

Flux and rate of change of flux with a Hall sensor and solenoids

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Why does a magnet rotate a current carrying loop placed close to it? Why does the secondary winding of a transformer carry a current even when it is not connected to a voltage source? How does a bicycle dynamo work? How does the Mangla Power House generate electricity? Let's perform a simple experiment to investigate some phenomena which can point us towards answers.

KEYWORDS

Faraday's Law · Magnetic Field · Magnetic Flux · Induced EMF · Magnetic Dipole Moment · Hall Sensor · Solenoid

1 Conceptual Objectives

In this experiment, we will,

1. understand one of the fundamental laws of electromagnetism,
2. understand the meaning of magnetic fields, flux, solenoids, magnets and electromagnetic induction,
3. learn how to measure magnetic flux and induced e.m.f.,
4. interpret the physical meaning of differentiation and integration,
5. verify Faraday's law of electromagnetic induction.

2 Experimental Objectives

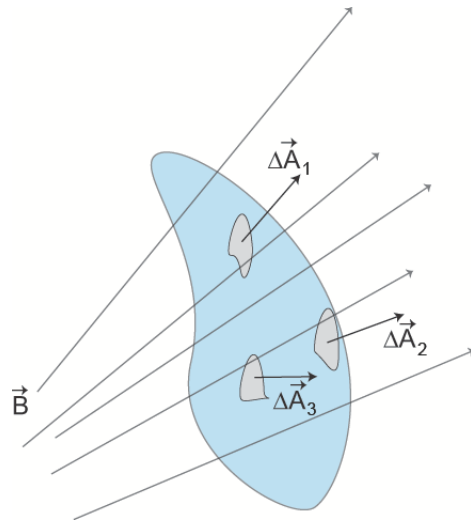
The experimental objective is to use a Hall sensor and a solenoid, to find the magnetic field, magnetization and magnetic flux from static and moving magnets. We will also gain practical knowledge of,

1. magnetic field transducers,
2. visually and analytically determining the relationship between induced emf and magnetic flux,
3. learn to use a data acquisition device (*e.g. PhysLogger*), and
4. dependence of emf on speed of flux change, hence a direct verification of Faraday's law.

3 The Magnetic Field \vec{B} and Flux Φ

Around 1821, Oersted found that a current carrying conductor produces a magnetic field. In the twentieth century, scientists determined the configuration of elementary particles in atoms and they realized that electrons inside atoms also produce tiny magnetic fields. This intrinsic magnetic field is found in all materials. The magnetic field is mapped out by the concept of *magnetic field lines*.

Magnetic field lines are like stretched rubber bands, closely packed near the poles. This is why the closer we get to the poles of a magnet, the higher the magnetic field. The number of magnetic field lines passing through an area is known as the *magnetic flux* ϕ .



Consider the arbitrarily shaped surface drawn above immersed inside a magnetic field \vec{B} . We want to find the flux through identical area elements $\Delta\vec{A}$; perpendicular and away from the

surface. A scalar product between the magnetic field vector \vec{B} and each $\Delta\vec{A}$ is formed and composed to give,

$$\phi = \vec{B}_1 \cdot \Delta\vec{A}_1 + \vec{B}_2 \cdot \Delta\vec{A}_2 \dots, \quad (1)$$

which can also be written as ,

$$\phi = \sum_i \vec{B}_i \cdot \Delta\vec{A}_i \quad (2)$$

In the limit of infinitesimal area elements, the flux is

$$\phi = \int \vec{B} \cdot \Delta\vec{A} \, dx \quad (3)$$

4 Electromagnetic Induction

Extensive work was done on current carrying conductors in the nineteenth century, the major ground work was set by Faraday (1831) and following him, Lenz (1834) [1]. Faraday discovered that a changing magnetic field across a conductor generates an electric field. When a charge moves around a closed circuit this electric field does work on the charge. Like the electromotive force (EMF) of a battery this *induced EMF* is capable of driving a current around the circuit.

