Electrical signals propagating in a transmission line

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Electromagnetic waves are a well-known phenomenon. These waves transmit signals laden with information and are at the heart of modern telecommunication systems. Hence their understanding is pivotal for any science and engineering student. These waves are responsible for transmitting information over optical fibers, coaxial cables and waveguides as well as through space. A typical example of such a medium is furnished by a coaxial cable, such as the one shown in Fig. 1. The properties of the electromagnetic transmission on this cable are determined by the physical characteristics of the cable such as its capacitance, inductance (circuit parameters) and electric and magnetic susceptibilities (material properties). These characteristics also affect the propagation speed of the wave.

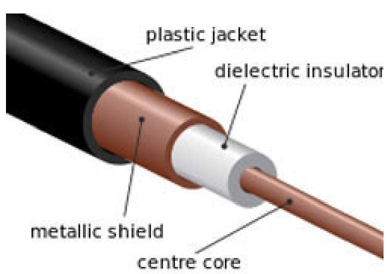

Figure 1: Cutaway of a general coaxial cable. The cable has two conductors separated by a dielectric. This construction imparts a characteristic impedance to the cable.

In the current experiment, we will observe and quantify the speed of propagating electromagnetic waves on the cable. Furthermore, the waves are also reflected whenever they encounter a change in the medium, such as when the cable makes contact with a dissimilar component. This reflection is indeed similar to how light bounces off as it hits the interface between two mediums with distinct refractive indices.

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What is the equivalent of "refractive index" for a transmission line? It is the "impedance". Every cable has a characteristic impedance Z_o which is a result of the material properties and its construction. The concept of impedance is straightforward. A voltage v at the end of the cable drives a current i through the cable and $Z_o = v/i$.

Additionally, The cable is meant to connect a signal source to a load; the basic scheme is shown in Figure 2. The signal source has an output impedance Z_S while the load has an impedance of value Z_L. The mismatch of impedances between the source and the cable and between the cable and the load results in reflection of signal. The transmitted and the reflected signals propagate along the length of the cable with some finite speed. If we probe the voltage at a particular point, shown as the point **A** in Figure 2, we will observe the resultant of the originally transmitted and reflected signals. These signals will be offset in time and the exact form can be predicted using a simple logical analysis involving the tracing of the progression of incident and reflected waves. This is precisely what we set out to do in this experiment!

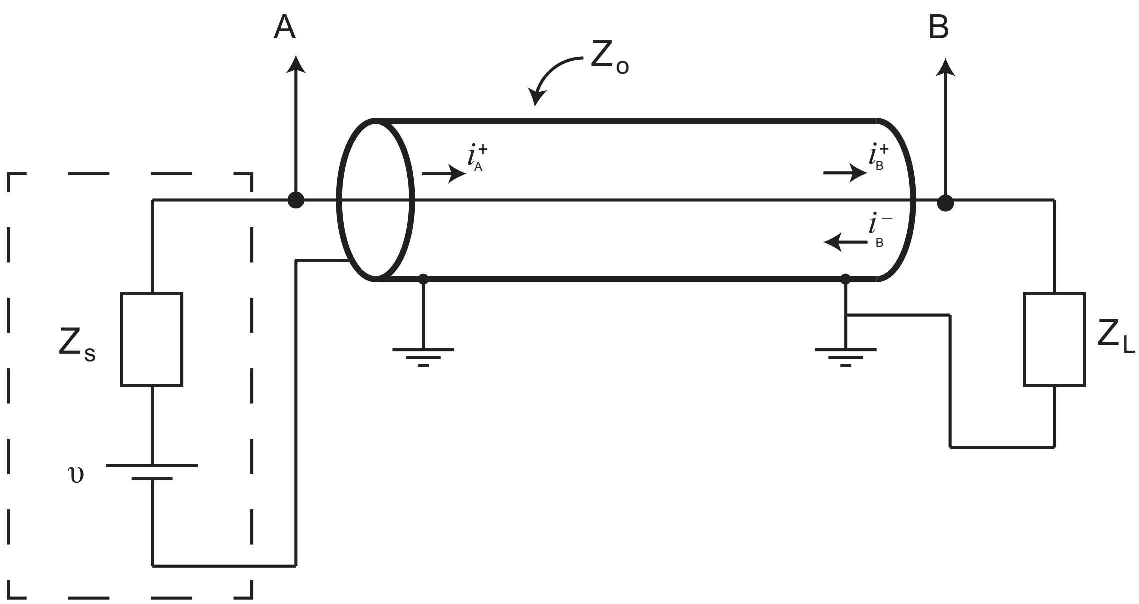

Figure 2: A diagrammatic overview of the experiment. The impedances Z_S, Z_o and Z_L are associated with, respectively the source, cable and the load. The point **A** shows a test point where we could place an oscilloscope test probe and observe the resultant voltage stemming from the transmitted and reflected waves. The point **B** is the interface between the cable and the load. The dashed box indicates the source, in our experiment this is a pulse generator.

The impedance mismatch has profound practical consequences and cannot be ignored. For example, for transmitting or receiving a signal in an RF communication task, the load is an antenna. It is desirable to let most of the RF power be transmitted to or from the antenna. This is possible if reflections are minimized and the impedances are perfectly balanced, which is a statement of the *maximum power transfer* theorem. In an NMR spetroscopic experiment, we are interested in picking up minute signals from a coil placed close to our sample. The coil's impedance must be precisely matched with the cabling. The matching also critically depends on the frequency. Any serious experimenter working with electromagnetic transmission or reception of signals, be it in the optical, radio or microwave realms cannot therefore remain oblivious of the nature of wave propagation in transmission lines. Before approaching the lab work, read the background material and attempt the questions that follow.

