

Rotational motion with a photogate

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The purpose of this experiment is to get familiarized with rotational motion. We normally deal with linear motion in our introductory physics class. Rotational motion is only glossed over, leaving holes in our students' understanding. This simple experiment looks at how a torque produces angular acceleration. Moment of inertia determines how much an object can accelerate in response to a torque. Angular acceleration is the change in angular velocity in unit time. How could we measure these quantities? Finally, just as in linear motion, there is the ubiquitous friction which results in loss of energy. What factors determine the amount of friction?

In the experiment we will use a photogate to determine angular velocity and all quantities will be derived from this basic signal. We will learn how this instrument works and how data can be processed on a computer to extract desired quantities. Rotational motion is also interlinked with linear motion through the relationship $\mathbf{v} = \boldsymbol{\omega}r$. In this experiment, we will see what is meant by this relationship. Furthermore, we will observe graphic depictions of differentiation and integration of numerical data.

KEYWORDS

Torque · Angular velocity · Angular acceleration · Moment of inertia · Angular displacement · Rotational friction

1 Conceptual Objectives

In this experiment, we will,

1. observe that torque produces rotational acceleration $\boldsymbol{\alpha}$;
2. derive acceleration and angular displacement θ from angular velocity $\boldsymbol{\omega}$;
3. understand and use the relationship $\mathbf{v} = \boldsymbol{\omega}r$ which connects rotational with linear motions;

4. understand that there are frictional losses in rotational motion, and
5. motivate oneself that moment of inertia I plays the same role as mass in linear motion.

2 Experimental Objectives

The experiment employs one kind of a sensor—a photogate. The photogate looks for obstructions in a path of light between its two arms. In our experiment, this obstruction is presented by a rotating disk. The rate of obstruction is measured and from it the rotational speed, called the angular velocity of the object is deduced. All other quantities such as α and θ can be subsequently inferred from differentiation and integration of the data. This is achieved by a computer program. So in the process, you will also learn some numerical techniques that are commonly used in experimental physics. Of course, at each stage, we will ask you to interpret all graphs generated and take that extra leap by converting your qualitative interpretation into quantitative inferences.

3 Apparatus

The apparatus for this experiment is quite simple and is shown in Figure 1. An axle is vertically mounted on a bearing. The axle can rotate. Two horizontal shafts can be inserted into the top of the axle through a connecting element. These identical shafts are on opposite sides of the axle. The shafts swirl in a horizontal plane as the axle rotates. The shafts have equidistant grooves spaced 1 cm apart and will be useful as positional markers in later parts of the experiment. Different kinds of weights can be added to the shaft, thus changing the rotor’s moment of inertia. Important dimensions of the various elements are enlisted in Table 1.

Axle	$D = (12.75 \pm 0.03)$ mm
Horizontal shaft	$L = (16.5 \pm 0.1)$ cm; $m = (57.4 \pm 0.1)$ g
Aluminum mass	$L = (1.00 \pm 0.05)$ cm; $m = (30.50 \pm 0.05)$ g
Small brass mass	$L = (1.50 \pm 0.05)$ cm; $m = (99 \pm 1)$ g
Large brass mass	$L = (3.00 \pm 0.05)$ cm; $m = (200.0 \pm 0.5)$ g

Table 1: Dimensions and specifications of the various elements used in the rotational apparatus.

The axle can be set into rotation by turning with hand. In fact, in the first part of the experiment, this is how you will set it into rotation. Alternatively, a vertically falling mass M can apply a torque on the axle, thereby rotating the horizontal shafts. A set of weights is connected to a hanger, and a thread which passes over a pulley and reaches the axle. As the weight is released it descends under the action of gravity, pulling the thread along and exerting a torque on the axle. As the descending weight accelerates in its downward linear motion, it also boosts the angular velocity of the axle and the shafts attached to it. We have provided weights of mass (100 ± 1) g. You can add them successively on a mass hanger which also weights (100 ± 1) g.

