

radius r as an Amperian loop and traverse it in the clockwise direction. Ampere's law (Eq. 29-14) yields

$$(B)(2\pi r) = \mu_0 iN,$$

where i is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and N is the total number of turns. This gives

$$B = \frac{\mu_0 iN}{2\pi r} \quad (\text{toroid}). \quad (29-24)$$

In contrast to the situation for a solenoid, B is not constant over the cross section of a toroid.

It is easy to show, with Ampere's law, that $B = 0$ for points outside an ideal toroid (as if the toroid were made from an ideal solenoid). The direction of the magnetic field within a toroid follows from our curled-straight right-hand rule: Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.

Sample Problem

The field inside a solenoid (a long coil of current)

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L . What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$\begin{aligned} B &= \mu_0 in = (4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.57 \text{ A}) \frac{5 \times 850 \text{ turns}}{1.23 \text{ m}} \\ &= 2.42 \times 10^{-2} \text{ T} = 24.2 \text{ mT}. \end{aligned} \quad (\text{Answer})$$

To a good approximation, this is the field magnitude throughout most of the solenoid.



Additional examples, video, and practice available at WileyPLUS

29-6 A Current-Carrying Coil as a Magnetic Dipole

So far we have examined the magnetic fields produced by current in a long straight wire, a solenoid, and a toroid. We turn our attention here to the field produced by a coil carrying a current. You saw in Section 28-10 that such a coil behaves as a magnetic dipole in that, if we place it in an external magnetic field \vec{B} , a torque $\vec{\tau}$ given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (29-25)$$

acts on it. Here $\vec{\mu}$ is the magnetic dipole moment of the coil and has the magnitude NiA , where N is the number of turns, i is the current in each turn, and A is the area enclosed by each turn. (*Caution:* Don't confuse the magnetic dipole moment $\vec{\mu}$ with the permeability constant μ_0 .)

Recall that the direction of $\vec{\mu}$ is given by a curled-straight right-hand rule: Grasp the coil so that the fingers of your right hand curl around it in the direction of the current; your extended thumb then points in the direction of the dipole moment $\vec{\mu}$.

Magnetic Field of a Coil

We turn now to the other aspect of a current-carrying coil as a magnetic dipole. What magnetic field does it produce at a point in the surrounding space? The problem does not have enough symmetry to make Ampere's law useful; so we must turn to the law of Biot and Savart. For simplicity, we first consider only a coil with a single circular loop and only points on its perpendicular central axis, which we take to be a z axis. We shall show that the magnitude of the magnetic field at such points is

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}, \quad (29-26)$$

in which R is the radius of the circular loop and z is the distance of the point in question from the center of the loop. Furthermore, the direction of the magnetic field \vec{B} is the same as the direction of the magnetic dipole moment $\vec{\mu}$ of the loop.

For axial points far from the loop, we have $z \gg R$ in Eq. 29-26. With that approximation, the equation reduces to

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3}.$$

Recalling that πR^2 is the area A of the loop and extending our result to include a coil of N turns, we can write this equation as

$$B(z) = \frac{\mu_0}{2\pi} \frac{NiA}{z^3}.$$

Further, because \vec{B} and $\vec{\mu}$ have the same direction, we can write the equation in vector form, substituting from the identity $\mu = NiA$:

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3} \quad (\text{current-carrying coil}). \quad (29-27)$$

Thus, we have two ways in which we can regard a current-carrying coil as a magnetic dipole: (1) it experiences a torque when we place it in an external magnetic field; (2) it generates its own intrinsic magnetic field, given, for distant points along its axis, by Eq. 29-27. Figure 29-21 shows the magnetic field of a current loop; one side of the loop acts as a north pole (in the direction of $\vec{\mu}$) and the other side as a south pole, as suggested by the lightly drawn magnet in the figure. If we were to place a current-carrying coil in an external magnetic field, it would tend to rotate just like a bar magnet would.

