Simple Properties of a Light Bulb

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The light bulb is a technology we take for granted, but if we explore further, we discover some amazing natural phenomena hidden inside the bulb. The bulb has a filament made of tungsten that becomes hot and starts to glow when a current flows through it. The temperature may easily reach up to 3000 K. Therefore the bulb is likely the hottest object you may ever come see on planet earth. To compare, the temperature of the Sun's photosphere is about 6600 K. So a bulb is half as hot as the sun's surface.

This experiment let's you use the light bulb to look at important ideas about energy, and how real objects behave, often differently from what appears in a textbook. If you are lucky, this is *your* chance to make a graph, observe data and obtain some nice and useful quantities from it. So let's get started.

First, a look at the apparatus provided.

The setup for this experiment consists of a commercially available light bulb (such as a 60 W bulb) that connects to the mains through a potential divider which varies the voltage across the bulb and that changes the current through the bulb. You can measure the voltage and the current whenever you desire. The intensity of light reaching a detector can also be measured. Our detector is a 'light-dependent resistor' (LDR). So what is this?

The LDR is a resistor. Its resistance R changes when the intensity of light falling upon it I changes. Higher the light intensity I, lower is the resistance R. This is called an inverse proportion.

$$R \propto \frac{1}{I} \tag{1}$$

$$\text{or } R = \frac{K}{I} \tag{2}$$

$$\text{which leads to } I = \frac{k}{R}. \tag{3}$$

In the second equation given above, the proportionality sign is replaced by an equality sign and a constant K has been added. It is an example of a constant of proportionality.

By looking at this relation in Eq. (2), can you predict how a plot between R and I would look like? How would a graph between R and $1/I$ look like? Which one of these plots is a *linear* graph? Linear graphs are straight lines and they are really easy to work with.

0.01	1.3	87	1123	13197	9567864	1634896
−2.0	0.1	1.9	3.1	4.1	6.0	7.2

Table 1: Top row are numbers and bottom row are their logs.

The key ideas is that the same data when plotted differently, can provide different shapes of graphs. One shape can be more intuitive than another. A straight line is considered to be the simplest.

There is one more method of making an inverse relationship appear linear. That is by the process of logarithms. Logarithms (or logs for short), are useful because they can convert very large or small numbers that are spaced far apart into middle-sized numbers that are all closer together. Look at the numbers in the top row of the Table. These numbers go from a hundredth of one to a million and a half. Plotting them on a graph would make several points clustered towards the bottom edge of the graph and then unequally spaced. On the other hand, the bottom row shows the logs of these numbers. These are all around one tenth to a ten, and are almost equally spaced apart. We say that the data has been 'linearized'.

It's also easy to plot the lof of the numbers on a graph. What do you see?

We can perform the linearization operation on Eq. (3), converting the equation into log form. Take the log of both sides,

$$\log I = \log \frac{K}{R} \tag{4}$$

and then use a property of logs to convert the fraction on the R.H.S. into a difference,

$$\log I = -\log R + \log K. \tag{5}$$

This looks similar to the equation of a straight line $y = mx + c$ with the slope m being -1 and the intercept being $\log K$. So plotting $\log I$ versus $\log R$ will appear as a straight line. This straight line makes it easy to relate the intensity to the measured resistance.

In fact, this process of linearization can apply to any mathematical model. For example, Consider the relationship between two variables x and y which is of the form,

$$y = Kx^n \tag{6}$$

where K is a constant of proportionality and n is an unknown constant. Applying the log to both sides and using another property gives us,

$$\begin{aligned} \log y &= \log\left(Kx^n\right) && (7) \\ &= \log x^n + \log K && (8) \\ &= n \log x + \log K, && (9) \end{aligned}$$

which again looks similar to a straight line $y = mx + c$ for which the slope of the line $m = n$. Hence, if data is expected to obey the model depicted in Eq. (6), which is also called a power law, a linearized, log-versus-log plot will produce a straight line whose slope provides an estimate of the power n and intercept can provide a value for K.

