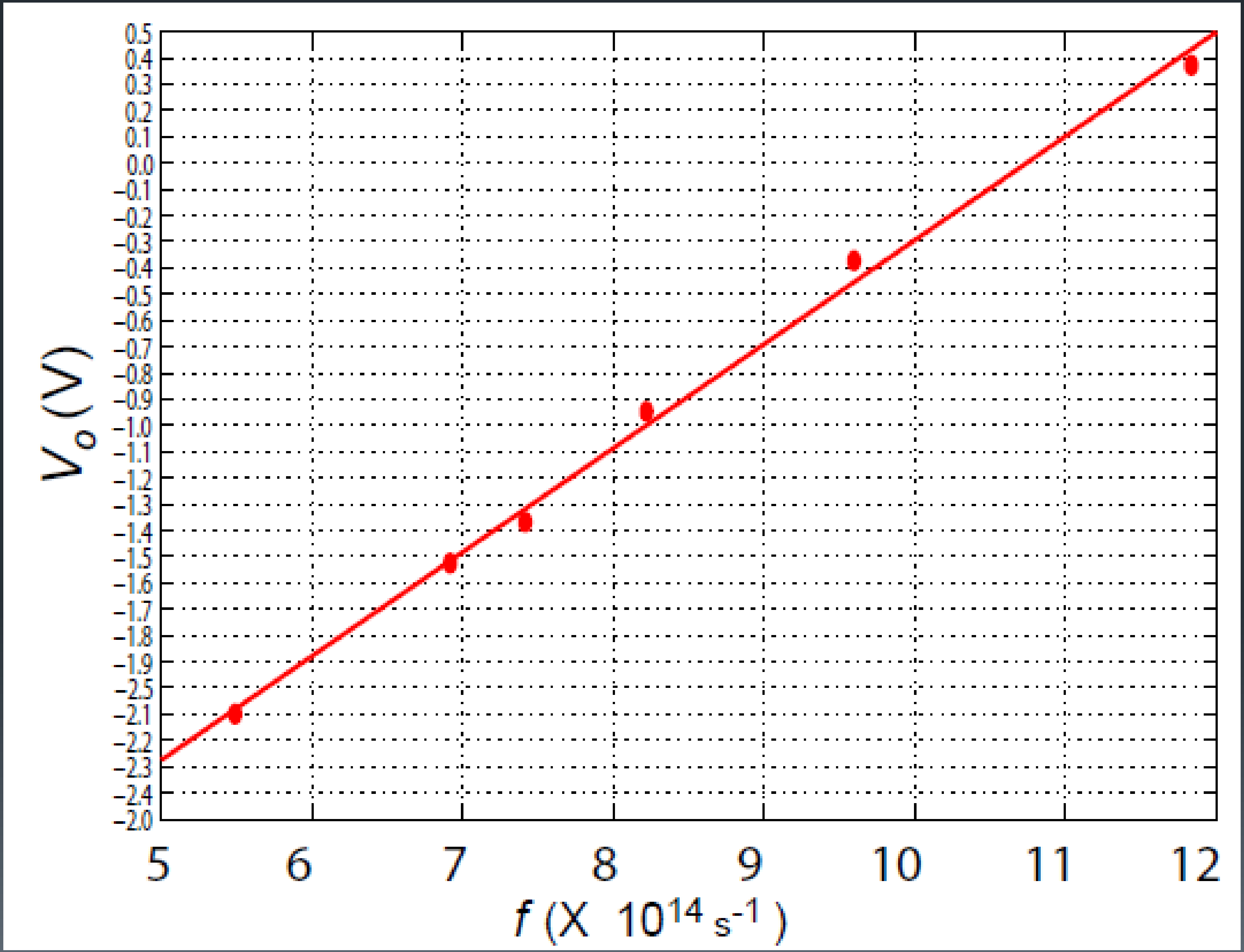
(c) Better



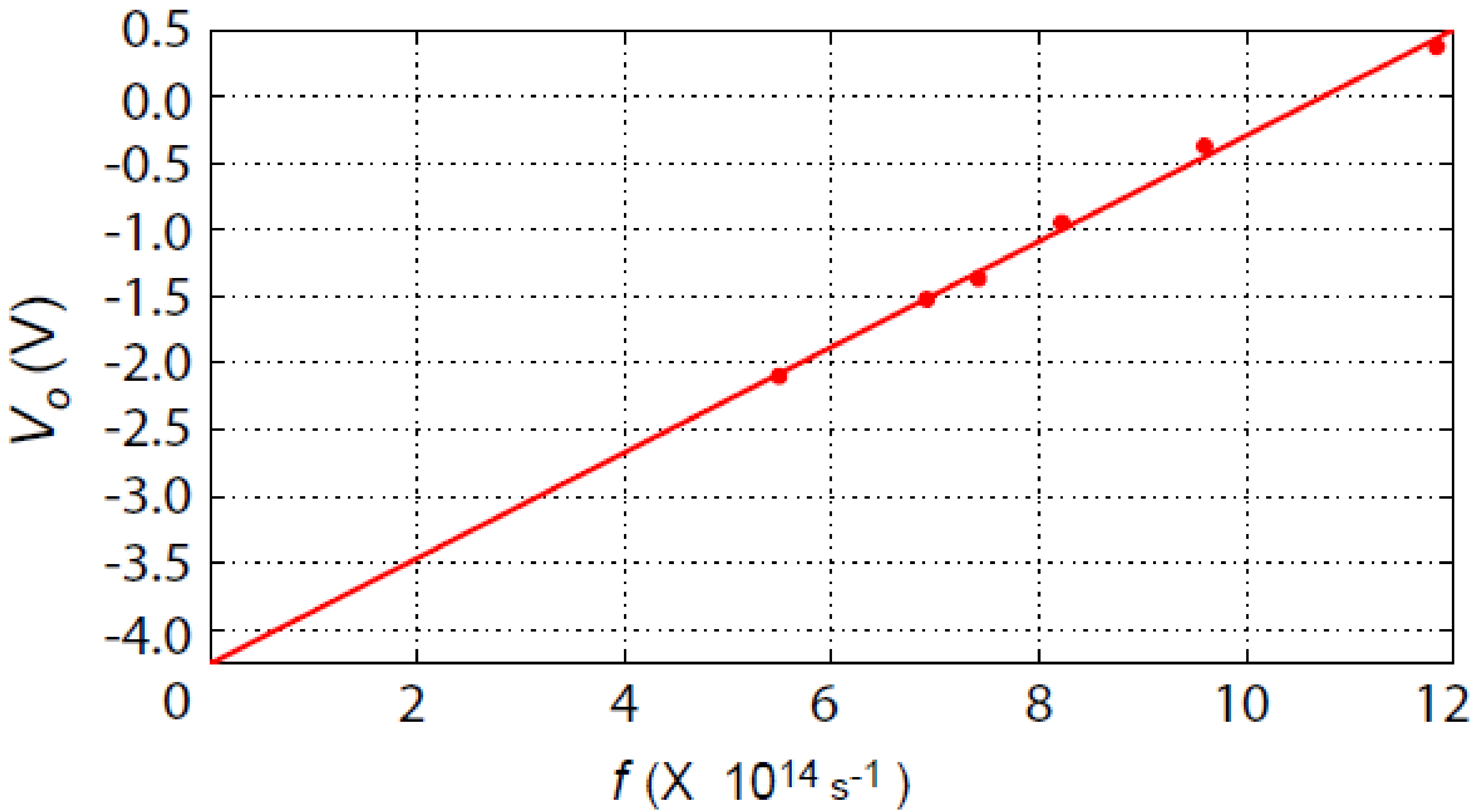
(d) Acceptable, good in all respects



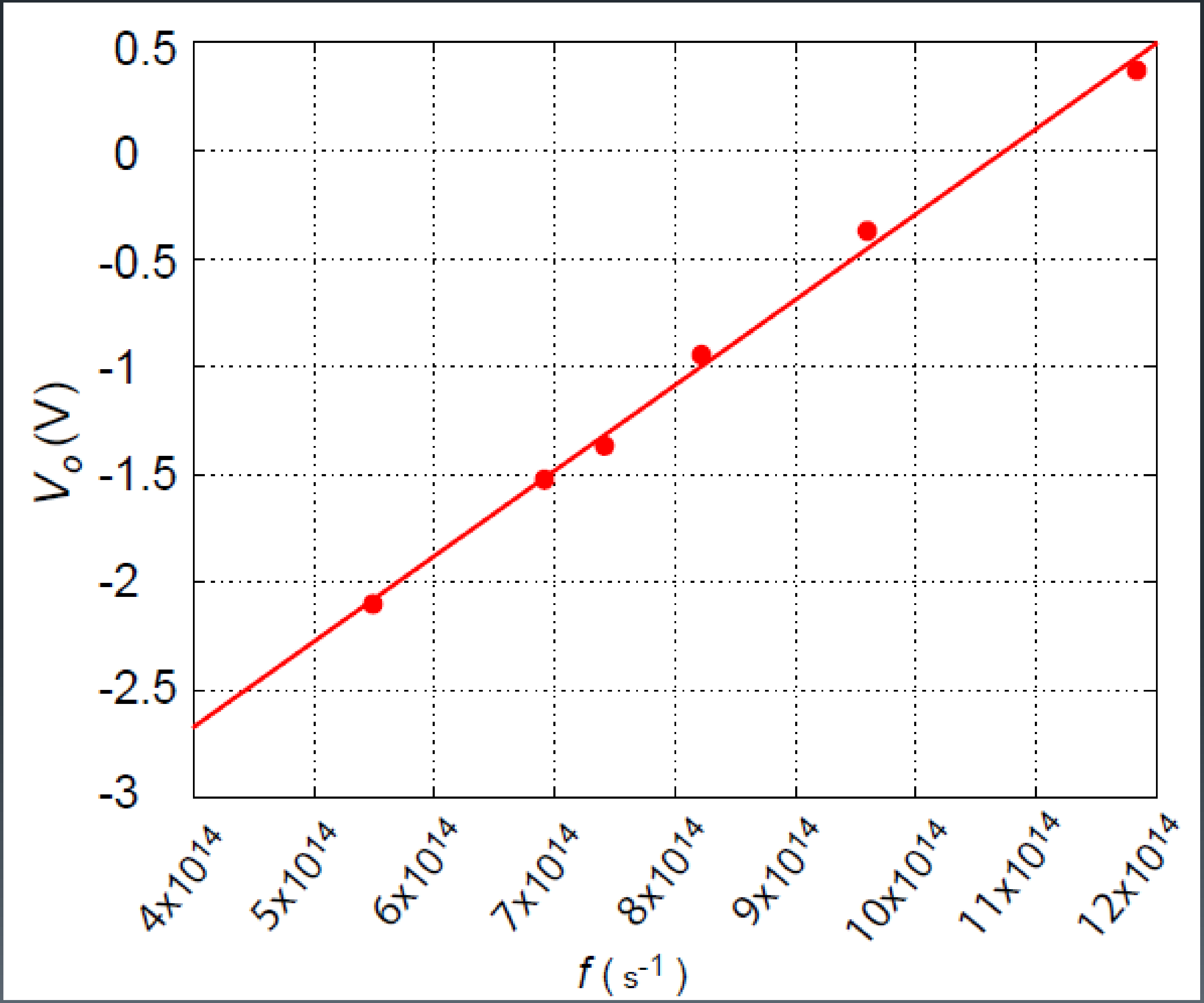
(e) Unnecessary detail or embellishment



(f) Axes too long

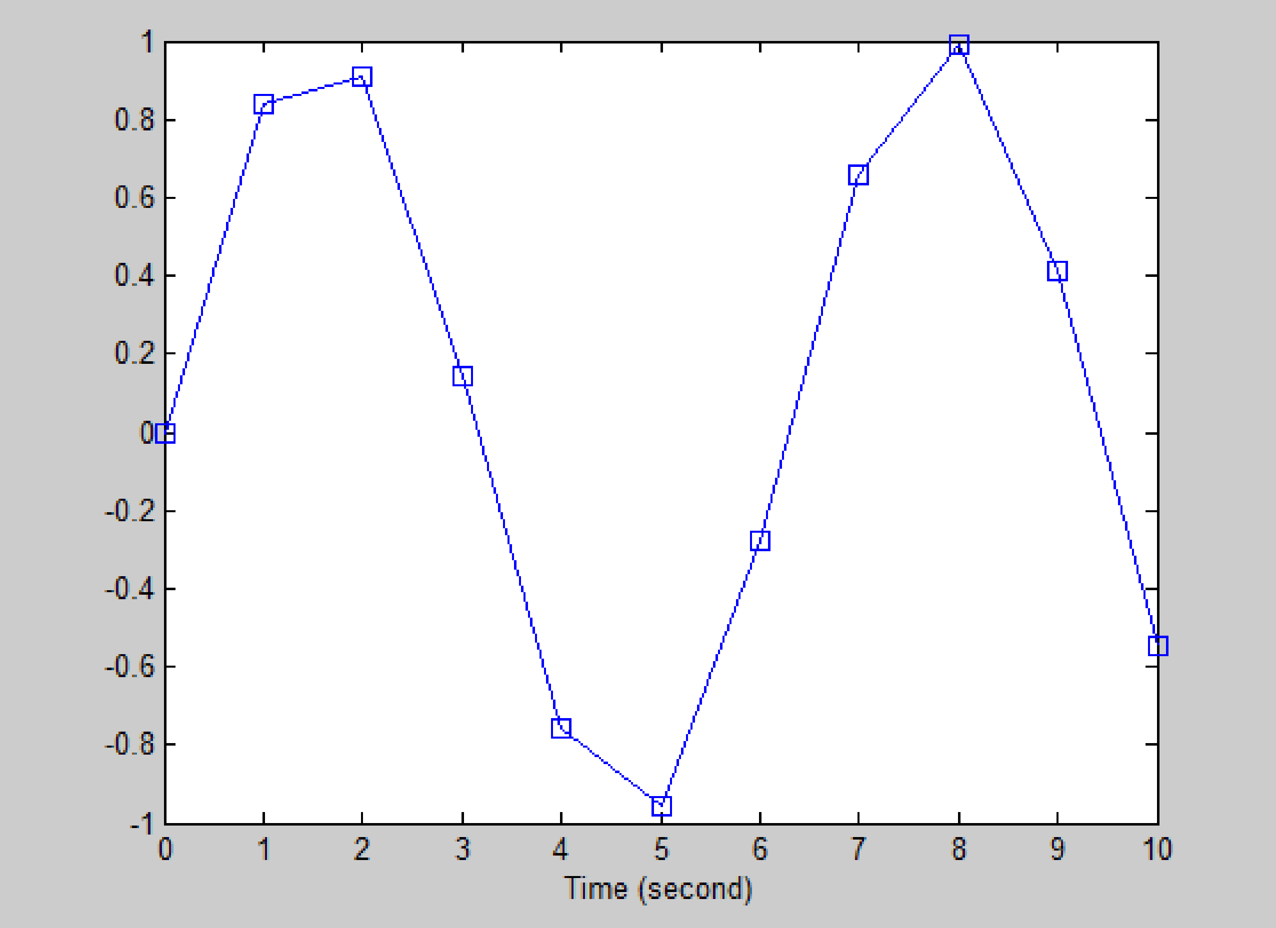


(g) Axes tick marks are inconsistent and clumsy



Resolution of Graph

Suppose we have a sine curve, sampled at interval of 1s for a duration of 10s, it means there are eleven data points contained within the sampled duration.



