two

# TRANSIENTS ON A LOSSLESS TRANSMISSION LINE

## 2-1 DISTRIBUTED CONSTANTS OF A LOSSLESS TRANSMISSION LINE

In order to determine the voltage and current along a transmission line, we must establish the electrical characteristics or the electrical equivalent of the line. The two most important parameters of the transmission line (the only parameters considered in a lossless line) are the inductance and capacitance. The current in the line sets up a magnetic flux around the conductors, which in turn induces a voltage in the conductors $\left(\text{the familiar } L\frac{di}{dt}\right)$. This distributed *inductance* is represented by the symbol $L$, having units of henries per unit length of line. The *capacitance*, which is also distributed (think of the lines as two parallel plates) is represented by $C$. This is measured in farads per unit length of line.

A simplistic schematic of the distributed $L$ and $C$ is given in Fig. 2-1. As a current–voltage wave (or an electromagnetic wave) travels down a transmission line, the currents must flow through the distributed inductors and a voltage must be set up across the distributed capacitance. Because he current cannot

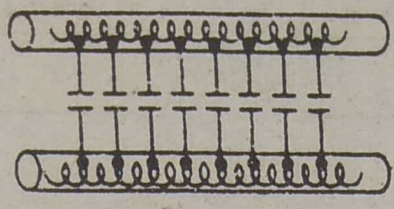


FIG. 2-1 Equivalent circuit of a lossless transmission line.

change instantaneously through an inductor, and a voltage cannot be immediately changed across a capacitor, it takes time for the current–voltage wave to travel down the line. Thus, there is a finite velocity for the wave propagation down the line.

If neither the series resistance of the line nor the finite conductance of the insulation material cannot be neglected, there is also an attenuation of the wave as it progresses down the lossy line. We shall neglect these parameters for the moment but will consider them later.

## 2-2 TRAVELING WAVES ON A LOSSLESS TRANSMISSION LINE

It is possible to mathematically derive the general solutions for the voltage and current along a uniform transmission line. We shall do this later for the sinusoidal steady-state case, but since the general solution involves solving a partial second-order differential equation, we shall for the moment merely describe the traveling-wave phenomenon of a transient voltage or current on a transmission line.

The convention used when dealing with voltages and currents on transmission lines is slightly different from that used in two-port circuit theory. The voltage is considered to be positive when the upper terminal or lead is at a more positive potential than the lower lead, and the currents are taken to be positive when traveling from the generator to the load. This is illustrated in Fig. 2-2, where the subscripts $S$ and $R$ refer to the sending end and the receiving end, respectively.

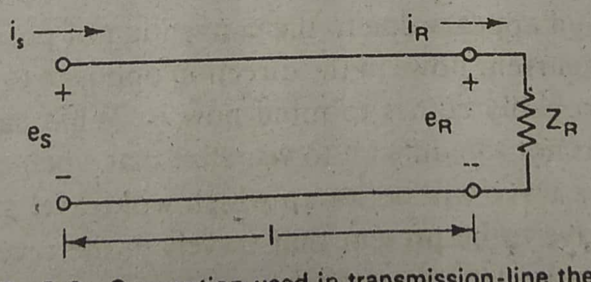



FIG. 2-2 Convention used in transmission-line theory.

Often, when a source is applied to such a line, two distinct waves are present, which are called the *incident wave* and the *reflected wave*. The incident wave propagates from the source to the receiving end, whereas the reflected wave propagates from the receiving end toward the sending end. This is illustrated in Fig. 2-3, where, by convention, a current traveling toward the generator is taken to be negative. Rigorous analysis, as well as experimental data, indicates that these traveling