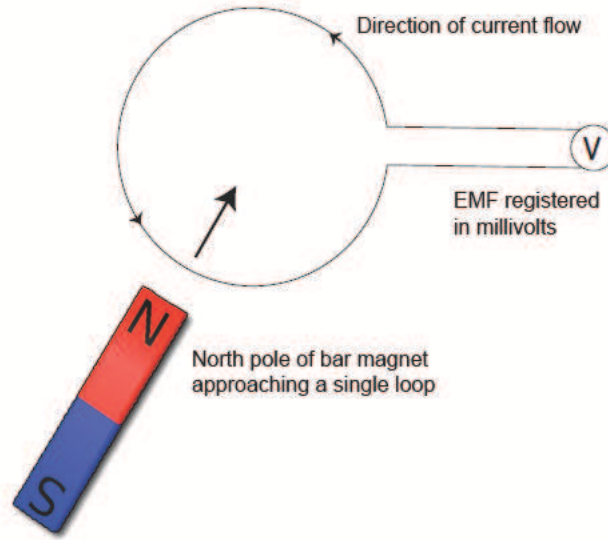


Figure 1: Principle of electromagnetic induction. As a dipole (such as a magnet) is pulled towards a loop of wire, the flux through the loop changes inducing an EMF and current through the loop.

Faraday's law asserts that the EMF produced is directly proportional to the rate at which the magnetic field lines per unit area or magnetic flux 'cuts' the conducting loop. Lenz's law is

incorporated into Faraday's Law through a negative sign indicating that the EMF produced opposes the relative motion between the conductor and magnet, it tries to resist the change in flux.

Mathematically both of these laws are expressed together as,

$$\varepsilon = -\frac{d\phi}{dt}, \quad (4)$$

for a single loop of conductor, where ε is the electromotive force induced, ϕ is the magnetic flux. $d\phi/dt$ is time rate of change of magnetic flux. The rate depends on the speed at which the magnet moves relative to the conductor loop, as well as the strength of the field.

Electric power plants or more commonly; generators, are a physical manifestation of the laws of induction. The principle is to change the magnetic flux over large stationary coils. The 'change' of flux is brought about mechanically, for example by falling water or by running a turbine. The changing flux induces an EMF in the coils.

Q 1. What are the units of ε and ϕ ?

Q 2. Rewrite Equation 4 for N number of loops. How does the EMF depend on N ?

5 Solenoids

It is a coil of wire wound around a core. Magnetically it behaves like a bar magnet, producing a magnetic field when it carries current. It remains a magnet till the time current is flowing through the conductor.

The mathematical expression for magnetic field generated inside an *ideal solenoid* is,

$$B = \mu_0 n I, \quad (5)$$

where μ_0 is the permeability in free space, value; $4\pi \times 10^{-7} \text{ Wbm}^{-1}\text{A}^{-1}$ outside, n is the number of turns of the conductor per unit length and I is the current through the conductor. The magnetic field B is measured in Tesla (T) or Gauss (G), where 1 G equals 10^{-4} T. In our experiment we will use a changing magnetic field near a solenoid to induce an EMF in it. This is the Faraday effect!

6 The Hall effect

Imagine a sea. There is a sea of electrons in a conductor. When we apply a potential this 'sea' flows from the higher to the lower potential. Further, if we place this conductor, in which current is flowing in a magnetic field the moving charges tend to interact with the

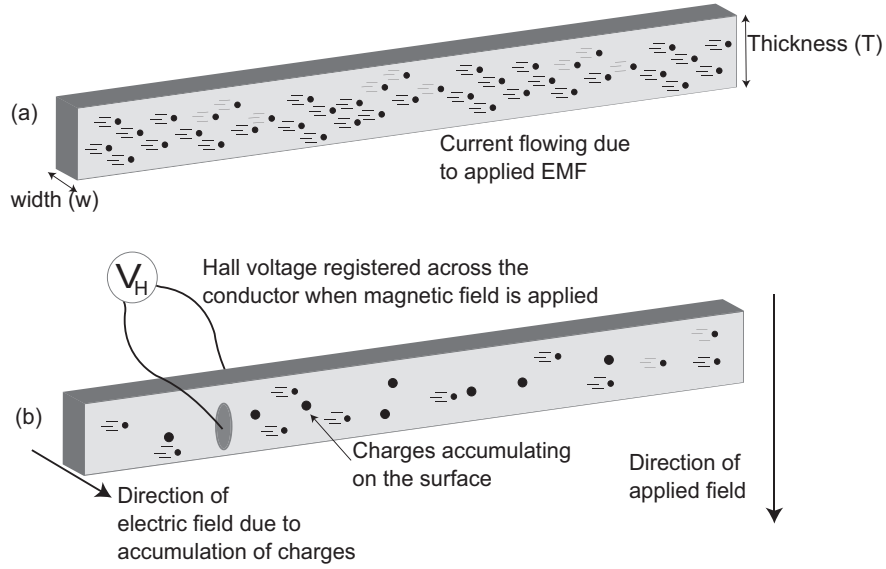


Figure 2: (a) Shows electrons flowing through the conductor. (b) Shows some charges accumulating on the front and back surfaces generating Hall voltage across the width w .