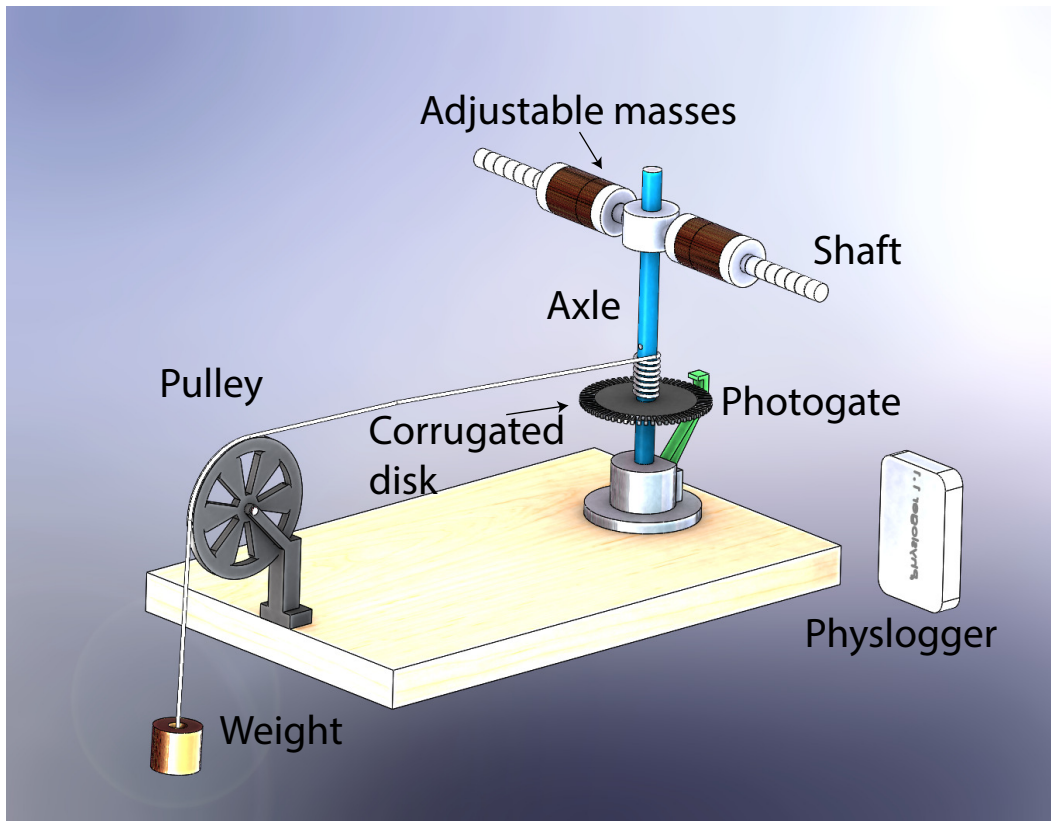


Figure 1: Overview of the experimental arrangement demonstrating various facets of rotational motion.

In this experiment, we will measure the angular velocity ω of the assembly. This is achieved through a disc that is coaxially attached to the axle and co-rotates with it. The circular disk has a series of carefully carved equidistant and identical spokes. These spokes intervene the sensitive area of a photogate. The photogate comprises an LED and a photosensor and can detect when the beam of light is interrupted by an intervening obstacle, which in our case is a particular spoke of the rotating disk. The signal is fed into the computer and analyzed.

The photogate's response is a binary pulse with two states of 1 V and 0 V corresponding to the light beam being interrupted and clear respectively. This signal is read through our data logging device called PhysLogger. The photosensor is connected with one of its analog channels. The pin configuration and other details of the PhysLogger data logging system can be read from our website [1]. The PhysLogger box is connected to the PC using a USB cable.

4 The Experiment

The experiment is divided into two phases and on top of it we have also included an optional part which the brightest students will be motivated to conduct. However, before moving on, we like to highlight a few noteworthy tips to get you started with PhysLogger. Your instructor will, hopefully, provide you with instruction and the user manual [1] if you are stuck somewhere.

After the connecting wires between the photogate and PhysLogger are in place and PhysLogger is connected to your PC, open the PhysLogger application. Click **Measure** and then **Presence**. Select the **channel** to which your photogate is connected to. The wizard will guide you how to calibrate the photogate and once that is done, you should immediately observe a waveform appear on the workspace. You are now set up for the next step.