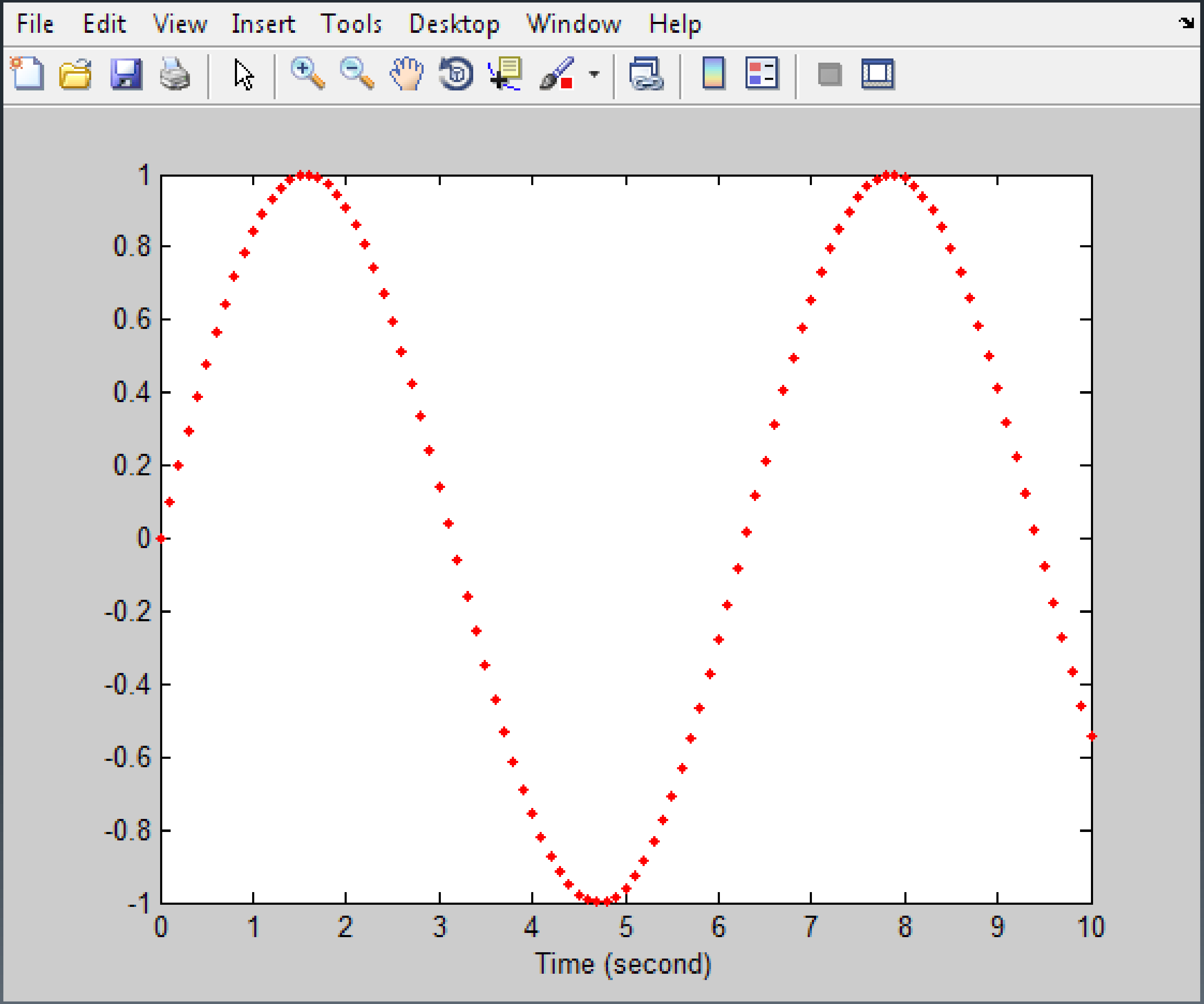
Resolution of Graph

- We know from experience that a plot of the sine function should be smooth.

- **Why is this discrepancy?**

The reason is that we have not sampled enough points. So, we decrease the sampling interval to 0.1 s and hence, increasing the number of samples to 101, we recover a smooth sine curve.

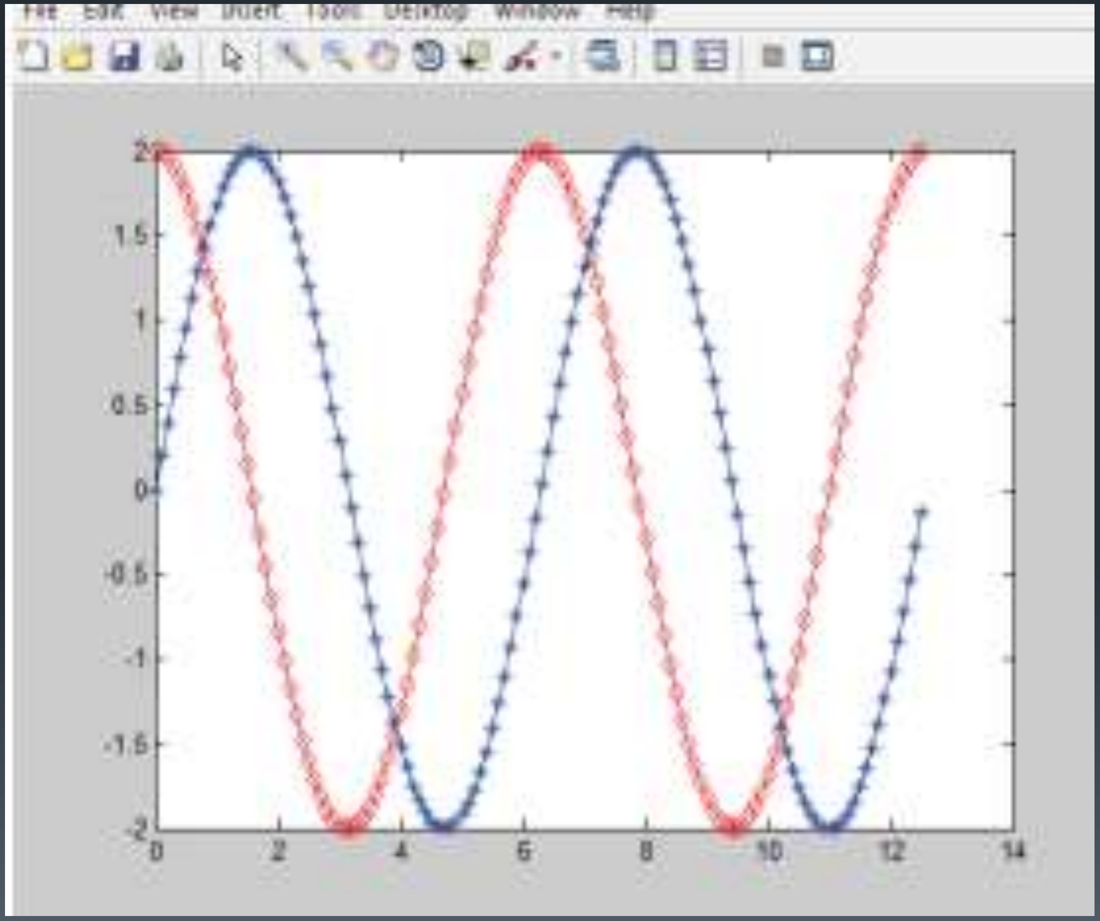
Resolution of Graph



Multiple plotting

Let’s define input vectors:

```
x=0:0.1:4*pi;
y=2*cos(x);
y1=2*sin(x);

figure; plot(x,y,`r-d')
hold on
plot(x,y1,`b-*')
```



Let’s practice!

Solve the second exercise …

Introduction to MATLAB

Curve Fitting

Contents

Example

- A student wants to check the resistance of a resistor by measuring voltage (V) across it and the resulting current (I) through it and then calculating the resistance through Ohm's Law.
- *Using slope of the relation*

$$V = R\,I$$

$$y = m\,x + c$$

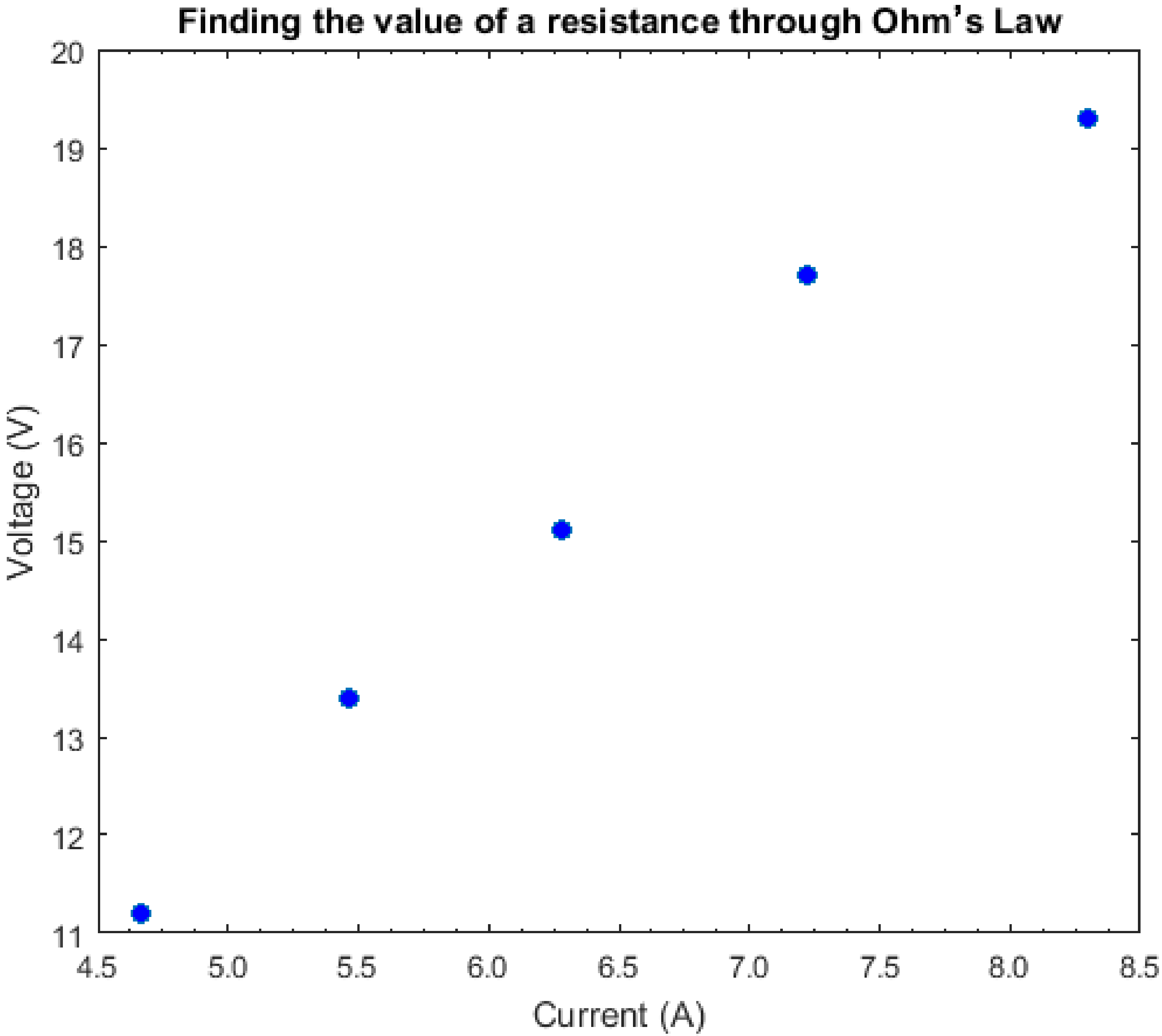
Curve Fitting Example

Plot the following experimental data:

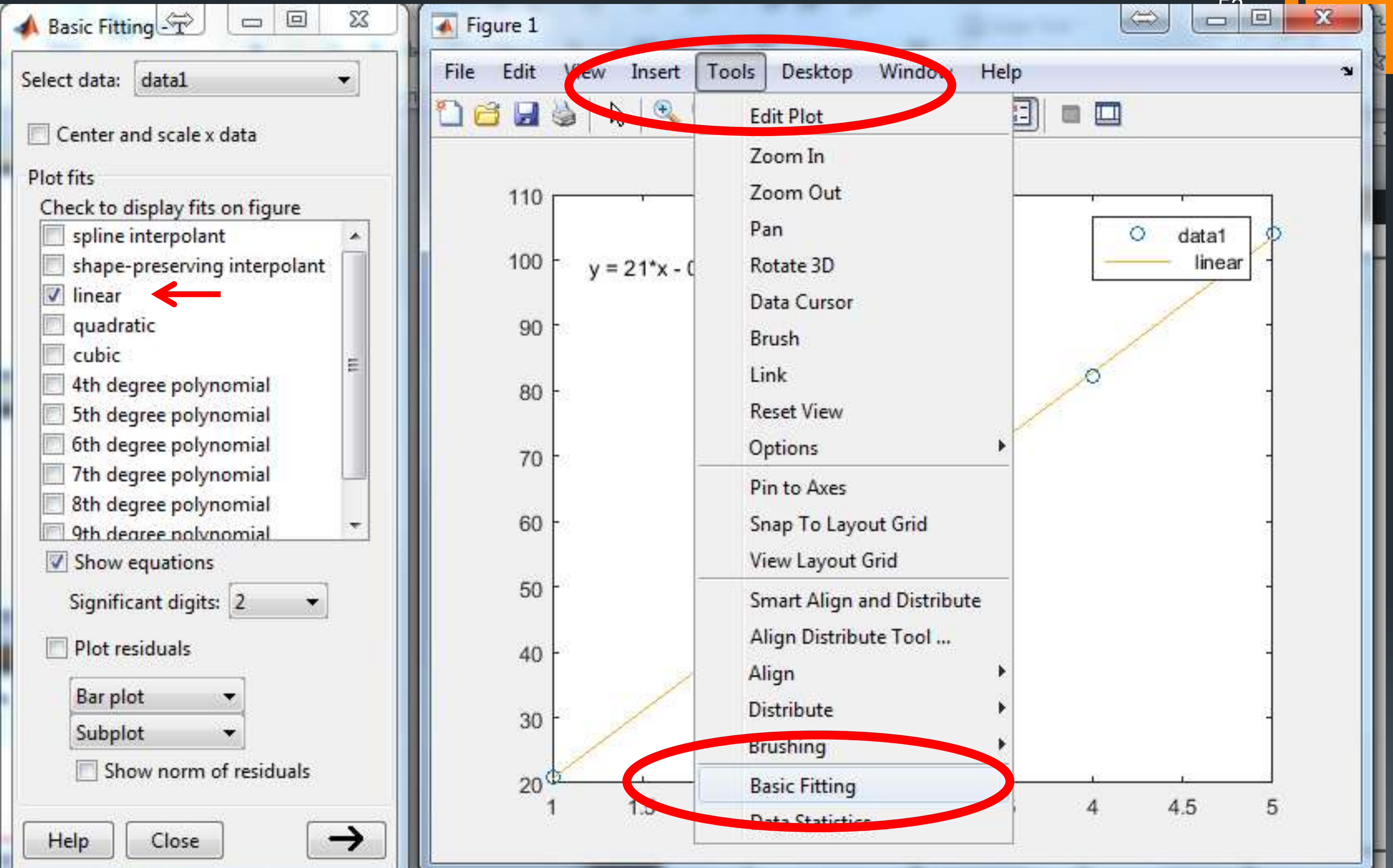
Voltage (V)	**11.2**	**13.4**	**15.1**	**17.7**	**19.3**
Current (A)	4.67	5.46	6.28	7.22	8.30