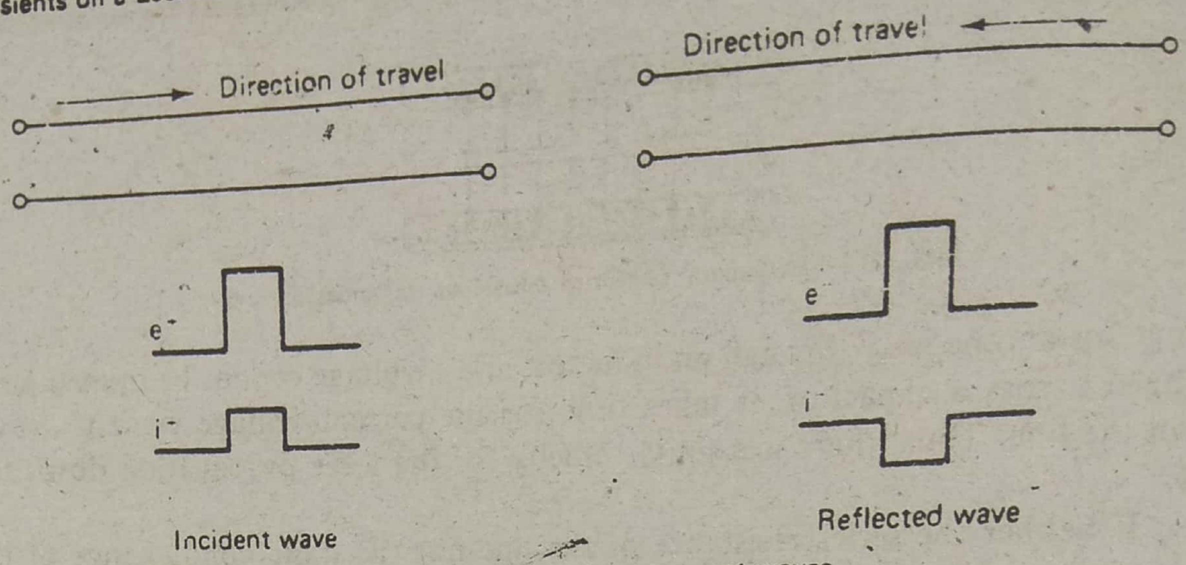



FIG. 2-3 Traveling voltage–current waves.

waves, whether incident or reflected, see what is called the *characteristic impedance* ($Z_0$) of the transmission line. The characteristic impedance of several types of transmission lines were given in Section 1-2. This impedance is independent of the line length or load and is a function of the line parameters only (i.e., size and spacing of conductors and type of insulation used).

We shall denote the incident voltage wave by $e^+$, which is a wave that travels from the source to the load. Similarly, the reflected voltage wave will be denoted by $e^-$. The incident and reflected current waves will be symbolized by $i^+$ and $i^-$, respectively.

Since a traveling wave sees the characteristic impedance of the transmission line, the incident current is related to the incident voltage by the relation

$$i^+ = \frac{e^+}{Z_0} \tag{2-1}$$

The reflected current can be expressed in terms of the reflected voltage as

$$i^- = -\frac{e^-}{Z_0} \tag{2-2}$$

where the negative sign appears due to the convention of positive current traveling toward the load (this current flows in the direction opposite to the incident current). The question that probably comes to mind now is: What causes a reflected wave to exist on a line? It is not too difficult to visualize that when a generator introduces a signal on the line, a wave will be set up which will travel away from the source; but what causes a wave to be present that travels in the reverse direction? To see this a little more clearly, let us consider a wave traveling toward the load. This wave, as it moves along the line, sees the characteristic impedance of the line. If the line is terminated with an impedance having a value equal to the characteristic impedance of the line ($Z_R = Z_0$), the wave will not notice any change as it reaches the termination, and the total voltage can be taken to be the incident voltage (the same holds for the current). In other words, no reflections need to occur, as Ohm's law is still validated. That is, when $Z_R = Z_0$,

$$\frac{e_R^+}{i_R^+} = Z_R = Z_0$$

The subscript $R$ is used here to denote currents, voltages, and so on, at the receiving end. *If, however, the load impedance is not equal to the characteristic impedance of the line, another wave must be set up to assure that Ohm's law is obeyed.*

The voltage at the load divided by the current through it must equal the load impedance. Since

$$\frac{e_R^+}{i_R^+} = Z_0 \neq Z_R$$

in this case, a reflection must occur.

At the termination we must have

$$\frac{\text{total } e}{\text{total } i} = Z_R \tag{2-3}$$

Unless $Z_R$ is equal to $Z_0$, the incident wave alone does not satisfy this relation and a reflected wave must occur. The total voltage and current at the load can be expressed as

$$\text{total } e = e_R^+ + e_R^- \tag{2-4}$$

$$\text{total } i = i_R^+ + i_R^- = \frac{e_R^+}{Z_0} - \frac{e_R^-}{Z_0} \tag{2-5}$$

Substituting equation (2-5) into equation (2-3), we obtain

$$\frac{\text{total } e}{\text{total } i} = Z_R = Z_0 \frac{e_R^+ + e_R^-}{e_R^+ - e_R^-} \tag{2-6}$$

From this equation, the relation between the reflected voltage and the incident voltage can be found:

$$\frac{e_R^-}{e_R^+} = \frac{Z_R - Z_0}{Z_R + Z_0} = \Gamma_R \tag{2-7}$$

The ratio $\Gamma_R$ is called the *voltage reflection coefficient*. If $Z_R = Z_0$, it is noted that $\Gamma_R$ goes to zero, and no reflected wave is present:

$$\Gamma_R = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

If $\Gamma_R$ is not equal to zero, a reflected wave does occur, having a value equal to

$$e_R^- = e_R^+ \Gamma_R$$

Similarly, it can be proven that the current reflection coefficient ($i_R^-/i_R^+$) is the negative of that for the voltage reflection coefficient. Some examples will now be considered to illustrate the concepts just outlined. *All the transmission lines are assumed to be lossless.*