Twist the axle and you should see a series of pulses appear on the screen. A sample is shown in Figure 2. In order to save data available in the current session, you simply click **Save Data** and convert your saved file to **.txt** format. Data will be saved as two columns, one for time and the other with the voltage readings. This file can then be loaded in a software of your choice for subsequent analysis. We choose Matlab and this is how we will describe the remainder of this experiment.

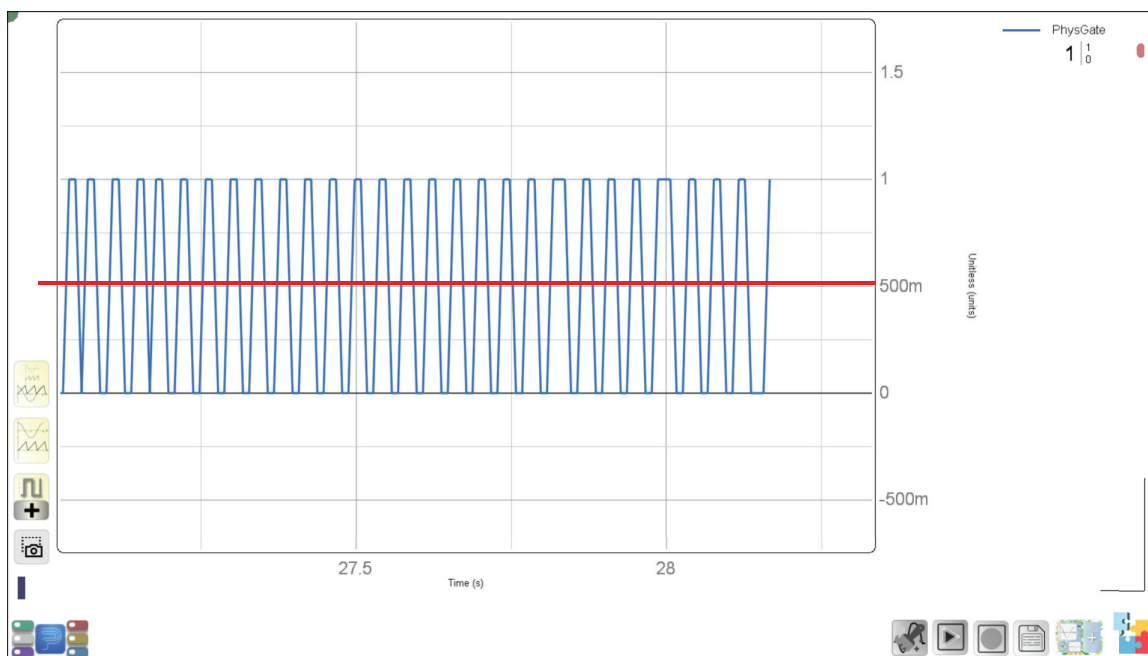


Figure 2: A sample waveform acquired from the photogate. The red line shows the setting of the threshold. Values above the threshold will be read as high (light interrupted) and below the red line will be interpreted as low (light passes through). The time between two successive edges is calculated and from the geometry of the corrugated disk, the angular velocity is computed. For higher angular velocities, the signal may appear rugged; in these cases we generally keep a smaller value for the threshold in the code, say 0.3.

4.1 Phase 1: Measuring angular velocity and observing how it decays

Remove the string from the axle. Attach the set of four weights to the outer end of each horizontal shaft. Keep the aluminum masses with screws on the outermost and innermost sides (Fig. 3) and the brass masses sandwiched in between. The two sets should be symmetric across the two shafts. Tighten the weights strong enough that they do not fly off when the axles rotates. Start the PhysLogger app, make the necessary configuration and start a new

session. Give the axle a twist with your hand and observe the waveform developed on the PhysLogger's desktop. Once the axle has slowed down and approached rest, save your data and open Matlab.

Note: To correctly, run the code when saving data as a `.txt` file, you may need to delete the header column and the first row of the data set (which has the Sr. Nos).