applied magnetic field and also deflect. This deflection results in a potential difference across or perpendicular to the conduction path, known as the *Hall voltage*.

Figure 2 illustrates how moving charges are deflected due to the applied magnetic field. The magnitude of this force (F_B) is given by,

$$F_B = Bqv, \quad (6)$$

where q is the charge and v is the velocity. The build-up of charges on one side generates an electric field (E_{\perp}) perpendicular to the current as shown in Figure 2. These charges continue to accumulate till the time force (F_E) due to electric field,

$$F_E = qE_{\perp}, \quad (7)$$

is equal to the force due to the magnetic field (F_B). Mathematically this equilibrium means that,

$$F_E = F_B, \quad (8)$$

or

$$Bqv = qE_{\perp}. \quad (9)$$

The voltage developed due to E_{\perp} is,

$$V_H = E_{\perp}w, \quad (10)$$

where (V_H) is the *Hall voltage* and w is the width of the conductor. Combining Equation 9 and Equation 10 we get,

$$V_H = vwB. \quad (11)$$

We know that the average velocity of electrons in terms of current (I) is given by,

$$v = \frac{I}{neA}, \quad (12)$$

where n is the volume density of electrons and A is the cross-sectional area, a product of width (w) and thickness (T).

Combining Equation 12 and Equation 11 we obtain the Hall voltage in terms of applied magnetic field,

$$V_H = \frac{BI}{neT}. \quad (13)$$

The Hall effect is important in the study of materials, for example it helps us to find the number of conducting particles in a wire and their charge. In our experiment, this effect holds a central importance as we will use sensors developed using this principle to probe the magnetic fields generated by magnets. Read heads in tape recorders and magnetic disk drives utilize this principle too.

Q 3. A strip of copper $150 \mu\text{m}$ thick is placed inside a magnetic field $B = 0.65 \text{ T}$ perpendicular to the plane of the strip, and a current $I = 23 \text{ A}$ is setup in the strip. What Hall potential difference would appear across the width of the strip if there were 8.49×10^{28} electrons/ m^3 ?

6.1 Interrelationship between the operation of the solenoid and the Hall probe

The Hall probe and the solenoid are both transducers, they convert one form of energy to another. Figure 3 shows that Hall probes generate a measurable potential which varies with the direction and magnitude of the field and flux. This electric potential is then converted to magnetic field using the simple relation provided by the manufacturer of the Hall sensor.

On the other hand, a solenoid, directly measures the EMF which depends on the *rate of change* of flux and also depends on the number of turns of the solenoid.

7 The Experiment

7.1 Understanding the Hall sensor

Hall sensors are used to measure magnetic fields. In our case, we've provided you with a sensor mounted on the tip of a probe. The output voltage of a Hall sensor V_H is proportional to the magnetic field B being measured. The measured voltage is then converted to magnetic field using a calibration scheme provided by the manufacturer of the Hall sensing chip. The

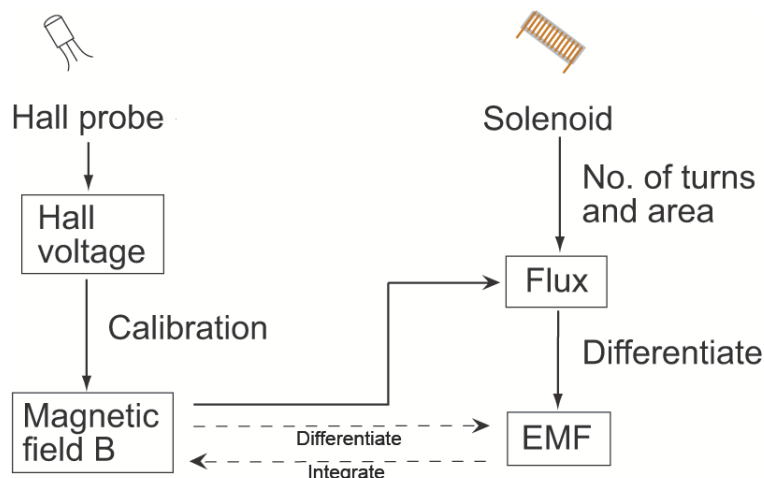


Figure 3: An inter-relationship between the operation of the solenoid and Hall probe.