Commands:

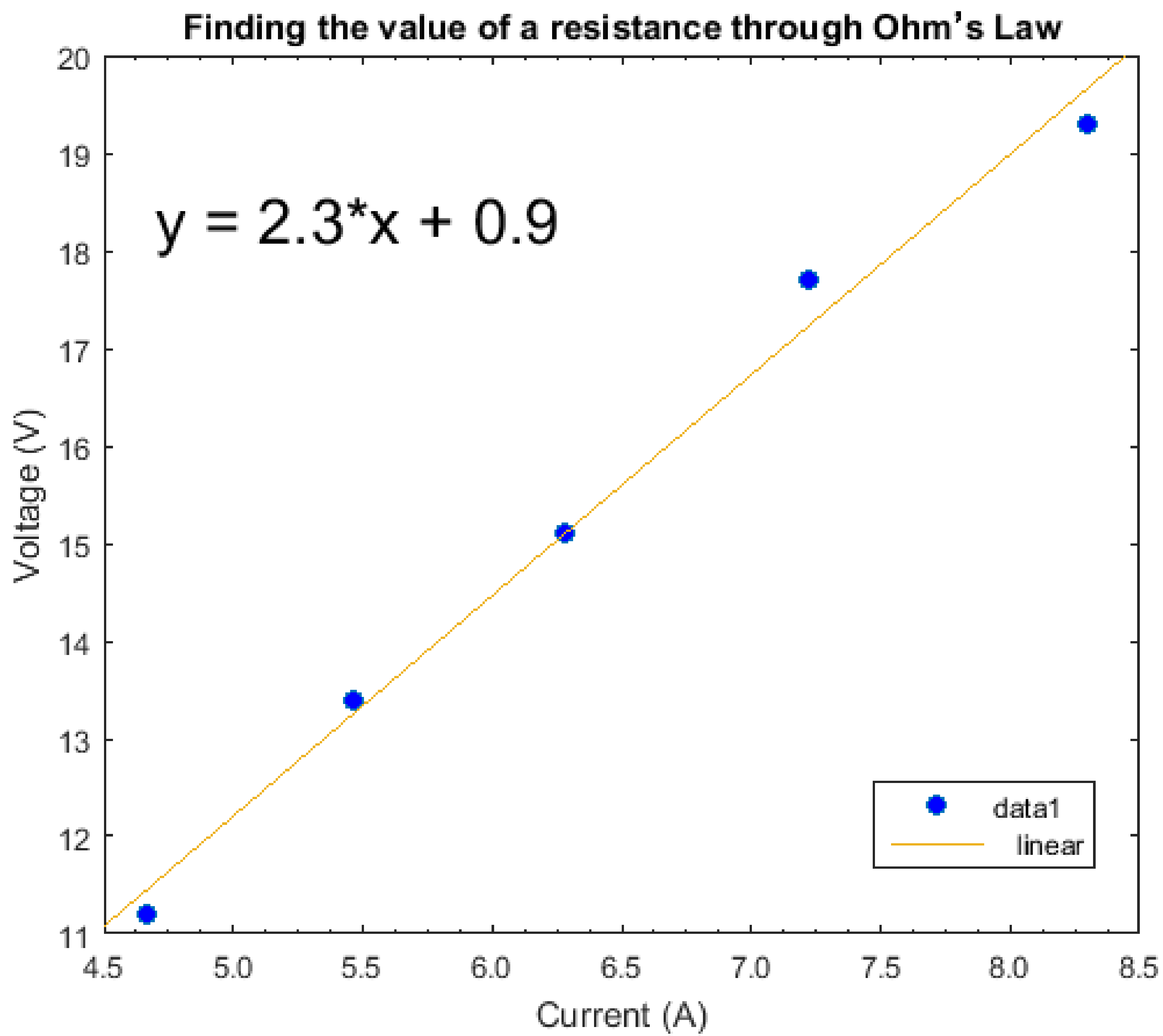
```
v=[11.2 13.4 15.1 17.7 19.3];
a=[4.67 5.46 6.28 7.22 8.30 ];
plot(a,v,’o’)
xlabel(`Current (A)')
ylabel(`Voltage (V))')
title(‘Finding the value of a resistance through Ohm’s Law')
```



Finding the value of a resistance through Ohm's Law
Voltage (V)
Current (A)
20
19
18
17
16
15
14
13
12
11
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5



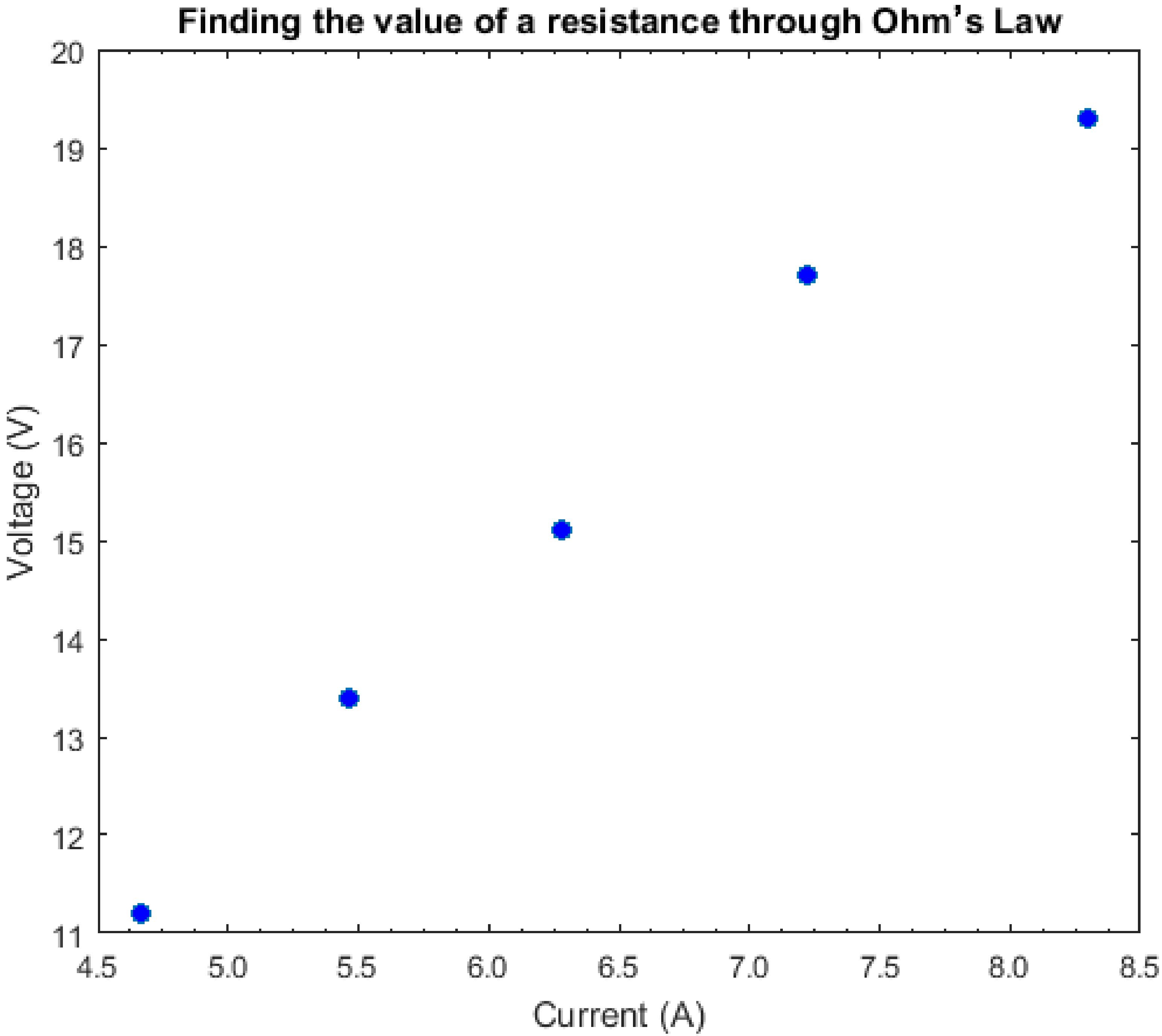
Basic Fitting
Select data: data1
Center and scale x data
Plot fits
Check to display fits on figure
spline interpolant
shape-preserving interpolant
linear
quadratic
cubic
4th degree polynomial
5th degree polynomial
6th degree polynomial
7th degree polynomial
8th degree polynomial
9th degree polynomial
Show equations
Significant digits: 2
Plot residuals
Bar plot
Subplot
Show norm of residuals
Help
Close
Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Edit Plot
Zoom In
Zoom Out
Pan
Rotate 3D
Data Cursor
Brush
Link
Reset View
Options
Pin to Axes
Snap To Layout Grid
View Layout Grid
Smart Align and Distribute
Align Distribute Tool ...
Align
Distribute
Brushing
Basic Fitting
Data Statistics
110
100
90
80
70
60
50
40
30
20
1
4
4.5
5
data1
linear



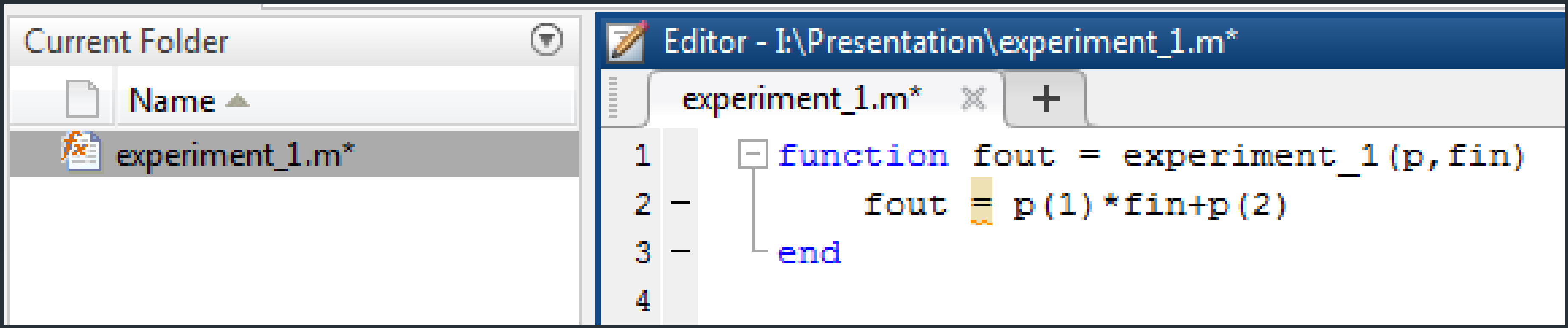
Finding the value of a resistance through Ohm's Law
y = 2.3*x + 0.9
data1
linear
Voltage (V)
Current (A)
20
19
18
17
16
15
14
13
12
11
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5

Curve Fitting

via lsqcurvefit command


Finding the value of a resistance through Ohm's Law
Voltage (V)
Current (A)
20
19
18
17
16
15
14
13
12
11
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5

1. Create a function file



2. Call the function via lsqcurvefit

experiment_1.m

```matlab
function fout = experiment_1(p,fin)
    fout = p(1)*fin+p(2)
end
```

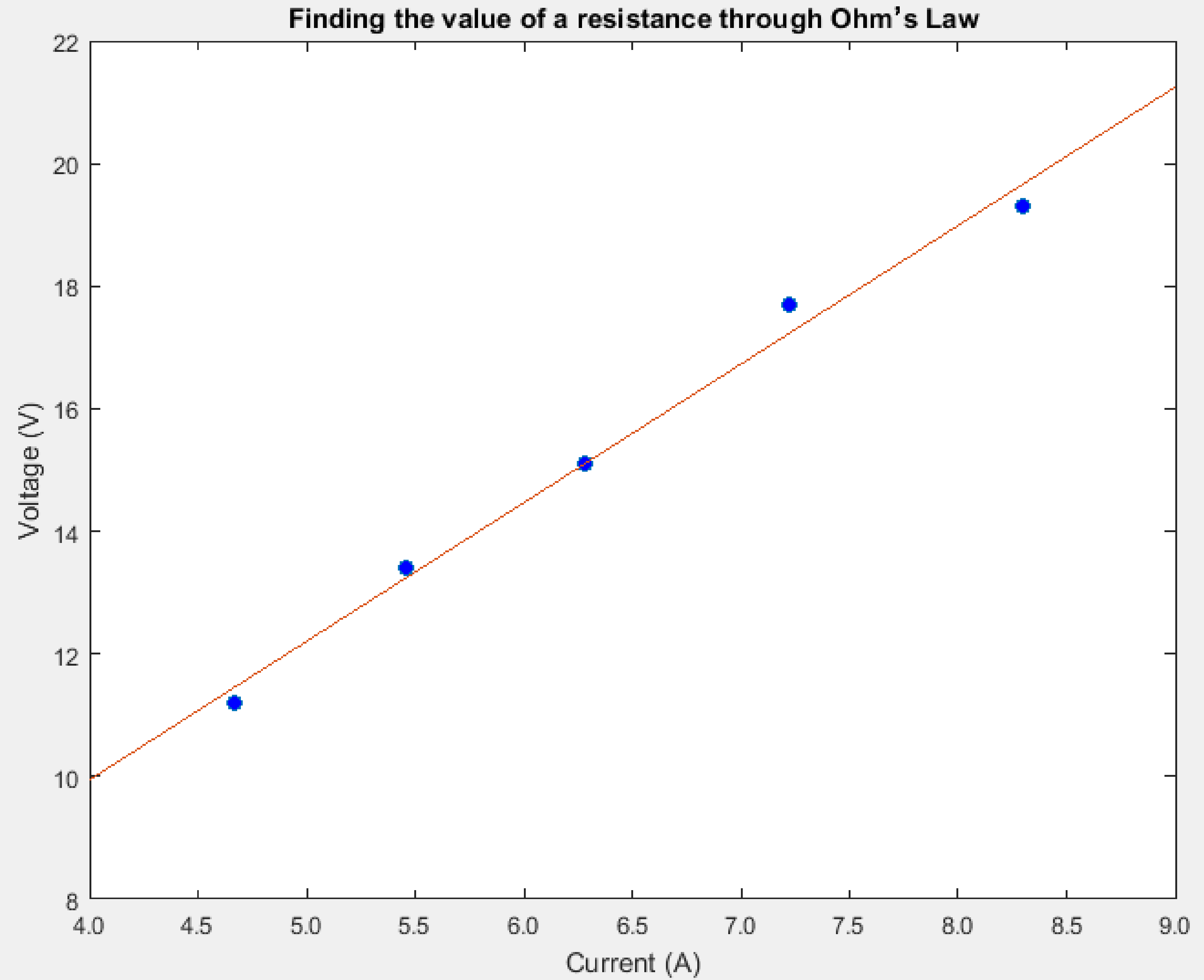
Command Window

New to MATLAB? See resources for Getting Started.

```matlab
>> lsqcurvefit(@experiment_1, [1 1], I,V)
```