We now turn to our experiment. The voltage, current and intensity can all be measured. We can also vary and measure the distance d between the LDR and the bulb. So here is a list of possible investigations.

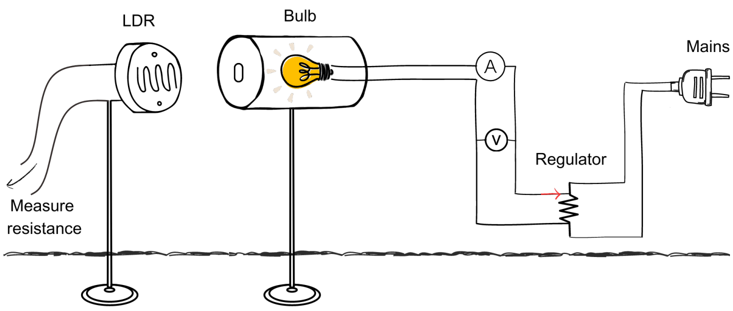

Figure 1: Scheme of the experiment to explore the light bulb's properties.

1. Ohm's law
$$V = iR \tag{10}$$
is a linear formula, which relates the voltage V through an element with the current through it i (not to be confused with the light intensity I). The proportionality constant is the resistance of the element R. The law means that higher the voltage across an element, charges will flow at a higher speed through it, resulting in bigger currents. An analogy would be water flowing downhill through a sharp slope acquires higher speeds than flow down a gentler slope. So our friends, does the light bulb follow Ohm's law? If not could you explain why? What kind of data will you acquire and analyze, to help explain your answer?

2. The power flowing through the bulb's resistive element can be calculated by taking the product of voltage and current
$$P = Vi \tag{11}$$
and the optical intensity can be measured by the LDR's resistance as described earlier. How does the optical intensity change with the electrical power supplied to the bulb?

3. A point source of light is a source whose dimensions are much smaller compared to the distance from the detector (see the figure too). For such an arrangement the intensity I follows an inverse square law with the distance between source and detector
$$I = \frac{K}{d^2} = Kd^{-2}. \tag{12}$$
Does the combination of the bulb and the detector follow an inverse square law? What kind of measurements will you make to show whether Eq. (12) is valid or not. If the law is replaced by another power law,
$$I = \frac{K}{d^n} = Kd^{-n} \tag{13}$$
what is your best estimate of n?

4. Last, this is a bit of a challenge. Stefan-Boltzmann's law states that the total power emitted P from a hot object at temperature T is

$$P = KT^4 \tag{14}$$

where K is another constant that depends on the shape and material of the object. If the power that is emitted is considered to be proportional to the electrical power $(V \times i)$ input into the bulb, we can write

$$Vi = KT^4. \tag{15}$$

The power $(V \times i)$ is measurable, but T is not directly measured. So we rely on a mathematical model that relates a measurable quantity to the temperature. You must have read in your textbook that resistance changes with temperature. If we assume a power-law relationship between resistance and temperature,

$$T \propto R^\gamma, \tag{16}$$

where γ is some unknown number, then we can write,

$$\frac{T}{T_o} = \left(\frac{R}{R_o}\right)^\gamma, \tag{17}$$

where R_o is the resistance at room temperature T_o. Insert the value of T from Eq. (17) into Eq. (15), linearize the relationship and from the data, draw a graph whose slope should give you the value of γ. Try this out! Finally, using Eq. (17), estimate the temperature of the light bulb.

Here are some questions you would like to discuss with mentors?

1. Does the bulb's filament obey Ohm's law? When does it deviate from this law?
2. Is the optical power proportional to the electrical power supplied to the bulb?
3. How do you measure the temperature of bulb's filament and how accurate is it? What can be the cause of inaccuracies?

References

[1] D. MacIsaac, G. Kanner and G. Anserson, Basic Physics of the Incandescent Lamp (Lightbulb), The Physics Teacher **37**, 520, Dec. 1999.