## EXAMPLE 2-1

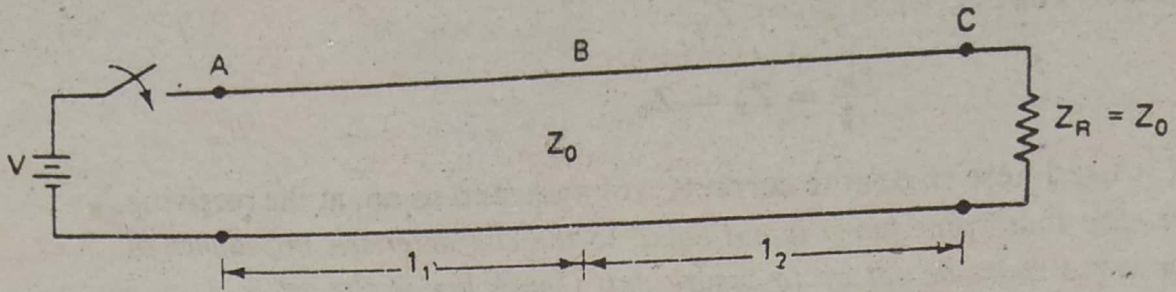



FIG. 2-4 Step wave applied to a matched transmission line.

A battery, having a voltage of magnitude $V$, is applied to a transmission line at time $t = 0$. Determine the waveforms that will be present at locations A, B, and C. Assume the velocity of propagation to be the velocity of light, $c$.

***Solution:*** When the switch closes, a voltage wave will commence to move down the line at a velocity $c$. It arrives at point B in $l_1/c$ seconds and at point C in $(l_1 + l_2)/c$ seconds. No reflection is present as the line is properly terminated (i.e., $Z_R = Z_0$).

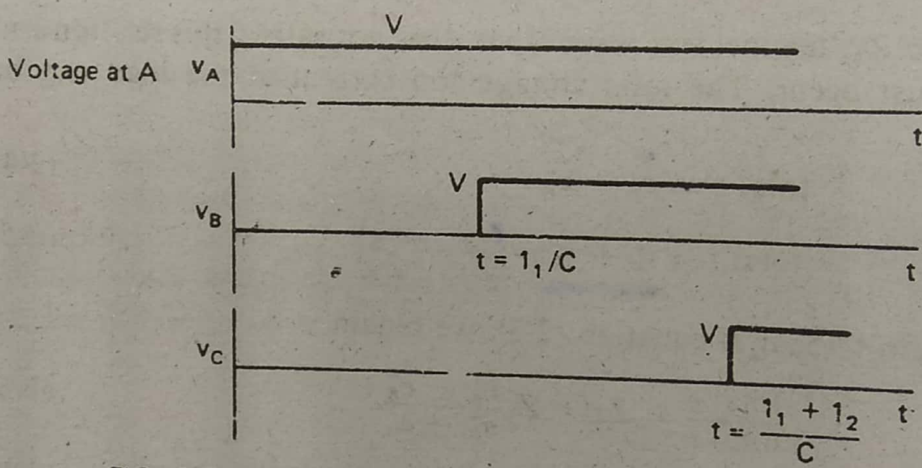



FIG. 2-5 Waveforms on the transmission line of Fig. 2-4.

## EXAMPLE 2-2

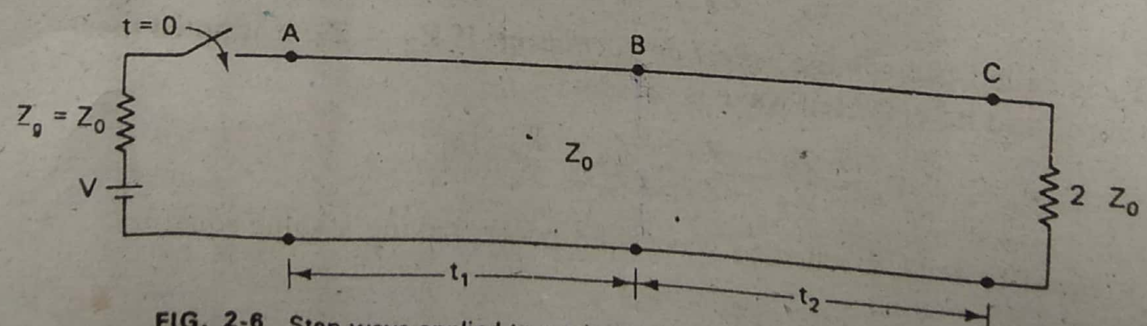



FIG. 2-6 Step wave applied to a mismatched transmission line.

A battery, having a voltage $V$ and an internal impedance of $Z_0$, is applied to a transmission line at time $t = 0$. Determine the waveforms that will be present at locations A, B, and C.

Assume $t_1$ to be the time taken for the wave to arrive at point B and $t_2$ to be the time taken to travel the distance from B to C.

***Solution:*** When the switch closes, a voltage wave commences moving down the transmission line, seeing an impedance $Z_0$. Since there will be a voltage drop across the generator impedance, the magnitude of the voltage moving down the line is not equal to the battery voltage. The equivalent circuit for the wave as it just enters the line consists of a generator with its internal impedance $Z_0$ and the characteristic impedance of the line $Z_0$.