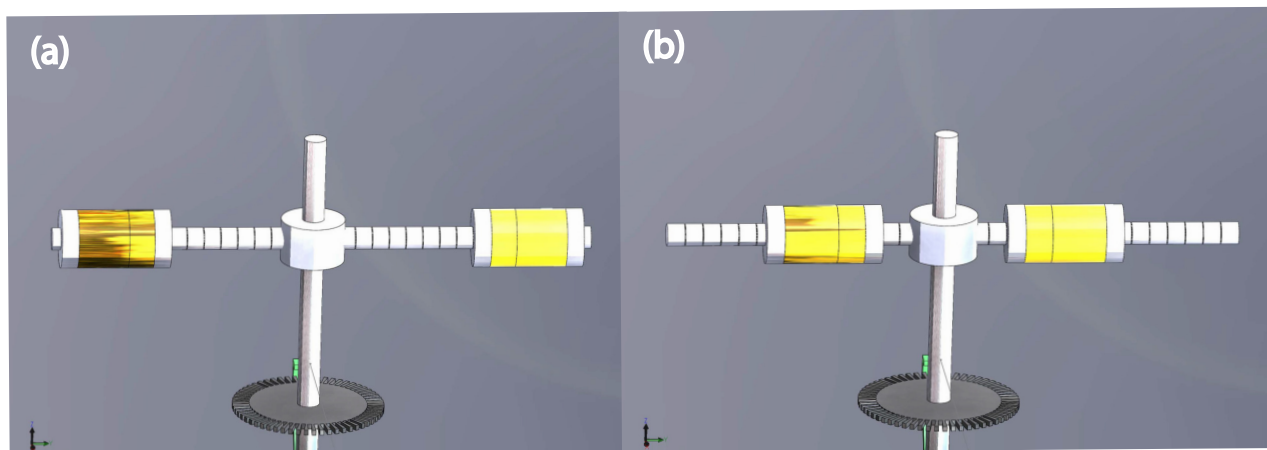


Figure 3: One technique for varying the moment of inertia of the rotational apparatus is to change the position of the masses attached to the axles representing (a) larger and (b) smaller moments of inertia.

In Matlab, move to the directory where you have saved the downloaded codes. On the Matlab prompt, enter the name of the script `findomega`. This script will take your file comprising the timing information and the photogate voltage signal and extract the angular velocity ω for you. What is the maximum value of ω and how does it decay? Fit your data to an exponential decay curve and quantify the rate of the decay? For curve fitting, we generally use Matlab's toolbox `cftool`. What could be the possible reasons for the decay of ω ?

You can experiment on how the decay rate changes with the moment of inertia. A mass m at a distance d from the axis of rotation possesses a moment of inertia md^2 . You can change the moment of inertia by varying the masses attached to the shafts or changing their positions. The grooves on the shafts serve as convenient markers for the distance. Change the moment of inertia and see how the decay rate changes. Note down all your observations and discuss these with your instructor.

Furthermore, if you would like to process and observe individual data sets, here is a list of variables generated by the script.

4.2 Phase 2: Quantifying energy loss due to frictional torque

In this phase of the experiment, you will perform a quantitative measurement of how much energy is lost in a single revolution of the axle and will correlate it with the angular speed. In linear motion, the frictional force f_{fr} may depend on the speed v of the object. A commonly

<code>time</code>	Time in seconds
<code>photo</code>	Raw signal generated by the photogate
<code>photo2</code>	Binary form of the photogate signal
<code>dsfilt</code>	Time intervals between successive edges
<code>time_avg</code>	Times at which ω is computed
<code>omega</code>	The angular frequency ω

Table 2: Variables generated as a result of the script `omegafind.m`.

used model is the Stokes model:

$$\mathbf{f}_{\text{fr}} = -b\mathbf{v}. \quad (1)$$

Analogously in rotational motion, the motion is impeded by a frictional *torque*. I wonder if this torque depends on the angular velocity ω ,

$$\tau_{\text{fr}} \stackrel{?}{=} -b\omega. \quad (2)$$

For this investigation, we apply a known torque to rotate the axle and the shafts. A weight of approximately 500 g is connected with a length of string. One end is attached to the weight bearing hanger, passes over the pulley and the other end is strung through the holes in the axle and tied to the screw fastened on the axle. The length of the string is such that when fully extended, the weight does not touch the ground. The weight can be ascended by turning the axle by hand; this will wrap the thread around the axle and lift the weight. As the weight descends, the thread unwraps.