3. The function outputs the optimized values of parameters

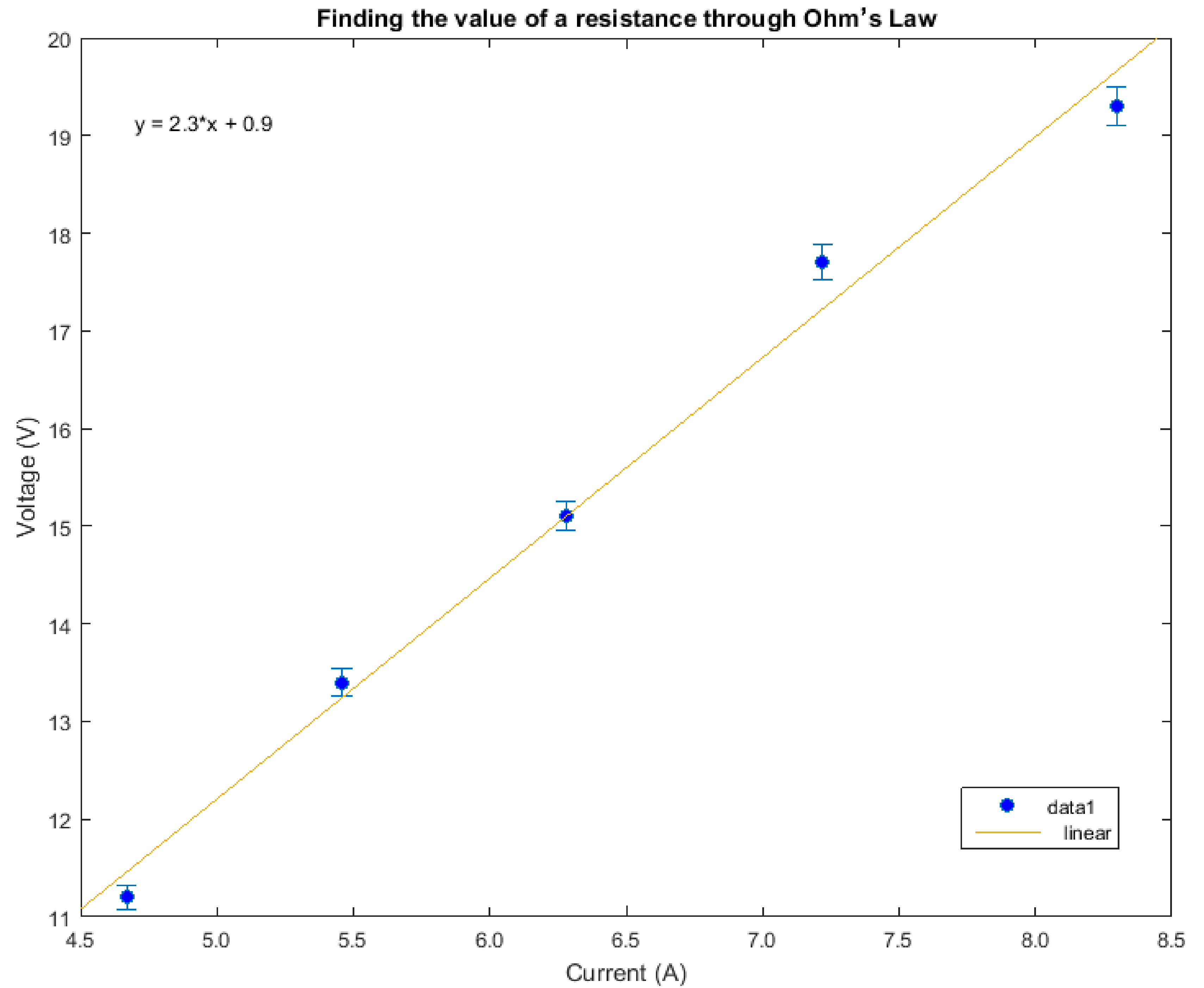
```
Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the function tolerance.

<stopping criteria details>


ans =

    2.2605    0.9042

>>
```



Finding the value of a resistance through Ohm's Law
Voltage (V)
Current (A)
22
20
18
16
14
12
10
8
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0

Uncertainties in Voltage and Current

Voltage (V)	Uncertainty due to resolution (U_s)	Uncertainty due to rating (U_r)	Total Uncertainty in Voltage (U_v)
11.2	0.03	0.11	0.12
13.4	0.03	0.13	0.14
15.1	0.03	0.15	0.15
17.7	0.03	0.18	0.18
19.3	0.03	0.19	0.20
Current (A)	**Uncertainty due to resolution (U_s)**	**Uncertainty due to rating (U_r)**	**Total Uncertainty in Voltage (I_v)**
4.67	0.003	0.047	0.047
5.46	0.003	0.055	0.055
6.28	0.003	0.063	0.063
7.22	0.003	0.072	0.072
8.30	0.003	0.083	0.083

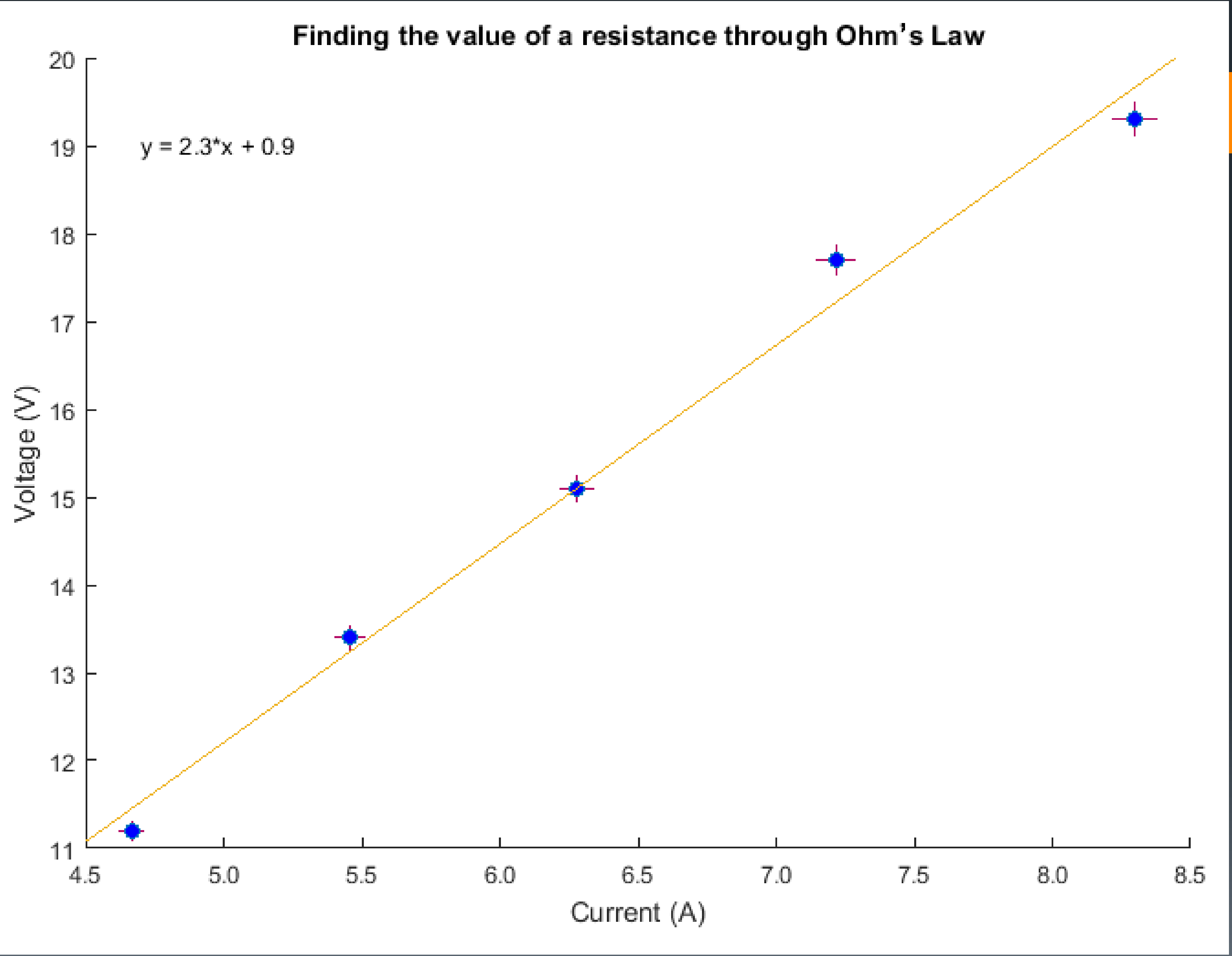

Finding the value of a resistance through Ohm's Law
y = 2.3*x + 0.9
Voltage (V)
Current (A)
20
19
18
17
16
15
14
13
12
11
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
data1
linear

What if we want to plot uncertainties for both axis?

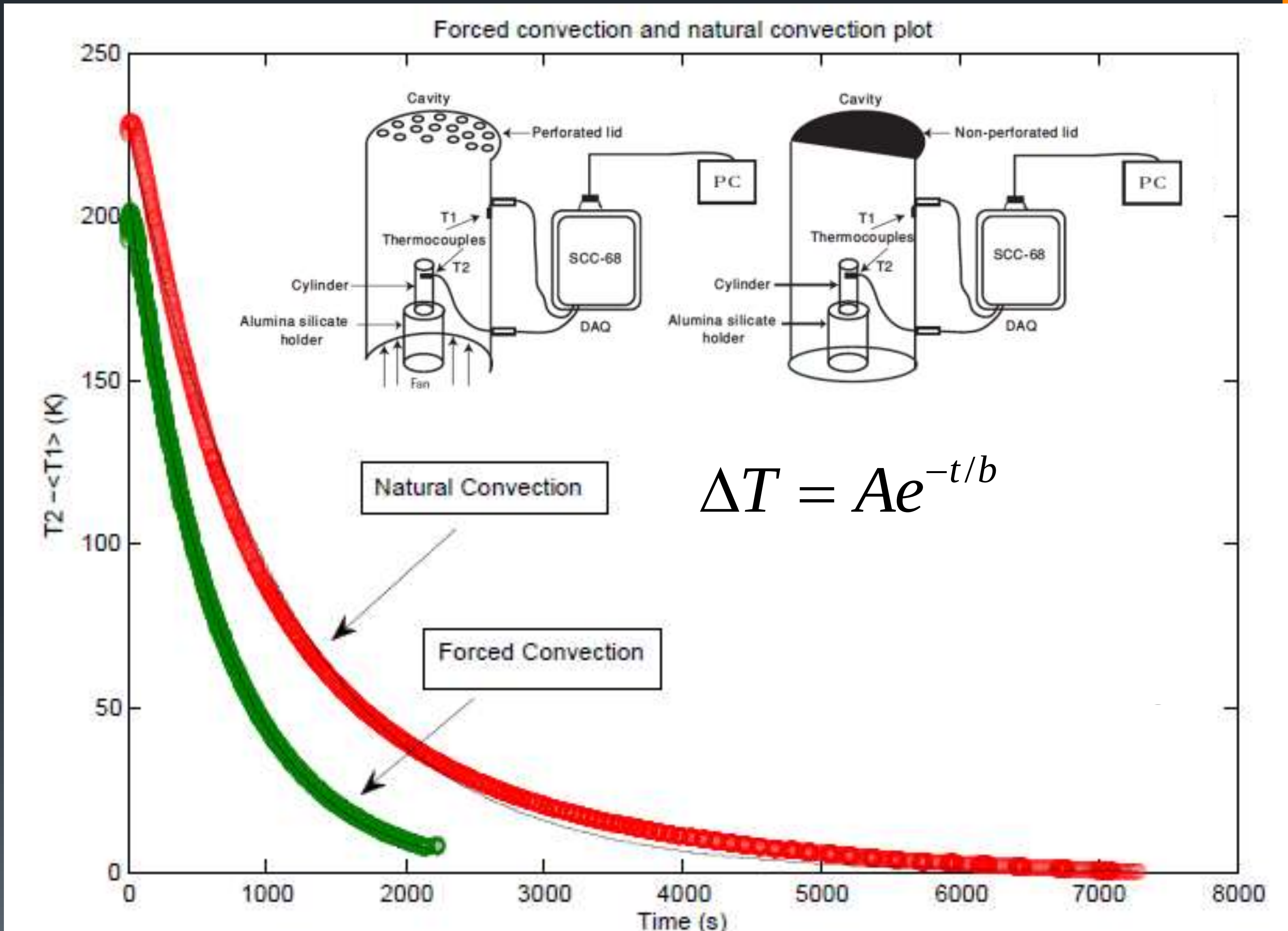
We'll use the "xyerrorbar" function file available on Physlab website.