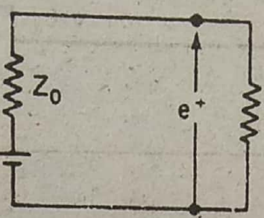



**FIG. 2-7** Equivalent input circuit at $t = 0$.

The incident voltage has a magnitude of

$$e^+ = V\frac{Z_0}{Z_0 + Z_0} = \frac{V}{2}$$

The reflection coefficient at the receiving end has a value of

$$\Gamma_R = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3}$$

The reflected voltage has a magnitude of

$$e^- = \Gamma_R e^+ = \frac{1}{3} \times \frac{V}{2} = \frac{V}{6}$$

The waveforms are shown in Fig. 2-8. The receiving end is matched, and there will be no further reflections.

## EXAMPLE 2-3

A battery is applied to a transmission line at time $t = 0$, as indicated in Fig. 2-9. Determine the waveforms at A, B, and C. Assume $t_1$ and $t_2$ to be the times it would take a wave to travel from point A to B and from point B to C, respectively.

***Solution:*** As explained in Example 2-2, a voltage of magnitude $V/2$ commences to move down the line upon closing the switch. When this wave reaches point B, it experiences a mismatch, since it effectively sees the shunting resistor ($R$) in parallel with the $Z_0$ of the continuing section of transmission line. The impedance at point B as seen by the wave is $Z_0/2$; and the reflection coefficient at this point, noted by $\Gamma_B$, is equal to

$$\Gamma_B = \frac{Z_0/2 - Z_0}{Z_0/2 + Z_0} = -\frac{1}{3}$$

The reflected wave from this point will then have a value of

$$e_B^- = \Gamma_B e_B^+ = -\frac{1}{3} \times \frac{V}{2} = -\frac{V}{6}$$

This wave will be reflected toward the generator.

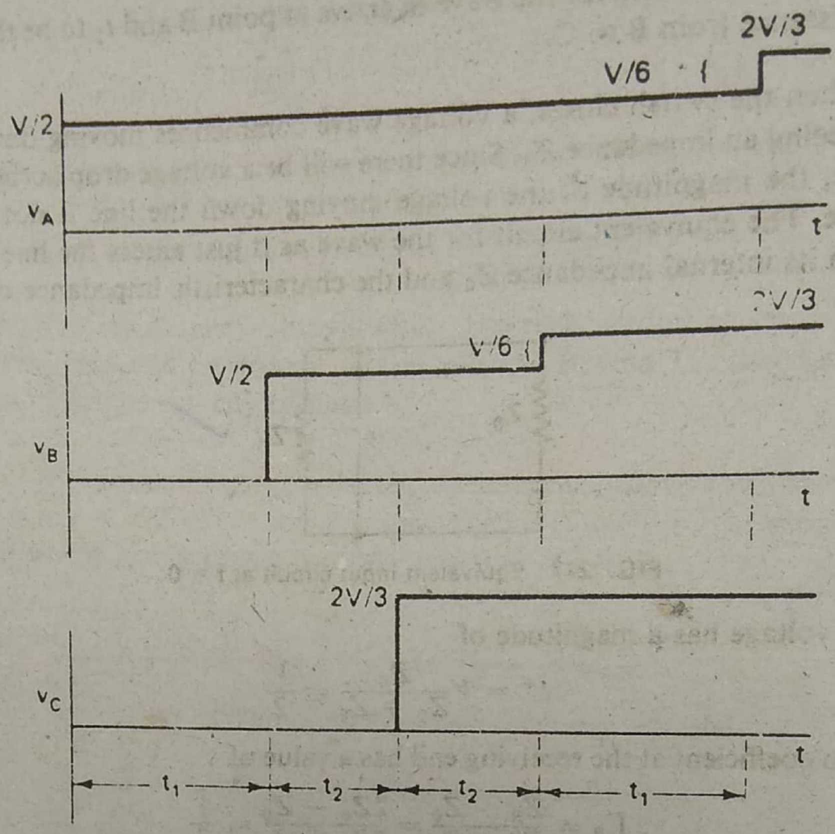



FIG. 2-8 Waveforms on the transmission line of Fig. 2-7.

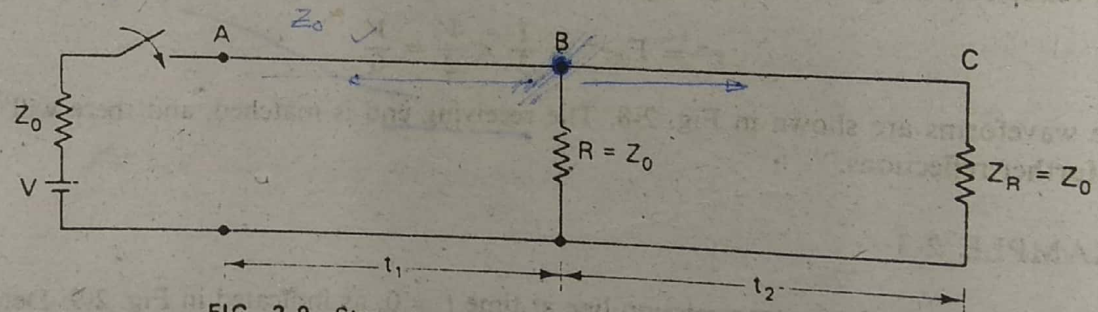



FIG. 2-9 Step wave applied to a shunted transmission line.