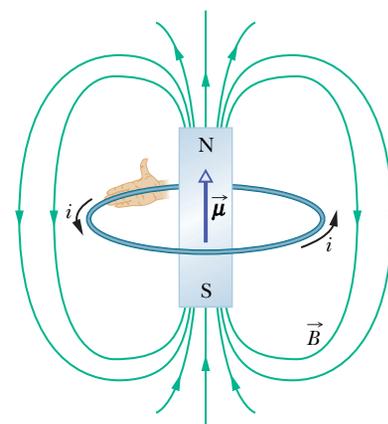
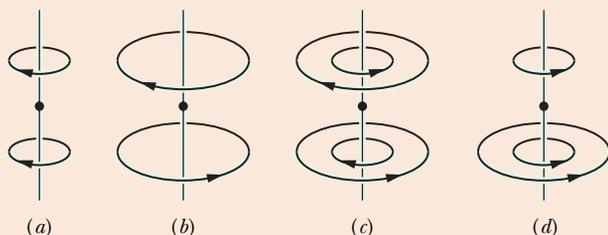


Fig. 29-21 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field \vec{B} within the loop.

CHECKPOINT 3

The figure here shows four arrangements of circular loops of radius r or $2r$, centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.



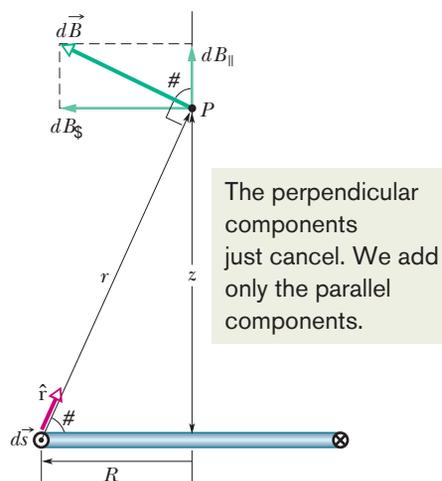


Fig. 29-22 Cross section through a current loop of radius R . The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point P on the central perpendicular axis of the loop.

Proof of Equation 29-26

Figure 29-22 shows the back half of a circular loop of radius R carrying a current i . Consider a point P on the central axis of the loop, a distance z from its plane. Let us apply the law of Biot and Savart to a differential element ds of the loop, located at the left side of the loop. The length vector $d\vec{s}$ for this element points perpendicularly out of the page. The angle θ between $d\vec{s}$ and \hat{r} in Fig. 29-22 is 90° ; the plane formed by these two vectors is perpendicular to the plane of the page and contains both \hat{r} and $d\vec{s}$. From the law of Biot and Savart (and the right-hand rule), the differential field $d\vec{B}$ produced at point P by the current in this element is perpendicular to this plane and thus is directed in the plane of the figure, perpendicular to \hat{r} , as indicated in Fig. 29-22.

Let us resolve $d\vec{B}$ into two components: dB_{\parallel} along the axis of the loop and dB_{\perp} perpendicular to this axis. From the symmetry, the vector sum of all the perpendicular components dB_{\perp} due to all the loop elements ds is zero. This leaves only the axial (parallel) components dB_{\parallel} and we have

$$B = \int dB_{\parallel}.$$

For the element $d\vec{s}$ in Fig. 29-22, the law of Biot and Savart (Eq. 29-1) tells us that the magnetic field at distance r is

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin 90^\circ}{r^2}.$$

We also have

$$dB_{\parallel} = dB \cos \alpha.$$

Combining these two relations, we obtain

$$dB_{\parallel} = \frac{\mu_0 i \cos \alpha ds}{4\pi r^2}. \quad (29-28)$$

Figure 29-22 shows that r and α are related to each other. Let us express each in terms of the variable z , the distance between point P and the center of the loop. The relations are

$$r = \sqrt{R^2 + z^2} \quad (29-29)$$

and

$$\cos \alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}. \quad (29-30)$$

Substituting Eqs. 29-29 and 29-30 into Eq. 29-28, we find

$$dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} ds.$$

Note that i , R , and z have the same values for all elements ds around the loop; so when we integrate this equation, we find that

$$\begin{aligned} B &= \int dB_{\parallel} \\ &= \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \int ds \end{aligned}$$

or, because $\int ds$ is simply the circumference $2\pi R$ of the loop,

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}.$$

This is Eq. 29-26, the relation we sought to prove.