relation between the hall voltage V_H and the magnetic field for our particular sensor is,

$$B = 320V_H - 800 \quad (14)$$

where V_H is in volts and B is in Gauss.

The voltage $V_{cc} = 5V$ is provided to the Hall sensor using PhysLogger data acquisition device. The PhysLogger has a 5 V regulated output port which is used as power supply to the Hall sensor. Complete pin connections are shown in the Appendix.

7.2 Magnetic field of a disk magnet

In the first step, we will map the field of a disk magnet using the Hall probe. You are provided with small disk magnets.

Q 4. Keeping the hall probe away from the magnet, run the PhysLogger software, go to **Measure > Voltage > Channel D**. Ensure that the Hall probe is connected to the logger device. Note that the software will display 2.5 V as you connect the Hall probe. Using Equation (14), determine what magnetic field does this voltage correspond to.

Q 5. Place a disk magnet on to the provided wooden ruler with the circular groove and vary the distance between the disk magnet and the flat face of the Hall probe. Make sure that the flat face of the probe is perpendicular to the magnetic axis of the disk magnet as illustrated in Figure 4(b). Vary the distance from 0 to 3.5 cm in steps of, say, 0.5 cm.

Q 6. Record the output voltage displayed on the PhysLogger software as you move the Hall probe along the magnetic axis, and make a table as suggested in Table 1.

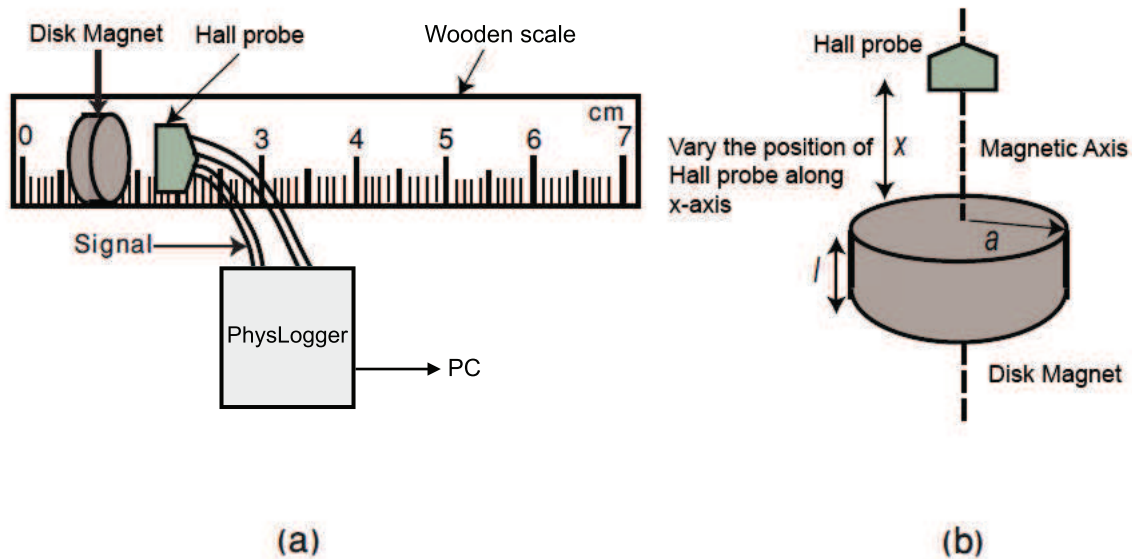


Figure 4: Schematic diagrams. (a) Hall probe voltage measurement, (b) disk magnet field mapping using a Hall probe.

Q 7. Find the corresponding magnetic field strength using the calibration Equation (14).