The total voltage remaining at point B, which will also begin moving toward the load is

$$e_B^+ + e_B^- = \frac{V}{2} - \frac{V}{6} = \frac{V}{3}$$

Both the receiving and generator ends are matched and there will be no further reflections. Figure 2-10 shows the resulting waveforms.

The final steady-state voltage or dc condition can be checked by assuming the transmission line to be merely a pair of connecting leads (Fig. 2-11). The dc voltage checks out to be

$$V_{dc} = V_A = V_B = V_C = V \times \frac{Z_0/2}{Z_0 + Z_0/2} = \frac{V}{3} \qquad \text{volts}$$

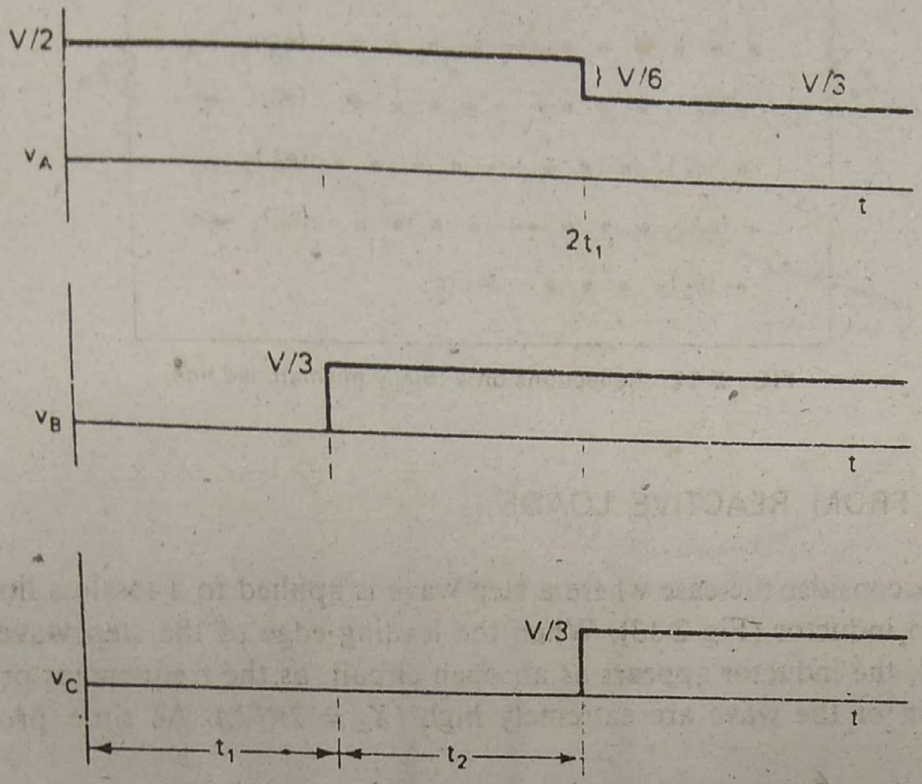



FIG. 2-10 Waveforms on the transmission line of Fig. 2-9.

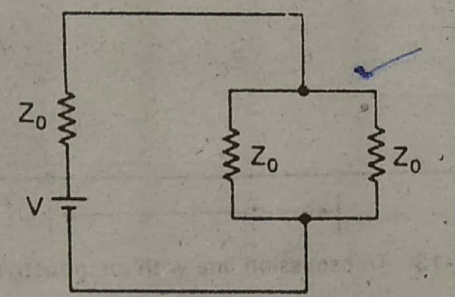



FIG. 2-11 Steady-state voltage on the transmission line of Fig. 2-9.

In either Example 2-2 or 2-3, if the generator impedance is also mismatched to the line, multiple reflections would occur. The reflected voltage at the generator end would have a value of

$$e_s^{-\prime} = \Gamma_s e_r^+$$

where $e_r^+$ is the voltage reflected back from the receiving end and $\Gamma_s$ is the reflection coefficient the generator would present to the line:

$$\Gamma_s = \frac{Z_g - Z_0}{Z_g + Z_0}$$

Figure 2-12 indicates the sequence of reflections that would occur on such a line, where the second subscript, after the parentheses, represents the number of reflections that have occurred at the respective end.

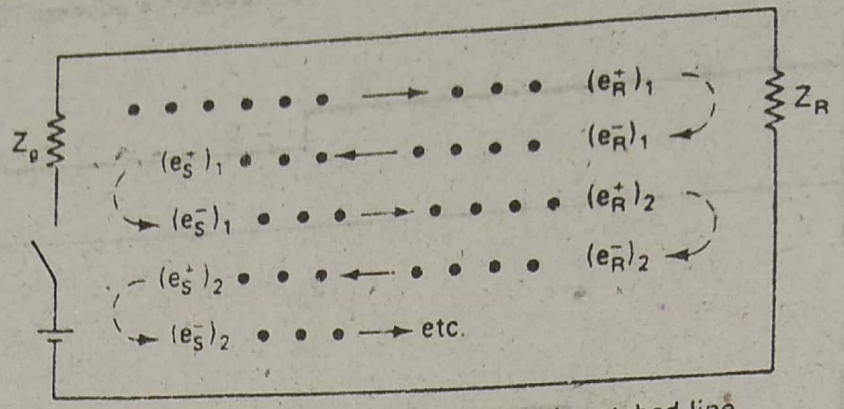



FIG. 2-12 Reflections on a totally mismatched line.