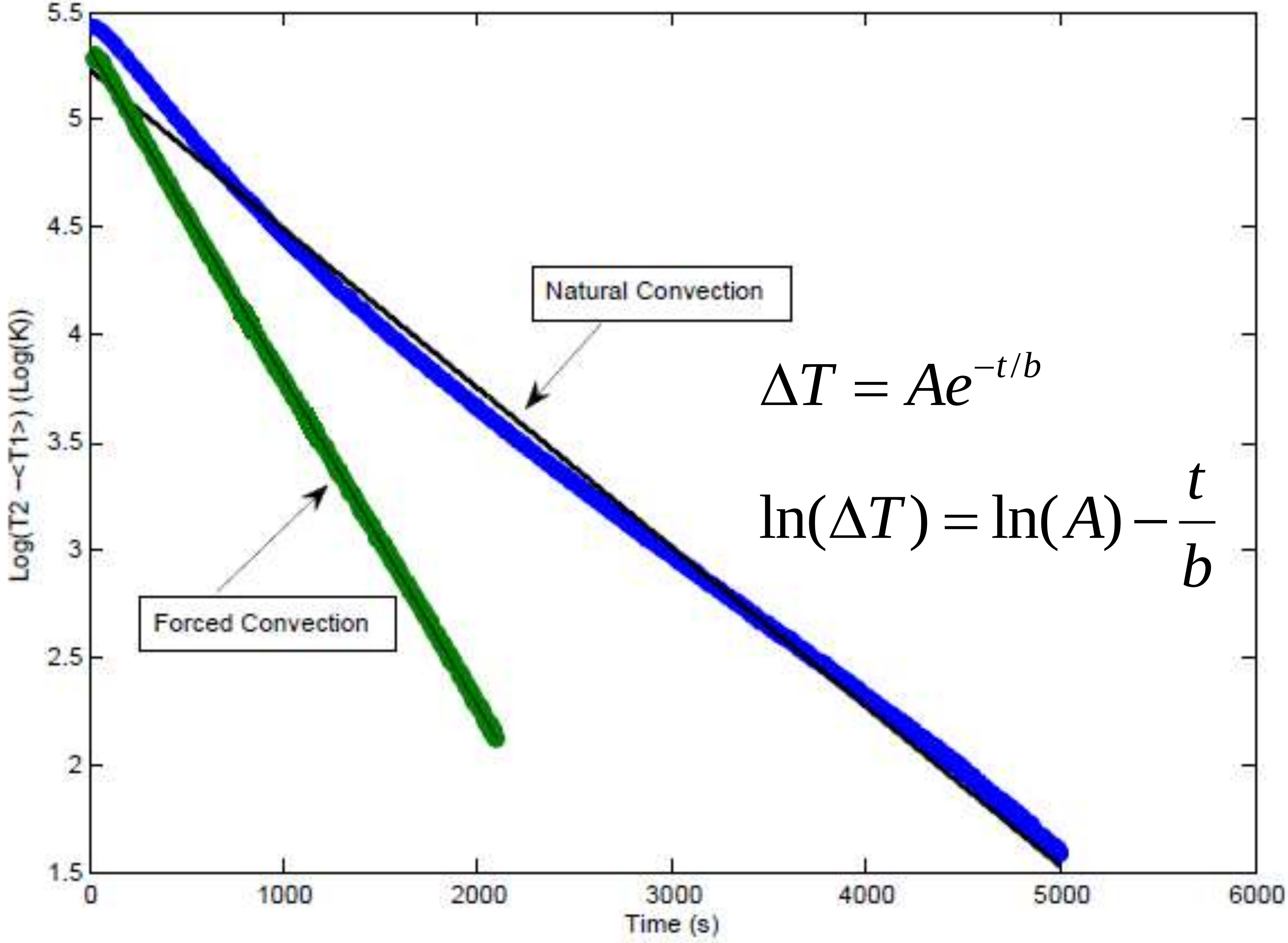
Syntax:

```
xyerrorbar(x_vector, y_vector, u_x, u_y, 'o')
```


Finding the value of a resistance through Ohm's Law
y = 2.3*x + 0.9
Voltage (V)
Current (A)
20
19
18
17
16
15
14
13
12
11
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5

Linearizing Plots: Cooling Objects

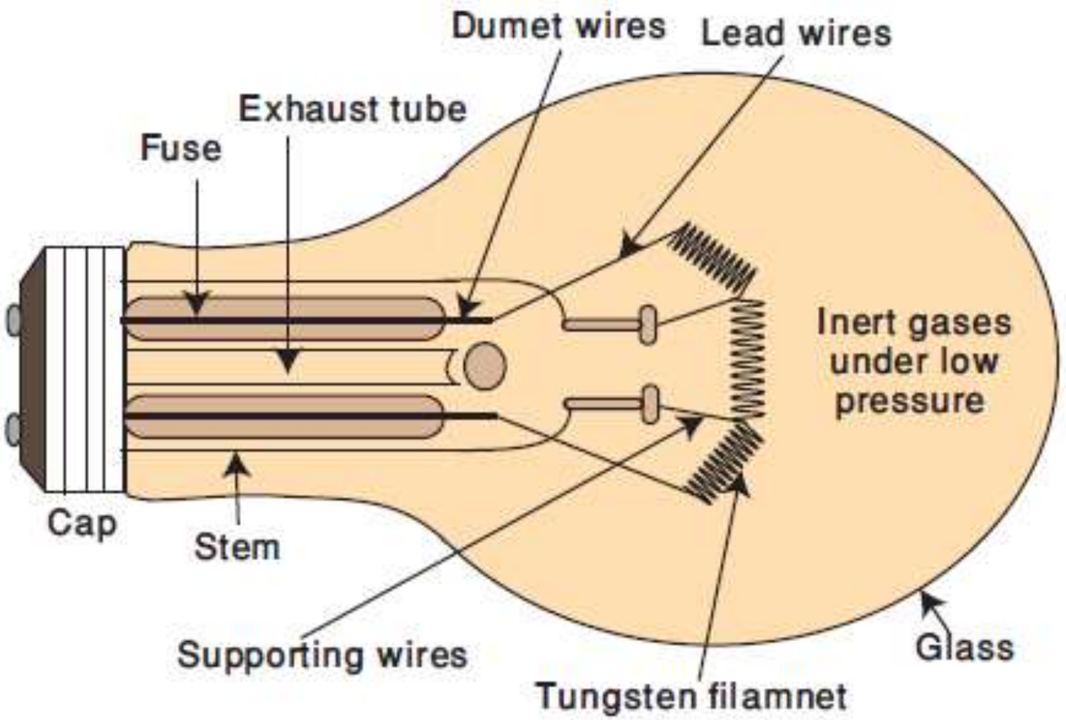




$$\Delta T = Ae^{-t/b}$$

$$\ln(\Delta T) = \ln(A) - \frac{t}{b}$$

Light Bulb (Power Law)

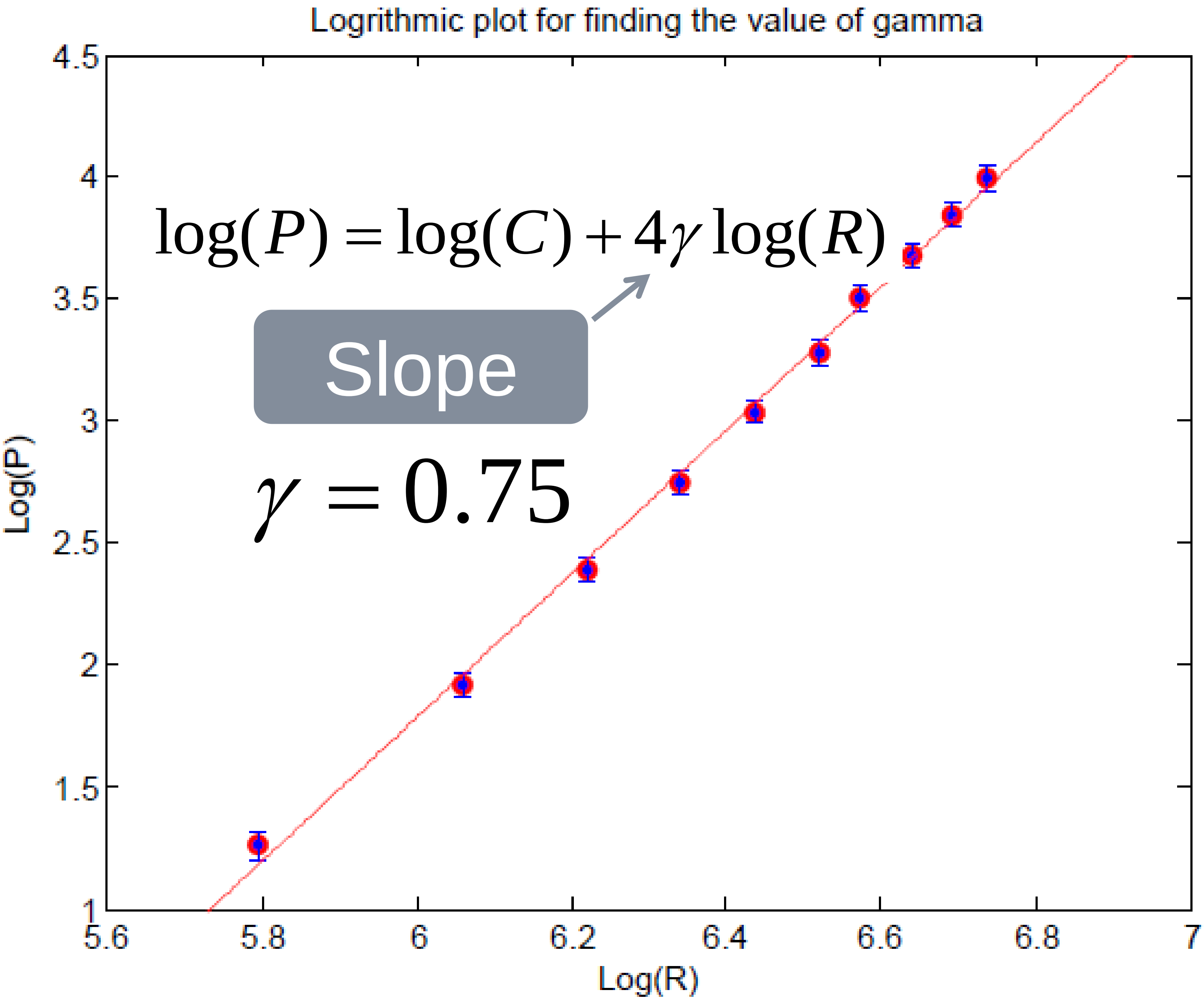


$$P_{\text{elec}} = \sigma A T^4$$

$$T = R^{\gamma}$$

$$P_{\text{elec}} = \sigma A R^{4\gamma} = C R^{4\gamma}$$

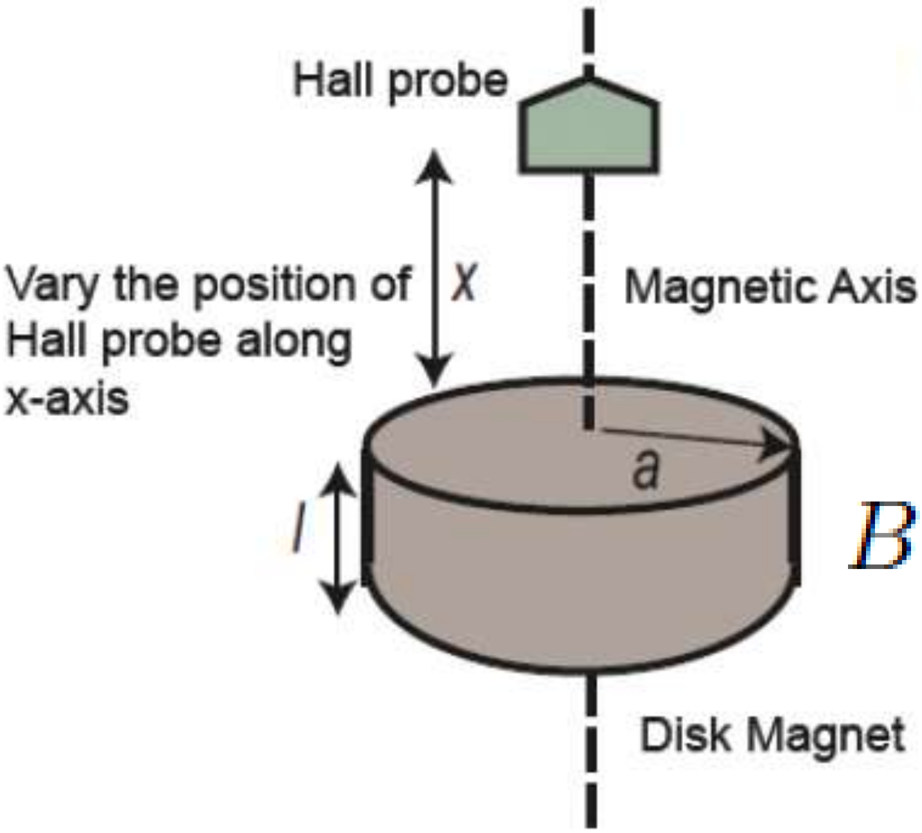
$$\log(P) = \log(C) + 4\gamma \log(R)$$



$$\log(P) = \log(C) + 4\gamma \log(R)$$

$$\gamma = 0.75$$

When does a model breakdown?