Distance (mm) x	Output Voltage (V)	Measured magnetic field $B_{measured}$ (G)	$B_{measured}$ (T)	$f(x)$ (T m/A)
0				
2				
.				
.				

Table 1: Mapping the field of a disk magnet. For voltage to field conversion use calibration Eq. (14). For $f(x)$, see section 7.3. Note that $1 \text{ G} = 10^{-4} \text{ T}$.

Q 8. Find the uncertainty in the measured magnetic field $B_{measured}$.

Q 9. Plot a graph between magnetic field strength $B_{measured}$ and distance x . Interpret and describe the plot.

7.3 Magnetization of a disk magnet

In this part of the experiment, we will use the obtained data to estimate the magnet's magnetization. Magnetic materials are made up of atoms which have *magnetic dipole moment*

$\vec{\mu}_B$. These randomly aligned dipoles have a resultant magnetic dipole $\vec{\mu}$,

$$\vec{\mu} = \sum \vec{\mu}_B. \quad (15)$$

We define magnetization \vec{M} as

$$\vec{M} = \frac{\sum \vec{\mu}_B}{V} \quad (16)$$

where V is the volume of the material.

For a disk magnet the expression for the magnetic field strength as a function of distance is approximately,

$$B(x) \approx \frac{\mu_0 M}{2} \left(\frac{x + l/2}{\sqrt{(x + l/2)^2 + a^2}} - \frac{x - l/2}{\sqrt{(x - l/2)^2 + a^2}} \right), \quad (17)$$

where M is the magnetization of the disk magnet, x is the distance along its magnetic axis, l is its thickness and a is its radius [4].

The term in brackets needs some mathematical detail in which we will not delve. However, it is important to tell that it is a geometrical term which is the result of an integral depending upon the dimensions of the magnet and solved over the distance at which we are measuring the field.

For the sake of simplicity lets replace

$$\frac{\mu_0}{2} \left(\frac{x + l/2}{\sqrt{(x + l/2)^2 + a^2}} - \frac{x - l/2}{\sqrt{(x - l/2)^2 + a^2}} \right) \quad (18)$$

with the geometrical function $f(x)$, obtaining

$$B(x) = Mf(x). \quad (19)$$

The goal is to find the magnetization of the disk magnet using Equation (19).

Q 10. For this purpose, perform the following procedure.

1. Find the thickness of the disk magnet with vernier callipers.
2. Find its radius.
3. Run Matlab.
4. Construct a vector showing the values of x at which the magnetic field is measured. Suppose the vector is labeled x . Type `>> magneticfield(x);`
5. The program prompts to enter the radius and thickness in mm.

6. The Matlab code returns the value of $f(x)$ given by the expression (17). Tabulate these values in your notebook. See last column of Table 1.

Q 11. Make a plot between $f(x)$ and $B_{measured}$ and find the magnetization. Use weighted fitting technique. You could assume zero uncertainty in x and $f(x)$. The typical magnetization of iron is in the range $\approx 10^6$ A/m [5].

7.4 Verification of Faraday's law of electromagnetic induction

In the second part, we will compare signals derived from the Hall sensor and the set of solenoids as disk magnets negotiate to and from beneath move in their vicinity. The experimental setup consists of a rotating disk and magnets that are placed at fixed intervals on top of it as shown in Figure 5. We consecutively use a Hall sensor and the provided set of three solenoids to compare the signals and observe the inter-dependence of speed of rotation, stacking magnets on top of the another etc. The provided apparatus comes with solenoids of 1100, 700 and 400 turns. EMF signals from these coils are routed to channel A, B and C, respectively, of the PhysLogger's physical channels; while the Hall sensor is connected to channel D. The connection diagram which has hardwired in the provided cable, is also shown in the Appendix.

7.5 Data acquisition using PhysLogger

In this experiment, data acquisition is done using PhysLogger, a low-cost home built data acquisition device that consists of 4 analog input channels that can be used to acquire differential voltage signals with sampling frequencies up to 20 KHz. The device is connected in parallel to the Hall probe and the set of solenoids. (See the appendix) This allows experimenter to consecutively observe the magnetic field strength from Hall sensor and the rate of change of flux generated from the solenoids. The logger is connected to the computer using the provided USB cable and the PhysLogger Desktop Application is used to acquire signals from the device, process and save them in the form of meaningful data for further analysis. See [6] for more details on PhysLogger.