## 2-3 REFLECTIONS FROM REACTIVE LOADS

Let us now consider the case where a step wave is applied to a lossless line terminated in an inductor (Fig 2-13). When the leading edge of the step wave arrives at the load, the inductor appears as an open circuit, as the frequencies present in this portion of the wave are extremely high ($X_L = 2\pi f L$). As time progresses,

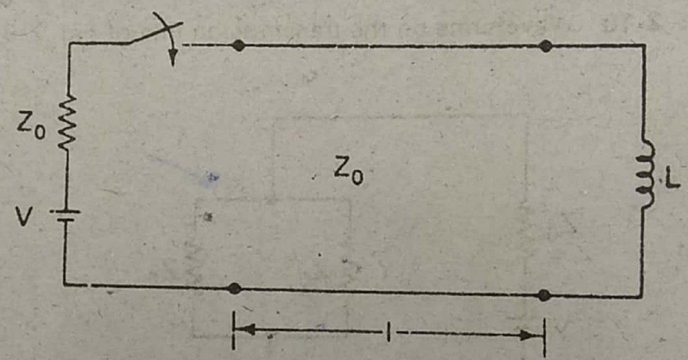



FIG. 2-13 Transmission line with an inductive load.

however, the step wave settles down to a dc voltage and the inductor then appears as a short. An oscilloscope sees a waveform at the sending end as that sketched in Fig. 2-14. The delay in Fig. 2-14 is due to the travel time of the wave up and down the transmission line.

If a capacitor forms the load, the opposite will occur. The high frequencies present at the leading edge of the wavefront see a very low reactance ($X_C = 1/2\pi f C$), whereas the later dc condition sees a very high impedance. Figure 2-15 shows the waveform that will be observed at the input of such a terminated line.

## 2-4 TIME-DOMAIN REFLECTOMETRY

Many test instruments are now being manufactured which employ a step voltage or a bell-shaped pulse to detect the location of a discontinuity in a transmission system. The Hewlett-Packard Model 1415A *time-domain reflectometer* (TDR), for

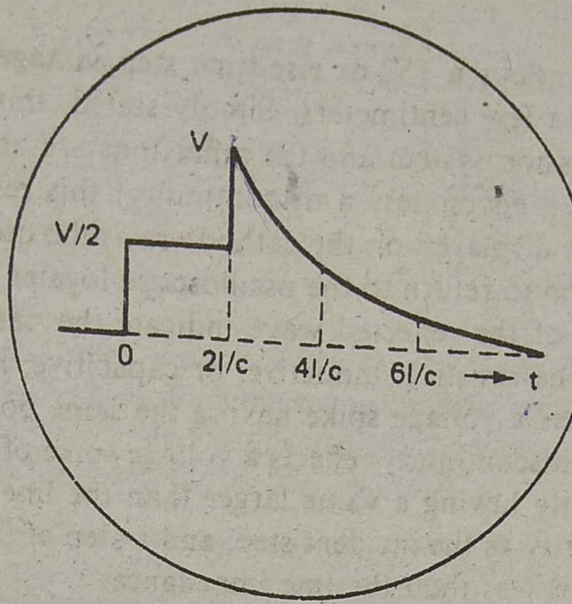
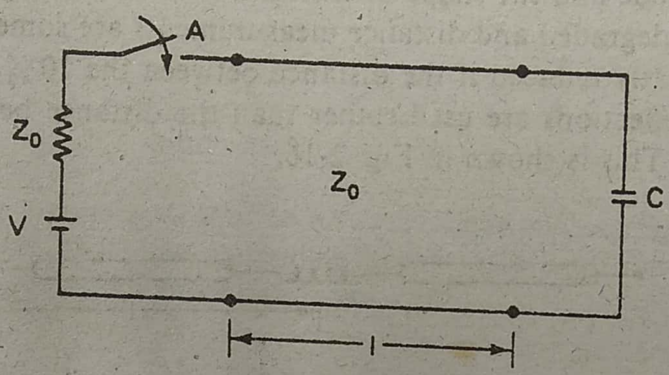
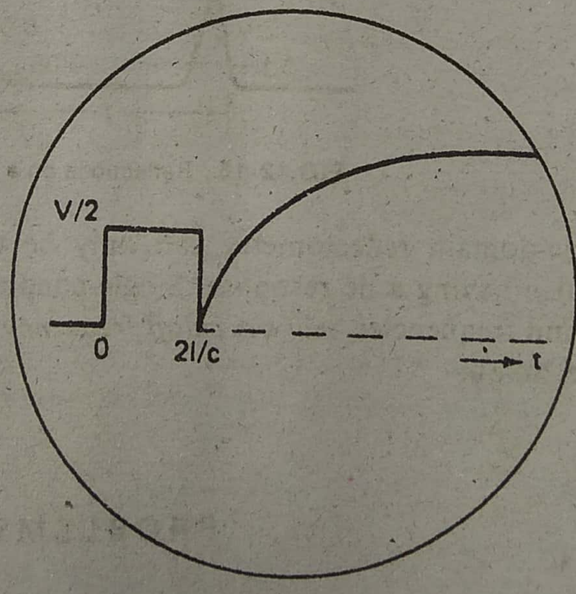