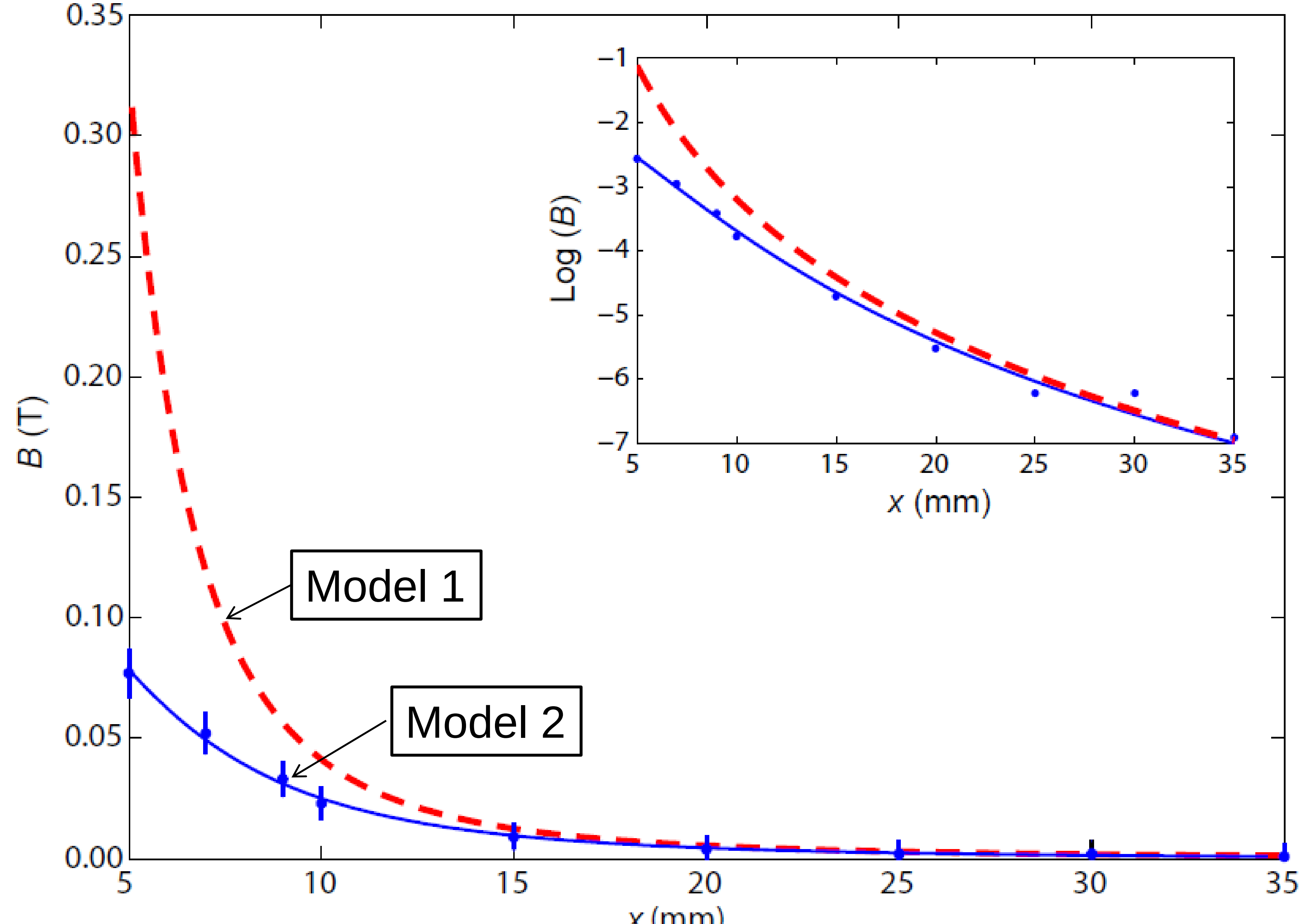


$$B = \frac{\mu_o m}{2\pi x_o^3}$$

Model 1

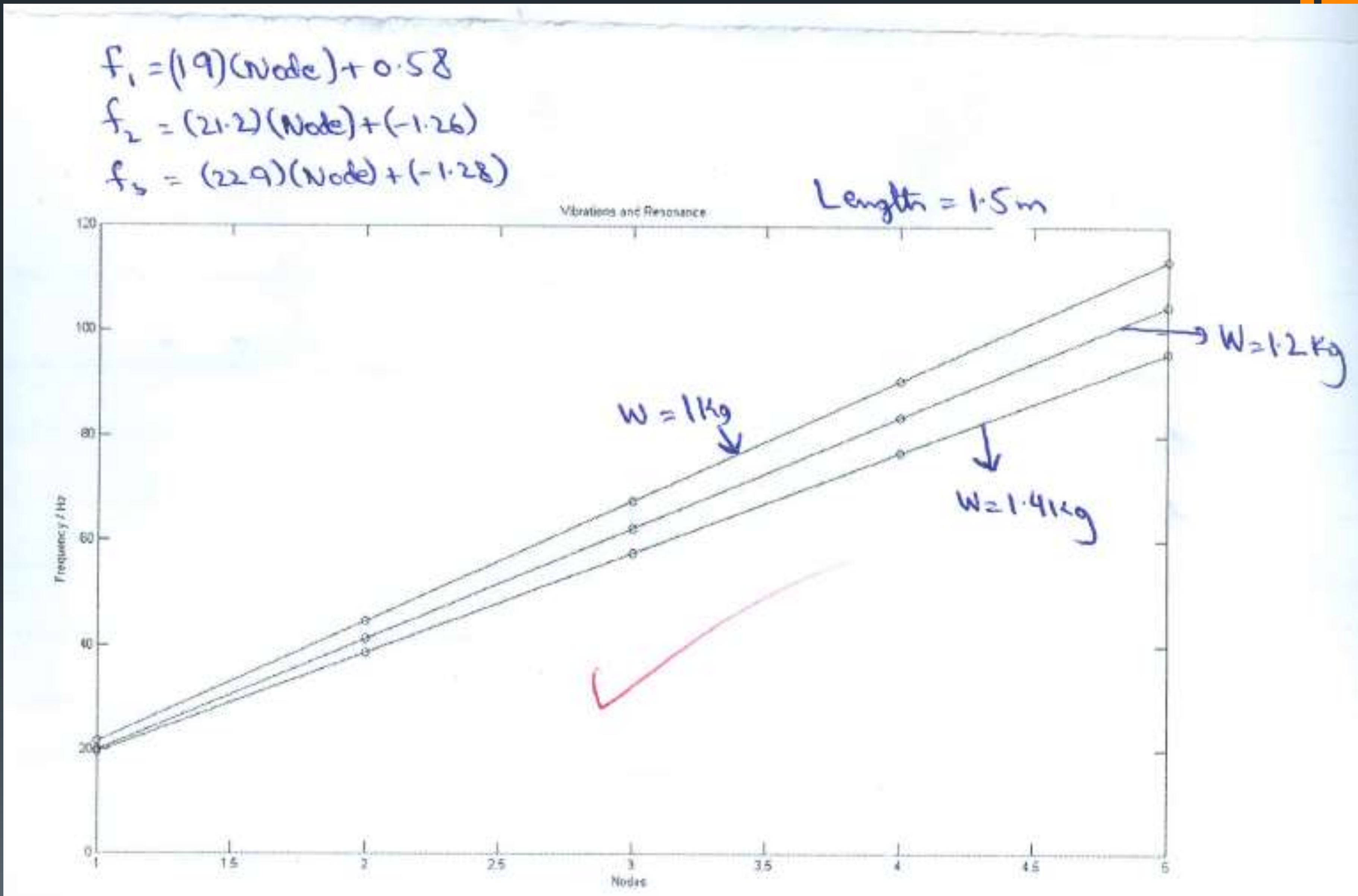
$$B = \frac{\mu_o}{2} M \left[\frac{x_o + d/2}{\left(a^2 + (x_o + d/2)^2\right)^{1/2}} - \frac{x_o - d/2}{\left(a^2 + (x_o - d/2)^2\right)^{1/2}} \right]$$

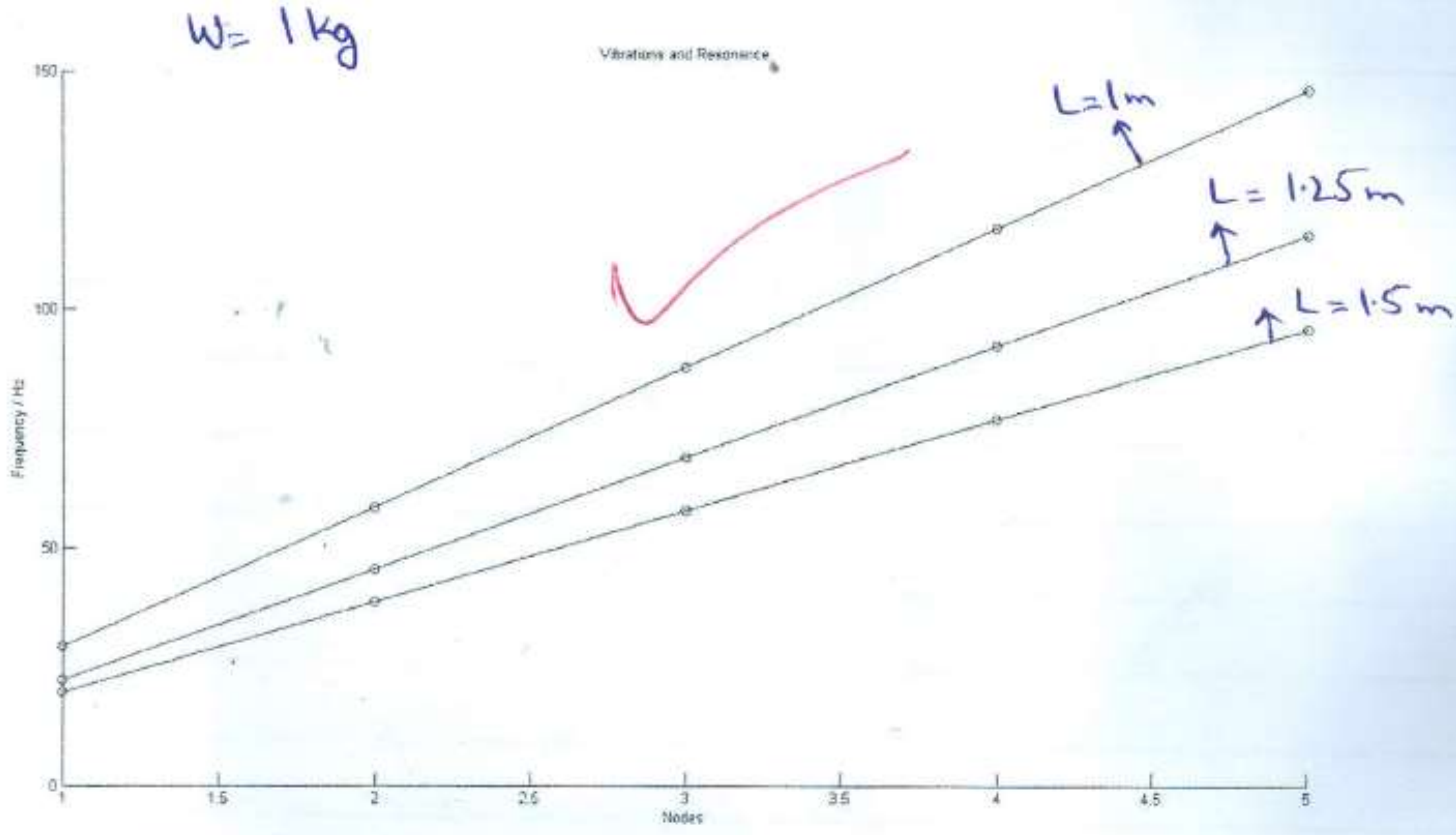
Model 2

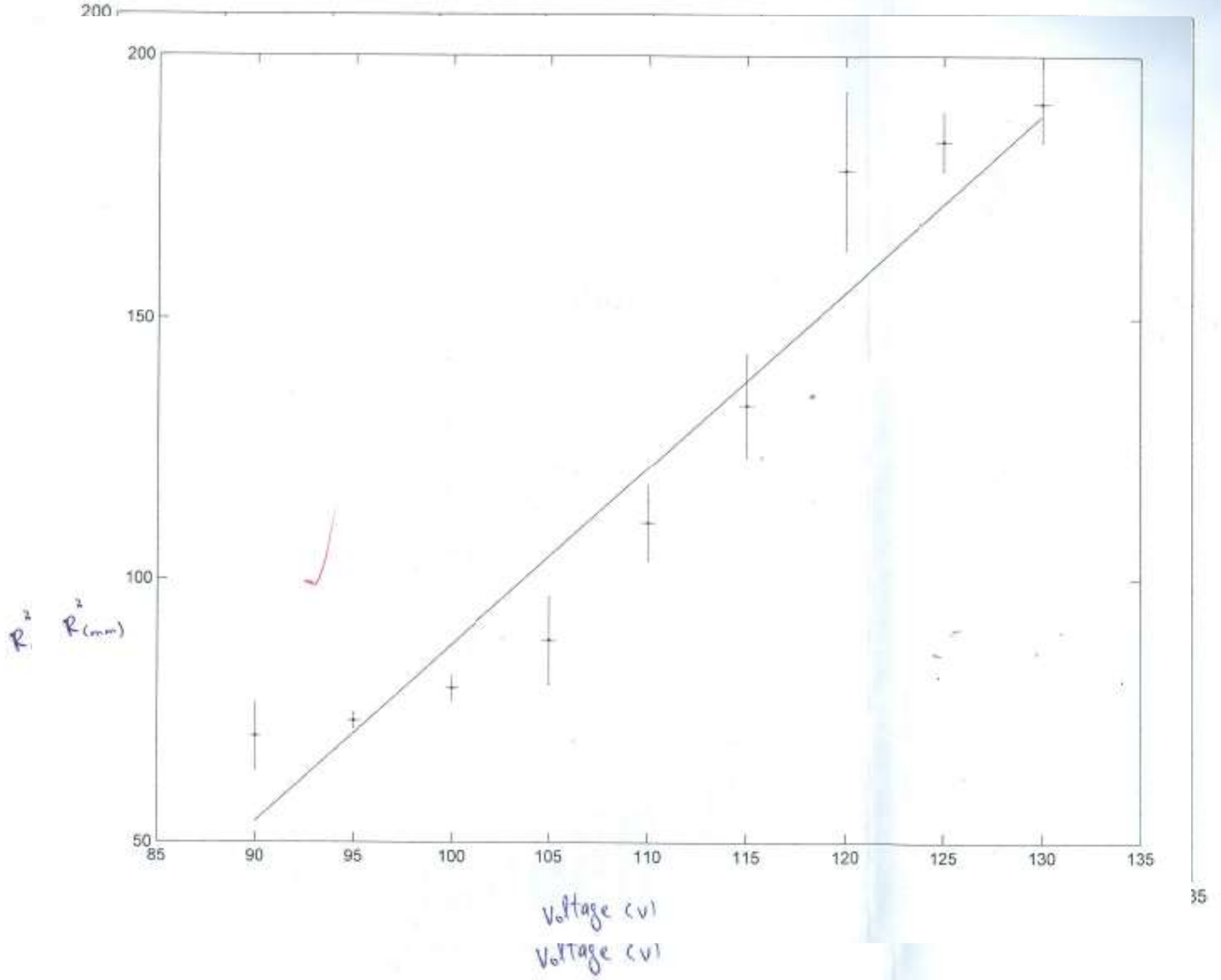


0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
B (T)
5
10
15
20
25
30
35
x (mm)
Model 1
Model 2
−1
−2
−3
−4
−5
−6
−7
Log (B)
5
10
15
20
25
30
35
x (mm)

Sample Work from Students of PHY100/200




W= 1kg
Vibrations and Resonance
L=1m
L = 1·25 m
L = 1·5 m
Frequency / Hz
Nodes
0
50
100
150
1
1.5
2
2.5
3
3.5
4
4.5
5



200
150
100
50
85
90
95
100
105
110
115
120
125
130
135
R^2 R^2 (mm)
Voltage (V)
Voltage (V)