KEYWORDS

Time domain reflectrometry · characteristic impedance · impedance matching · waveguides · Ohm's law

APPROXIMATE PERFORMANCE TIME 1 **Week**

References

[1] William Sinnema, *"Electronic Transmission Technology: lines, waves and antennas"*, Prentice-Hall Inc, New Jersey, pp. 20–32, (1979).

1 Learning Objectives

In this experiment, we will,

1. briefly review the mathematical formulation for time domain reflectrometry,
2. understand how the shape of the reflected wave indicates the nature of the load,
3. learn about the use of a cathode ray oscilloscope,
4. determine the speed of the signal propagating in the coaxial cable, and
5. learn how to compare theoretical predictions with experimental observations.

2 Theoretical background

We explain the basics of the experiment while referring to Figure 2. When a source of voltage v and internal impedancce Z_S is connected to a cable of impedance Z_o that is terminated by a load of impedance Z_L, a voltage magnitude $(Z_o/(Z_o + Z_S))v$ appears at point **A**. Suppose this voltage is denoted as v_A^+ and drives a current i_A^+ going into the cable. The subscript represents the location **A** while the superscript + denotes a signal travelling from source to load. The incident voltage is related to the incident current by Ohm's law (1),

$$i_A^+ = \frac{v_A^+}{Z_o}. \tag{1}$$

This voltage and current (that can be tied together and called a 'signal') travel down the coaxial cable with a finite speed and encounter the load impedance at point **B**. If $Z_L = Z_o$, the signal does not experience any change in medium, hence the current will continue into the load unimpeded and there will be no reflection. However, if $Z_L \neq Z_o$, the signal is reflected. Let the reflected current at this point be i_B^- which is driven by a reflected voltage v_B^-. The relationship is once again governed by Ohm's law,

$$i_B^- = -\frac{v_B^-}{Z_o}. \tag{2}$$

The impedance that the back-reflected wave going *into* the cable sees is still Z_o. The minus sign in the superscript shows the signal traveling from load to source. Furthermore, the overall minus sign in front in Eq. 2 shows a back-propagating signal. This signal will travel along the length of the cable and eventually at **A**, a sum of v_A^+ and v_A^- will be seen. In this experiment you will observe the resultant signal at **A**.

At point **B** we must also satisfy Ohm's law, where the voltage and current is a composite of right and left-propagating waves,

$$Z_L = \frac{\text{total } v}{\text{total } i} = \frac{v_B^+ + v_B^-}{i_B^+ + i_B^-}. \tag{3}$$

For a lossless line, signals travel along the cable without any change in the voltage or current, implying $v_B^+ = v_A^+$, $v_B^- = v_A^-$ and similarly for the currents. This allows us to write the preceding equation as,

$$Z_L = \frac{v_A^+ + v_A^-}{i_A^+ + i_A^-}. \tag{4}$$

Now substituting the expressions from Eqs. (1) and (2) into (4), we obtain,

$$\frac{Z_L}{Z_o} = \frac{v_A^+ + v_A^-}{v_A^+ - v_A^-}. \tag{5}$$

After manipulation, the incident and reflected voltages at point **B** can be seen to be related:

$$\frac{v_A^-}{v_A^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = \Gamma. \tag{6}$$

This ratio captures how the reflected voltage depends on the *mismatch* of the cable's and load impedances. It is also called the voltage reflection coefficient. Now answer the following questions before you move on to the experiment.

Q 1. Find the current reflection coefficient.

Q 2. A 1 V square pulse of width 10 ns is sent through a 1 metre long wire with a characteristic impedance of 75 Ω. The source impedance is 50 Ω and there is a mismatched terminal load of 50 Ω. Predict the shape and the amplitude of the reflected wave in relation to the incident wave.

Q 3. Solve the above question for a load of 200 Ω. What happens if the pulse width is increased to 20 ns?

3 The Experiment

3.1 The Equipment

1. 25 m long coaxial cable
2. Pulse generator 10 MHz **(TGP110 Aim-TTi).**
3. Digital Storage Oscilloscope **(BK Precision 2534 60 MHz).**
4. Variable resistors
5. Fixed resistive loads
6. 1 m long BNC cable
7. BNC T connector
8. Male-to-Female adapters.

3.2 Preparation

- Set the Pulse generator to `Run` and `Pulse` modes. Set the period at 1 ms and the pulse width at 50 ns.
- To begin, connect the pulse generator directly to the oscilloscope using the 1 m BNC cable and analyze the displayed signal on the digital oscilloscope. What are the `Single` and `Run` modes on the oscilloscope? What happens when you change the `Trigger` level? Explain the shape in the pulse displayed on the oscilloscope and why the pulse is not a perfect square.

3.3 Determining the characteristic impedance and measuring the speed of the reflected signal

Connect the variable resistor to one end of the long coaxial cable. The second end is connected to a BNC Tee which is affixed to the pulse generator. The third port on the tee makes connection with the oscilloscope using the shorter BNC cable. Use a screw driver to change the resistance. Use the aforementioned theory to determine the setting of the variable resistor where its resistance equals the characteristic impedance. Furthermore, record and analyze the shape of the reflected waves at different fixed loads, including an open (infinite) load and loads smaller than the characteristic impedance of the wire. Comment on the polarity and amplitudes of the reflected signals for these resistive loads. You can also save gather the oscilloscope date into the computer using the `Comsoft` application.

Attach a fixed load of 200 Ω at the long cable's termination. With the oscilloscope's time base set around 50 ns/div, use your observations to calculate the speed of the electric signal as it propagates on the cable. What if your load has a capacitive or inductive component? Explore.