8 Observing the induced EMF flux using the solenoid and the Hall sensor

Carry out the following procedure.

1. Make sure the solenoids are aligned with the magnetic field of the magnets (while you

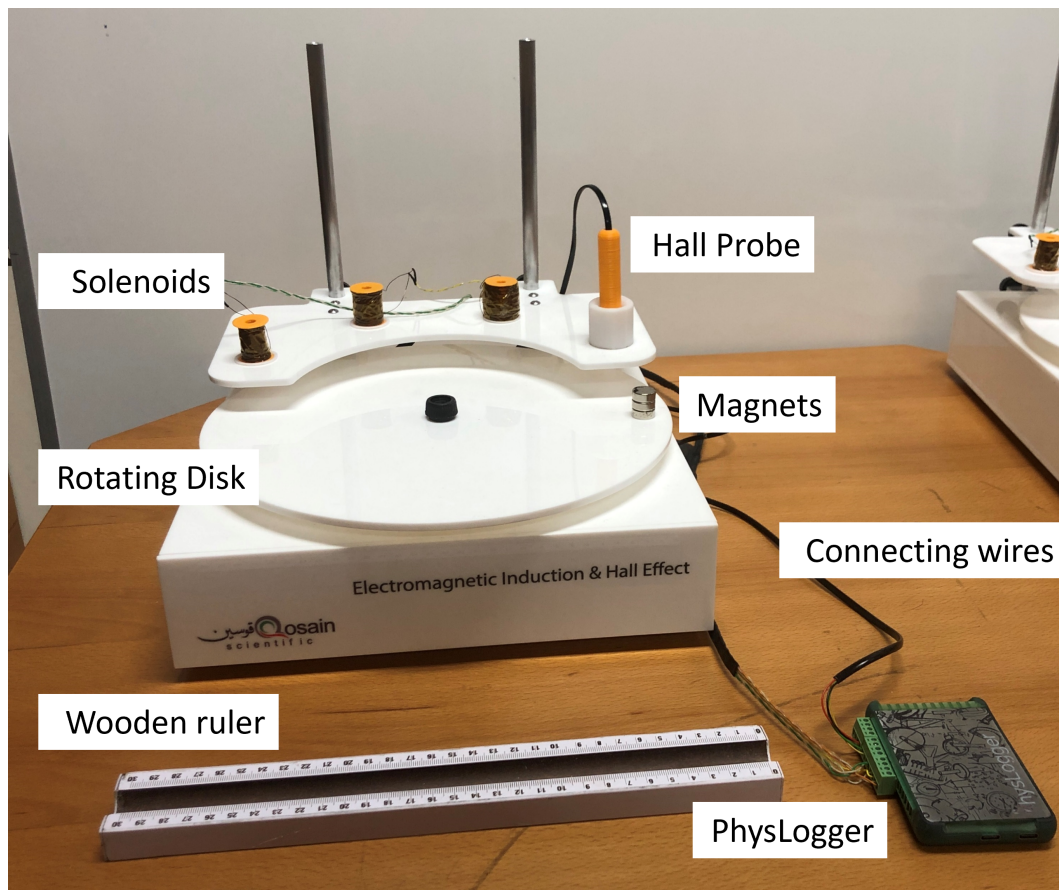


Figure 5: The experimental setup comparing a rotating disk and an assembly of solenoids and Hall probe.

pile magnets on the disk, place a magnet beneath the disk as well to prevent them from flying off while you rotate the disk.)

2. Connect the apparatus to PhysLogger using the provided connecting wires (as shown in appendix) and connect the logger device to computer using the USB cable.
3. Open PhysLogger Desktop App > Explore > Electromagnetic Induction Experiment Template.
4. Rotate the disk manually using the central knob and observe the four plots.

Now attempt to answer the following questions:

5. Why do you observe the particular shape of the EMF picked up from the coils? The signal goes up, down and then increases in the opposite direction. Explain the shape of this signal. What is the impact of reversing the direction of the disc magnets, the number of the magnets stacked on top of each other and how does the sharpness and height of the signal change as the rotating disk decelerates?

6. How do you compare the strength of the signals from the three solenoids? Are they in the ratio of 11:7:3? Why yes and why not?
7. Observe the signals from the Hall sensor and compare with the signal from the solenoids.