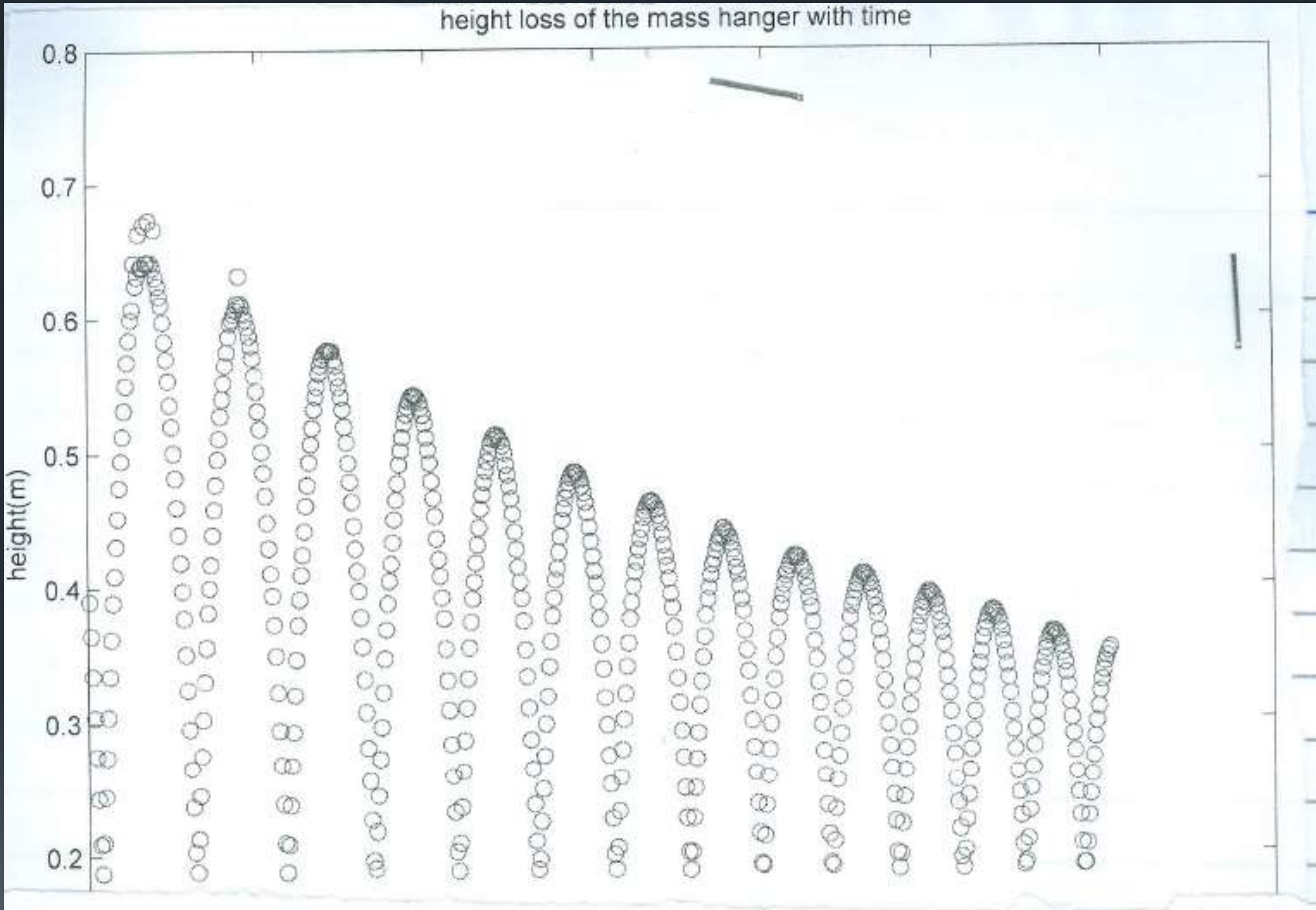
```
clc;
clear all;
close all
v=[90:5:130];
r= [8.379,8.555,8.908,9.408,10.525,11.554,13.3476,13.5534, 13.818];
r2 = r.^2;
plot(v,r2,'.')
dr = [0.382,0.0882,0.1323,0.441,0.3381,0.4263,0.5630,0.20433,0.26019]; %uncertainities
in r
a = v.*0.01;                              %Um ratings of v
p = sqrt((0.3).^2 + (a).^2);               %uncertainities in v
q= (2.*r).*dr;                         %uncertainities in r square by formula deltaq = 2rdeltar
Utrans = 0.32.*p;                      %Utrans 0.32 is slope from graph
Utotal = sqrt((q).^2+(Utrans).^2); %Uncertainities of rsquare after transformation
w = 1./(Utotal.^2)                   %weight of uncertainities

%calculation of final slope fm

v2 = v.*2;           %v square
sumw = sum(w);       %sum of weights
pr_vr2 = v.*r2;      %product of xy i.e v and r square
pr_w_vr2 = w.*pr_vr2; %product of weight and product of xy i.e v and r square
sumpr_w_vr2 = sum(pr_w_vr2); %sum of product of above two
pr_wv = w.*v ;               % product of weight and voltage
sumpr_wv = sum(pr_wv);  % sum of product of weight and voltage
pr_wr2 = w.*r2 ;        % product of weight and r square
sumpr_wr2 = sum(pr_wr2); % sum of p[roduct of weight and r square
pr_wv2 = w.*v2;              % product of weight and v square
sumpr_wv2 = sum(pr_wv2);    % sum of product of weight and v square
sumpr_wv_2 = (sumpr_wv).^2; % square of sum of product of weight and voltage


nume=((sum(w)).*(sum(w.*(v.*r2))))-((sum(w.*v)).*sum(w.*r2))
denom=((sum(w)).*(sum(w.*(v.^2))))-((sum(w.*v)).^2)

um=sqrt(sum(w)./denom)

final_m = nume/denom

% calculating c
nume_c = (((sum(w.*(v.^2))).*sum(w.*r2))-((sum(w.*v)).*(sum(w.*(v.*r2)))))
final_c = nume_c/denom

uc = sqrt((sum(w.*(v.^2)))/denom)

y = ((final_m).*(v))+ final_c;
hold on
```

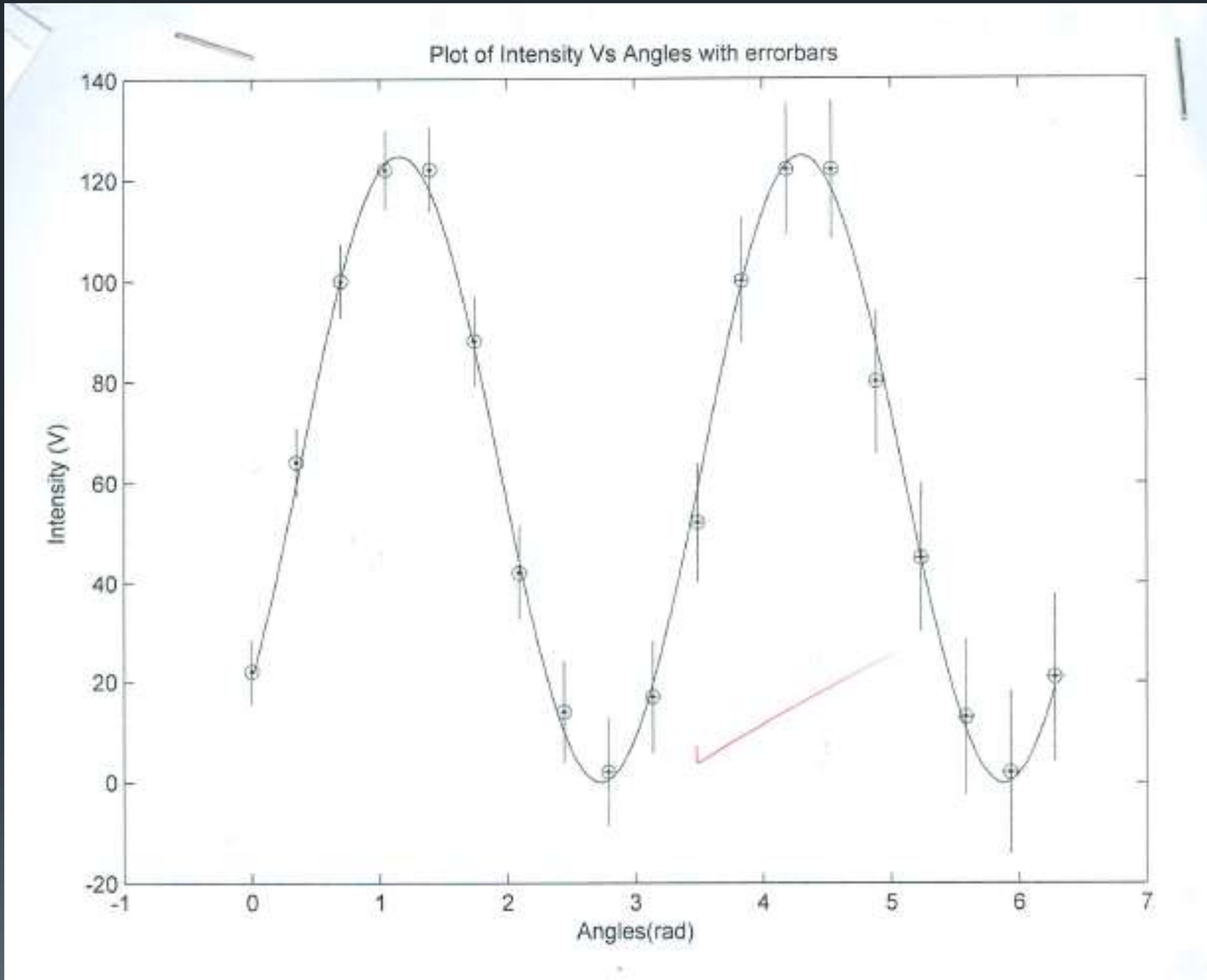
CALCULATIONS

cycle	Height (m)	Delta h (m)	N	Delta E (Joules)	W avg (rad/sec^2)
1	0.61420	0	7.797	0	5.065
2	0.57090	0.04300	7.247	0.04250	4.816
3	0.53480	0.03610	6.789	0.03540	4.646
4	0.50840	0.02640	6.454	0.02590	4.676
5	0.47060	0.03780	5.974	0.03710	4.267
6	0.45560	0.01500	5.784	0.01470	4.148
7	0.42510	0.03050	5.396	0.02990	1.904
8	0.41010	0.01500	5.206	0.01470	4.148
9	0.39180	0.01830	4.974	0.01800	3.569
10	0.37480	0.01700	4.758	0.01670	3.220

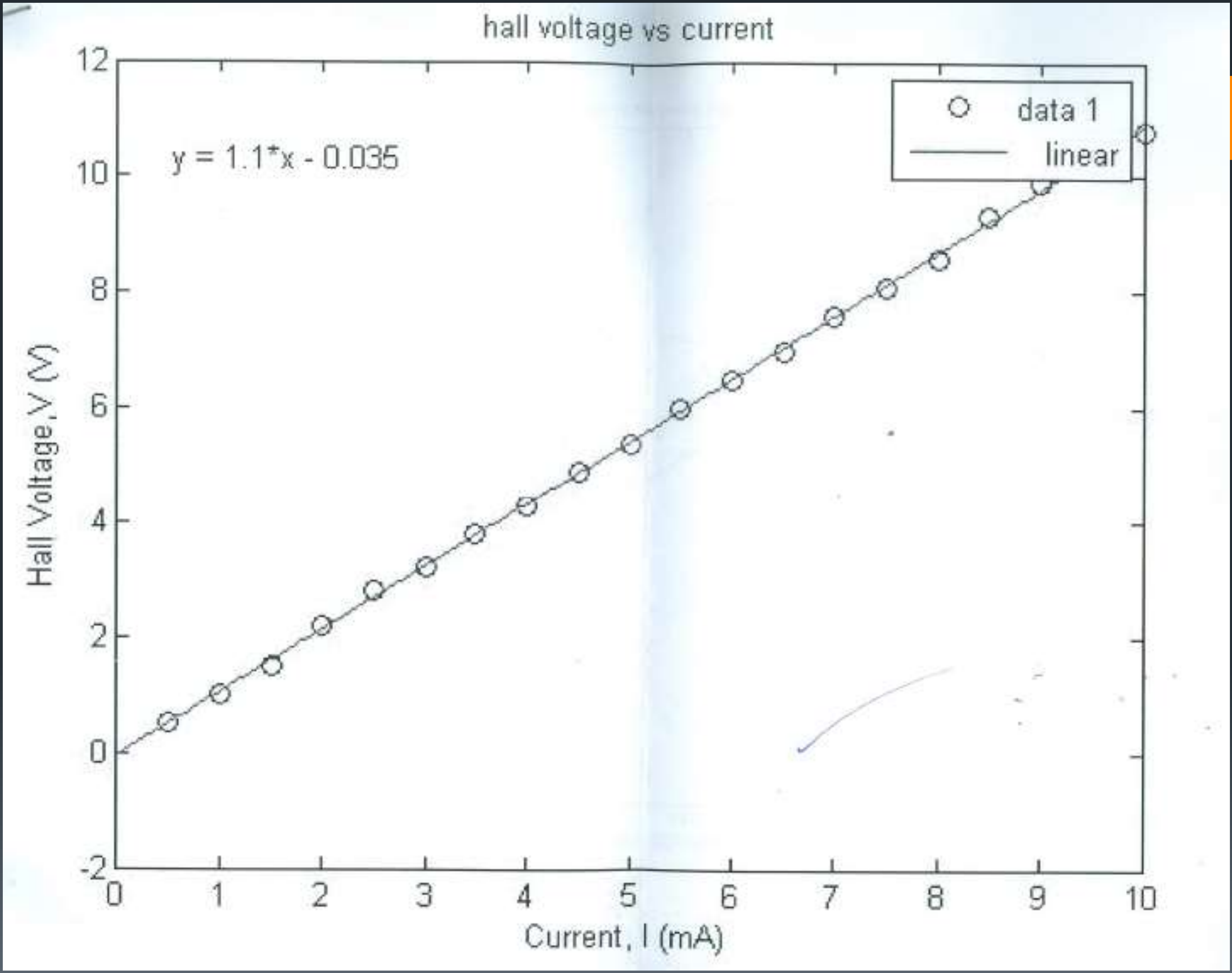
Uncertainty in height= 0.00005/(6)^0.5 (Analog instrument)