**FIG. 2-14** Waveform as seen at the input of an inductively terminated line.




(a)




(b)

**FIG. 2-15** (a) Transmission line with a capacitive load; (b) Waveform as seen at point A of Fig. 2-15(a).

instance, employs a 150-ps rise-time step voltage and can locate faults within a distance of a few centimeters. Simply stated, this TDR applies a voltage step to the transmission system and the reflections are observed. A reflection occurs each time the step encounters a discontinuity; this reflection is added to the incident wave and is displayed on the cathode-ray-tube oscilloscope. The time required for the reflection to return to the oscilloscope locates the discontinuity. The shape and magnitude of the reflected wave indicate the nature and value of the mismatch, which can be resistive, inductive, or capacitive. In general, an inductive discontinuity reflects a voltage spike having the same polarity as the incident step, and a capacitive discontinuity reflects a voltage spike of the opposite polarity. A resistive discontinuity having a value larger than the line impedance reflects a step of the same polarity as the incident step, and a step of the opposite polarity is reflected if the value is less than the line impedance.

When long lossy cables are tested with a time-domain reflectometer, both the amplitude and the shape of the reflections are changed. In general, the rise time is badly degraded and distance measurements are somewhat obscured. The error can be greatly reduced if the distance between the 10% points on the leading edge of two reflections are used rather than the distance between the peaks of the reflections. This is shown in Fig. 2-16.

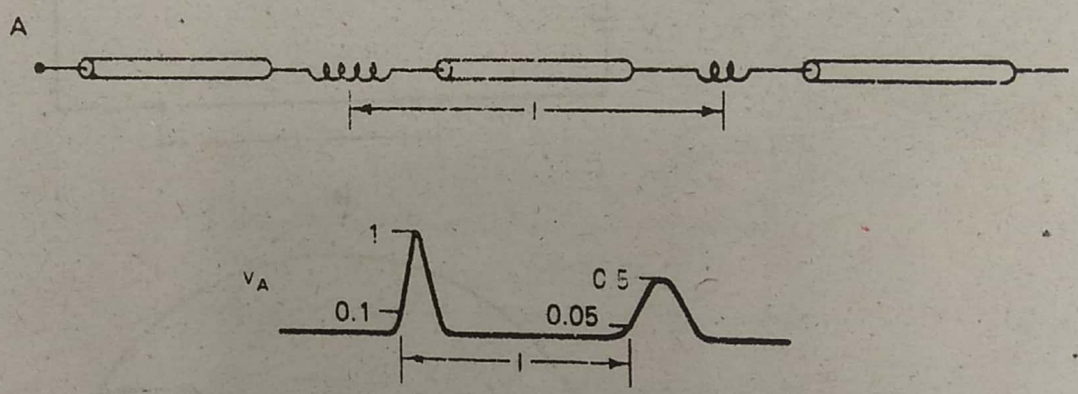



FIG. 2-16 Reflections on a lossy line.

Time-domain reflectometry can only be usefully employed in *broad-band systems* also having a dc response. Single-conductor waveguides, for instance, do not transmit frequencies below a *cutoff frequency* and therefore cannot be analyzed by this technique.

## PROBLEMS

**2-1.** Define the voltage reflection coefficient.

**2-2.** Originally the voltage is zero over the entire length of the transmission line. At $t = 0$ the switch closes. Find the voltage waveforms at A and B.

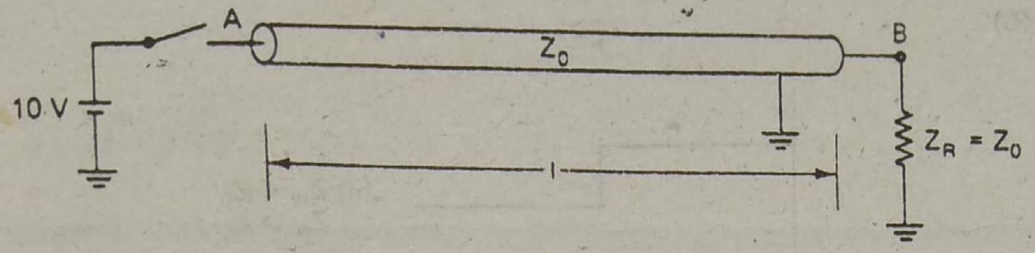



FIG. P2-2

2-3. (a) Originally, the voltage is zero over the entire length of the transmission line. At time $t = 0$ the switch closes. Find the voltage waveforms at A and B.