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A Appendix

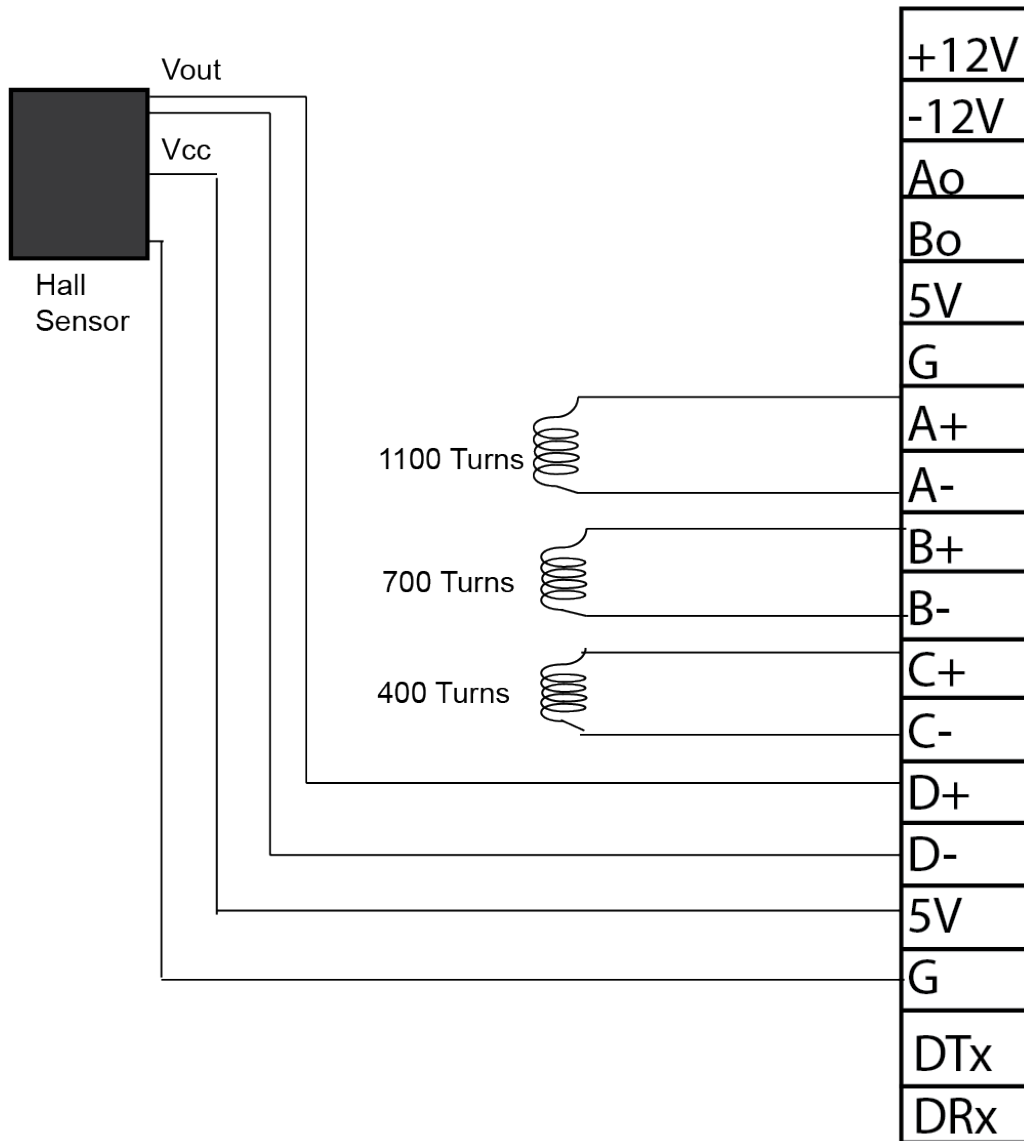


Figure 6: The Hall sensor circuit and pin diagram showing connections between the Hall cell and the Physlogger. The 5 V, GND, D+, and D- are connected to the VCC, GND and VOUT channels of the Hall sensor, respectively. The coils of 1100, 700 and 400 turns are connected to the differential inputs of the Physlogger i.e. A+ and A-, B+ and B-, C+ and C- respectively.