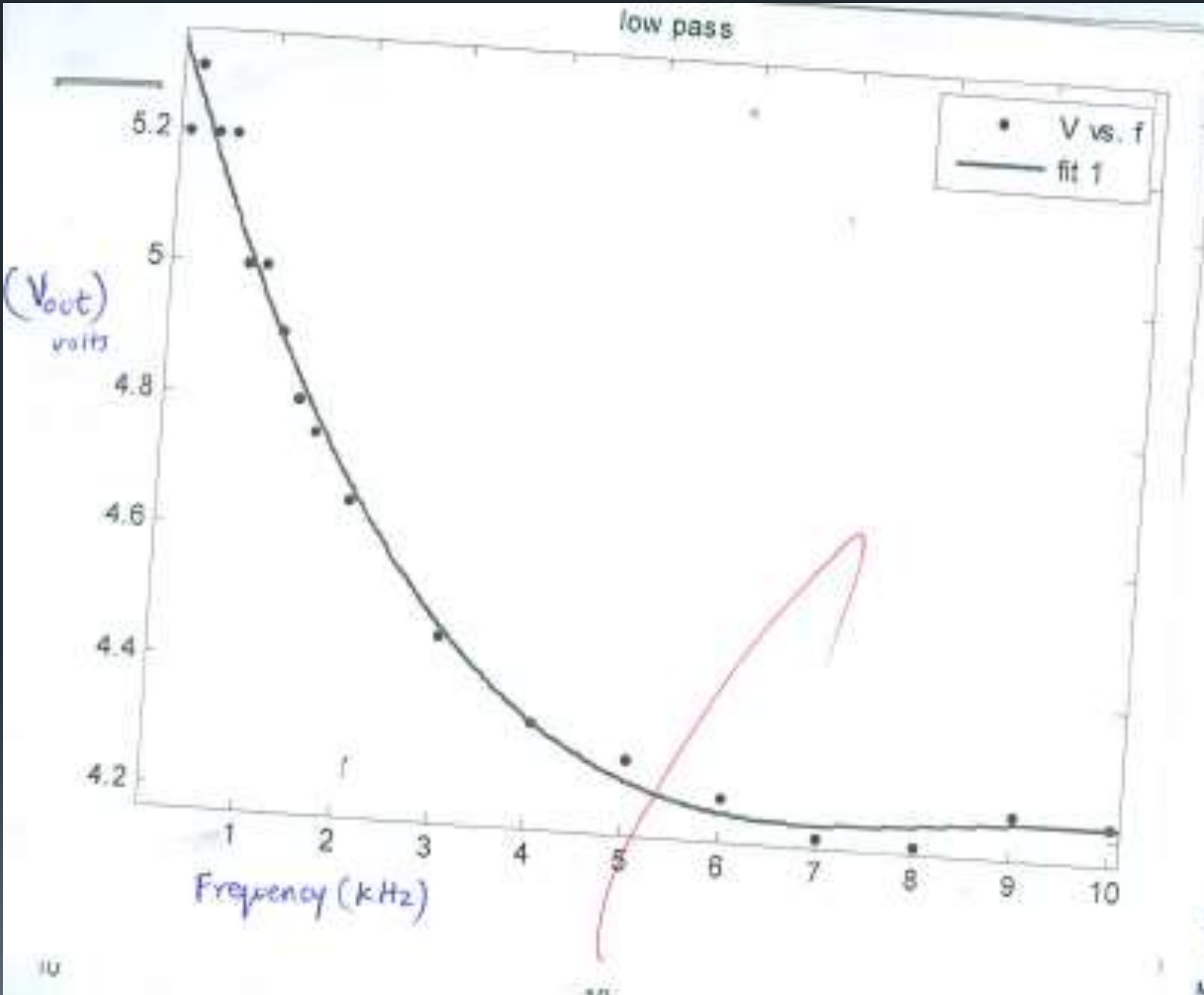
Uncertainty in radius= 0.005/(6)^0.5=0.002 (Analog instrument)


height loss of the mass hanger with time
height(m)
0.8
0.7
0.6
0.5
0.4
0.3
0.2

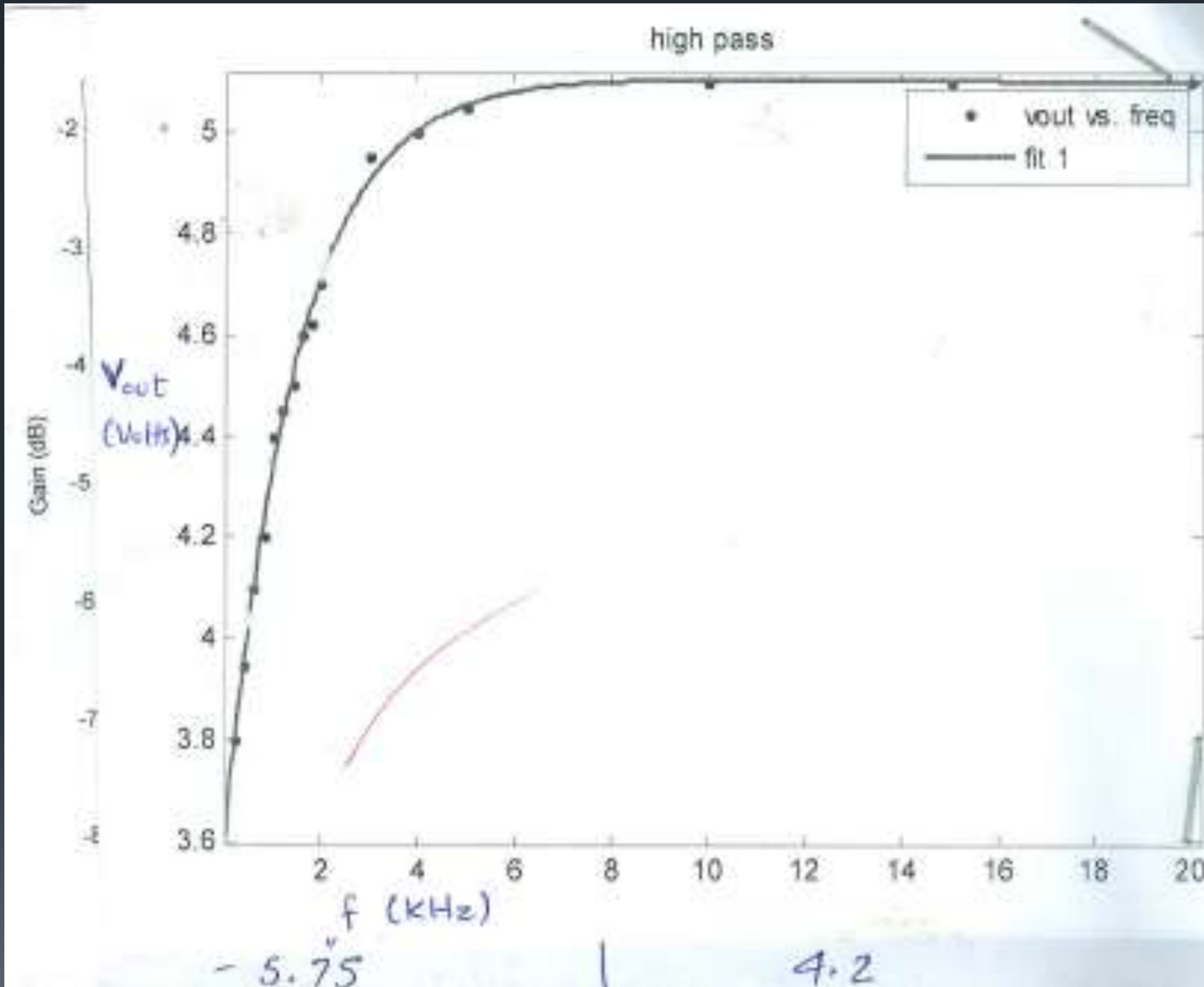


Plot of Intensity Vs Angles with errorbars
Intensity (V)
Angles(rad)
140
120
100
80
60
40
20
0
-20
-1
0
1
2
3
4
5
6
7


hall voltage vs current
y = 1.1*x - 0.035
data 1
linear
Hall Voltage, V (V)
Current, I (mA)
12
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9
10



low pass
V vs. f
fit 1
5.2
5
4.8
4.6
4.4
4.2
1
2
3
4
5
6
7
8
9
10
(Vout) volts
Frequency (kHz)


high pass
vout vs. freq
fit 1
Gain (dB)
Vout (Volts)
f (KHz)
-2
-3
-4
-5
-6
-7
5
4.8
4.6
4.4
4.2
4
3.8
3.6
2
4
6
8
10
12
14
16
18
20
- 5.75
4.2

Monday 10th Nov. 2014.

$\boxed{\text{Experiment \# 1.2A}}$

Q1.- Moment of Inertia of tennis ball.

$$I = \frac{2}{5} M \left(\frac{R_e^5 - R_i^5}{R_e^3 - R_i^3} \right)$$

First we find respective quantities needed to measure the moment of inertia of a hollow sphere and a composite ~~disk~~ cylinder

$\boxed{\text{For composite cylinder}}$

$R_1 = 4.39\ mm = 0.439\ cm$
$R_2 = 14.96\ mm = 1.496\ cm$
$h_1 = 5.78\ mm = 0.578\ cm$
$h_2 = 16.90\ mm = 1.690\ cm$
$T = 26.10/20 = 1.305\ secs$

$\boxed{\text{For tennis ball (hollow sphere)}}$

$R_e = 32.07\ mm = 3.207\ cm$
$R_i = 26.38\ mm = 2.638\ cm$
$M = 62.95g$
$T_{tennis} = \frac{38.71}{10} = 3.871\ secs$

$\boxed{\text{I for tall section}}$

$$I_{tall} = \frac{1}{2} \pi \rho h_1 R_1^4$$

$$I_{tall} = \frac{1}{2} (\pi) (2.7 \times 10^{-3}\ g/mm^3) (5.78) (4.39)^4$$

$$I_{tall} = \cancel{8.627\ g\ mm^2}\ \ 0.087\ g\ cm^2$$

I for short section

$$I_{shr} = \frac{1}{2}\pi\rho h_2 (R_2^4 - R_1^4)$$

$$I_{shr} = 0.5 \times \pi \times 2.7 \times \cancel{16.9}\ 1.69 (1.496^4 - 0.734^4)$$

$$I_{short} = 35.646 \text{ g cm}^2$$

$$I_{total} = I_{tail} + I_{short} = 35.733 \text{ g cm}^2$$

$$\boxed{I_{cylinder} = 35.733 \text{ gcm}^2}$$

We know that

$$T = 2\pi\sqrt{I/K}$$

or

$$K = \frac{4\pi^2 I}{T^2}$$

$$\boxed{K = 828.339} \text{ gcm}^2/s^2$$

Now

$$I_{tennis} = \frac{K\,T_{tennis}^2}{4\pi^2}$$

$$I_{tennis} = \frac{(828.339)(3.871)^2}{4\pi^2}$$

$$\boxed{I_{tennis} = 314.409\ g\,cm^2}$$

Theoretical value of moment of inertia of tennis ball

$$I_{tennis\ theoretical} = \frac{2}{5}(62.95)\left(\frac{32.07^5 - 26.20^5}{32.07^3 - 26.20^3}\right)$$

$$\boxed{I_{tennis\ theoretical} = 364.087\ g\,cm^2}$$

Uncertainities

* Uncertainities in measurements done with vernier calliper. (Δr)

$$\Delta r = \frac{0.02}{2 \times \sqrt{6}} = 4.08 \times 10^{-3}\ mm = 4.08 \times 10^{-4}\ cm$$

* Uncertainities in measurements done with stopwatch (ΔT)

$$\Delta T = \frac{0.01}{2 \times \sqrt{3}} = 2.887 \times 10^{-3}\ secs.$$

* Uncertainities in quantities measured with weighing balance (ΔM)

$$\Delta M = \frac{0.01}{2 \times \sqrt{3}} = 2.887 \times 10^{-3}\ grams.$$

Rating of digital devices not included in uncertainties.

$\boxed{\text{Uncertainity in } I_{tall}}$

$$\Delta I_{tall} = \sqrt{\left(\frac{\delta I_{tall}}{\delta h_1} \Delta h_1\right)^2 + \left(\frac{\delta I_{tall}}{\delta R_1} \times \Delta R_1\right)^2}$$

$$\Delta I_{tall} = \sqrt{\left(\frac{1}{2} \pi \rho R_1^4 \times \Delta h_1\right)^2 + \left(4 \times \frac{1}{2} \pi \rho h_1 R_1^3 \times \Delta R_1\right)^2}$$

$$\Delta I_{tall} = \sqrt{\left(0.5(\pi)(2.7)(0.951)^4(9.68 \times 10^{-4})\right)^2 + \left(2 \times (\pi)(2.7)(0.578)(0.951)^3(9.08 \times 10^{-4})\right)^2}$$

$$\Delta I_{tall} = \sqrt{3.77 \times 10^{-9} + 1.07 \times 10^{-7}}$$

$$\Delta I_{tall} = 3.33 \times 10^{-4} \text{ g cm}^2$$

$\boxed{\text{Uncertainity in } I_{short}}$

$$\Delta I_{short} = \sqrt{\left(\frac{1}{2} \pi \rho (R_2^4 - R_1^4) \times \Delta h_1\right)^2 + \left(\frac{1}{2} \pi \rho h_2 R_2^3 \times 4 \times \Delta R_2\right)^2 + \left(-\frac{1}{2} \pi \rho h_2 R_1^3 \times 4 \times \Delta R_1\right)^2}$$

$$\Delta I_{short} = \sqrt{7.4 \times 10^{-5} + 1.5 \times 10^{-3} + 9.1 \times 10^{-7}}$$

$$\Delta I_{short} = 0.04 \text{ g cm}^2$$

$\Delta I_{cylinder} = (3.33\times10^{-4} + 0.04)\ gcm^2$

$$\boxed{\Delta I_{cylinder} = 0.04\ gcm^2}$$

Uncertainity in $I_{tennis\ theoretical}$

$$\frac{\delta I_{tt}}{\delta R_e} = \frac{2}{5} M \left[\frac{-5R_e^7 + 5R_e^4 R_i^3 - 3R_e^7 + 3R_e^2 R_i^5}{(R_e^3 - R_i^3)^2} \right]$$

$$= -2075.99$$

$$\frac{\delta I_{tt}}{\delta R_i} = \frac{2}{5} M \left[\frac{(R_e^3 - R_i^3)(-5R_i^4) - (R_e^5 - R_i^5)(-3R_i^2)}{(R_e^3 - R_i^3)^2} \right]$$

$$= 102.83$$

$$\frac{\delta I_{tt}}{\delta M} = \frac{2}{5} \left[\frac{R_e^5 - R_i^5}{R_e^3 - R_i^3} \right] = 5.78$$

$$\Delta I_{tt} = \sqrt{\left(\frac{\delta I_{tt}}{\delta R_e}\Delta R_e\right)^2 + \left(\frac{\delta I_{tt}}{\delta R_i}\times\Delta R_i\right)^2 + \left(\frac{\delta I_{tt}}{\delta M}\times\Delta M\right)^2}$$

Ans 1:- $I_{tennis\ theoretical} = (364.09 \pm 0.85)\ g\,cm^2$

Now

$$\Delta K = \sqrt{\left(\frac{4\pi^2}{T^2} \times \Delta I_{cylinder}\right)^2 + \left(4\pi^2 I_{cylinder} T^{-1} \times -2 \times \Delta T\right)^2}$$

$\Delta K = 6.5\ g\,cm^2/s^2$

Ans 2:- Torsional constant $(K) = (828 \pm 6)\ g\,cm^2/s^2$

Now

$$I = \frac{KT^2}{4\pi^2}$$

10.11.2013

$$\Delta I_{tennis\ experiment} = \sqrt{\left(\frac{T^2}{4\pi^2} \times \Delta K\right)^2 + \left(\frac{2TK}{4\pi^2} \times \Delta T\right)^2}$$

$\Delta I_{tennis\ experimental} = 2.48\ g\,cm^2$

Ans 3:- $I_{tennis\ experimental} = (314.4 \pm 2.5)\ g\,cm^2$

Let’s practice!

Solve last exercise of this session…

Introduction to MATLAB

End of session!