(b) If the load $Z_R$ is removed such that an open-circuit condition results at point B, what would the new voltage waveforms be at A and B?

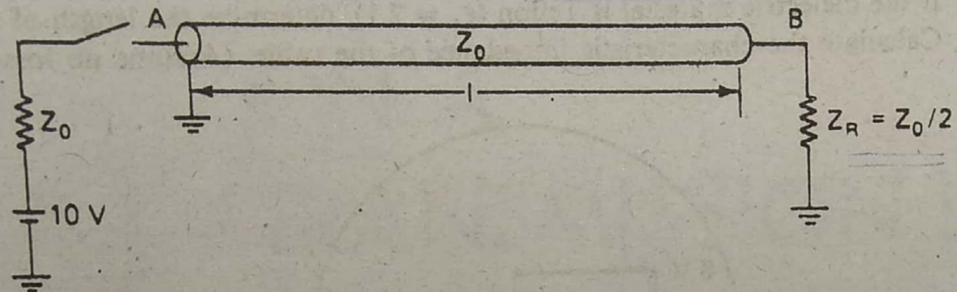



FIG. P2-3

2-4. The following voltage waveform is seen on an oscilloscope connected to the input of a length of faulted RG8A/U line. Determine the fault location (from the sending end) and the fault impedance. Ignore any losses.

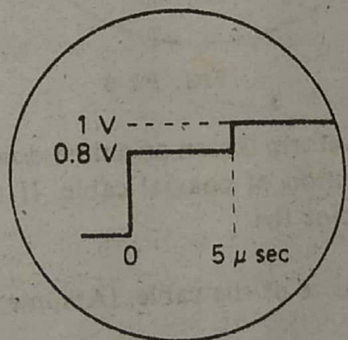



FIG. P2-4

2-5. The following waveforms are observed at an oscilloscope connected to the input of a cable loaded with an unknown impedance. Circle the correct answer.

(a)

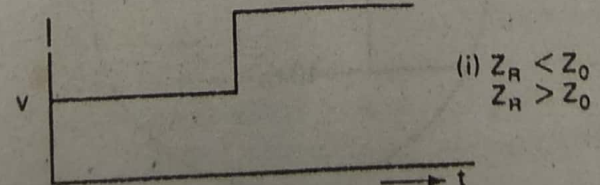



FIG. P2-5(a)

(b)

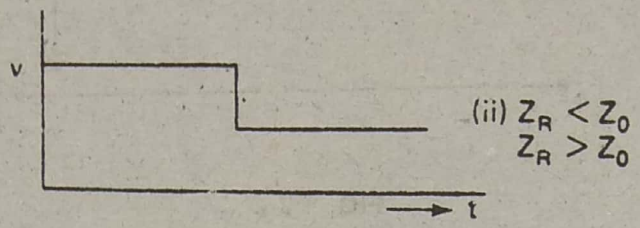



FIG. P2-5(b)

(c) What is the generator impedance relative to $Z_0$ of the cable for the above cases.

2-6. The following voltage waveform is seen on a time-domain reflectometer ($R_G = 50\ \Omega$) connected to a shorted section of coaxial cable.
(a) If the dielectric material is Teflon ($\epsilon_r = 2.1$), determine the length of cable.
(b) Calculate the characteristic impedance of the cable. (Assume no losses.)

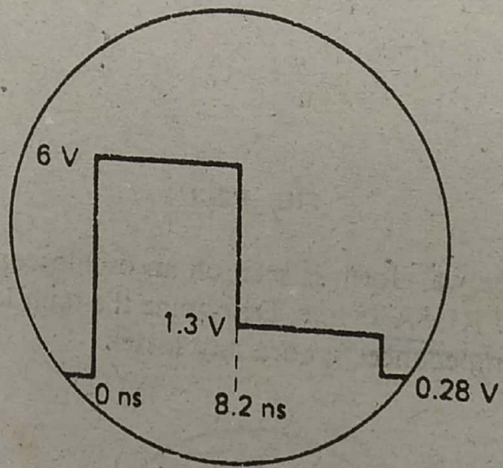



FIG. P2-6

2-7. The following voltage waveform is seen on a time-domain reflectometer ($R_G = 50\ \Omega$) connected to an open section of coaxial cable. If the dielectric material is polystyrene ($\epsilon_r = 2.5$), determine the:
(a) Length of the cable.
(b) Characteristic impedance of the cable. (Assume no losses.)

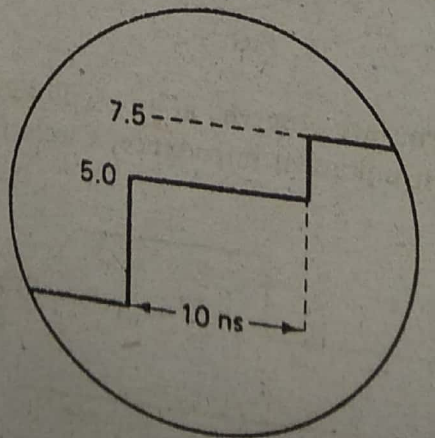



FIG. P2-7