Therefore, if the weight is lifted to its maximum height and released, it will descend while the axle starts rotating. When the weight reaches its lowermost position, it bounces up again, this time wrapping the thread in the opposite direction. In this manner, several wrapping-unwrapping cycles are observed and the weight bounces up and down before finally coming to rest at its lowest position. We will measure this bouncing and determine the energy lost per bounce and energy lost per single revolution of the axle. Later we will correlate the energy lost with the angular speed.

The weight of mass M tied to the end of the string in fact applies a torque $\tau = Mgr$ where r is the radius of the axle (See Table 1). This torque accelerates the system according to Newton's second law,

$$\tau = I \frac{d\omega}{dt} = I\alpha \quad (3)$$

hence changing the angular velocity. We will measure the changing angular velocity (called the angular acceleration α).

Lift the weight to its maximum height, start the PhysLogger app, configure the channel and start the session. Release the weight. The app will display the signal from the photogate. Physically observe the multiple bounces of the weight and the angular speed of the axle go up and down. Save your data. Note down all your observations.

In order to process data, you will need to run the Matlab script `bouncer`. This will generate all the required data, whose variable names are listed in Table 3 (in addition to the variable names defined in Table 2).

<code>omegafilt</code>	Filtered angular velocity
<code>omegadiff</code>	Angular acceleration
<code>omegadifffilt</code>	Filtered angular acceleration
<code>rev</code>	Number of revolutions of the axle
<code>height</code>	height of the bouncing weight

Table 3: Variables generated as a result of the script `bouncer.m`.

Bounce	Start time (s)	End time (s)	Before h (cm)	After h (cm)	ω_{avg} (s^{-1})	Before rev.	After rev.	Δh (cm)	N_{rev}	$\Delta E/N_{\text{rev}}$ (J)
1	19.02	31.04	-17.36	-25.66	6.23	19.7	32.36

Table 4: Sample data table for calculating frictional loss.

Here is a brief outline of what the script `bouncer` achieves. The angular acceleration is computed by numerically differentiating the angular velocity ($\alpha = d\omega/dt$). The linear displacement of the weight is correlated with the angular motion and determined in the following way. We know that $\mathbf{v} = \omega r$ where \mathbf{v} is the peripheral speed of the axle. Since the thread is attached to the axle, this variable also represents the speed of the thread and the speed of the weight. The displacement is computed from the integral,

$$x(t) = \int_0^t v(t') dt' = r \int_0^t \omega(t') dt'. \quad (4)$$

Hence numerical integration of the angular velocity yields the angular displacement. Multiplying the displacement with the axial radius r gives the linear displacement (height) and dividing by 2π gives the number of revolutions. So we have all the desired variables.

Plot these signals, draw a sketch of the signal in your notebook (or print) and explain why the signals look the way they are. Discuss your interpretations with the instructor. Recognize the points of maximum and minimum angular velocity, angular acceleration and the turning points of the bouncing weight.

You can plot a signal in Matlab using the following command:

```
figure; plot(time_avg, variable_name);
```

We would now like you to calculate the energy losses. If you observe the data for the height, you will notice that in successive bounces, the maximum height to which the weight rises keeps on dropping. Suppose the drop in each bounce is Δh . This will lead to a drop in energy $\Delta E = Mg\Delta h$. Determine the number of revolutions in each bounce. Hence compute the average energy lost per revolution. Furthermore, calculate the average angular velocity per bounce. This can be approximated by $\omega_{\text{max}}/2$.

We have found it useful to make a table of our observations; something similar to the following will help.

After you've populated the table, plot the average energy lost per bounce as a function of the angular speed. What does your relation look like? Fit your data to a suitable model. Change

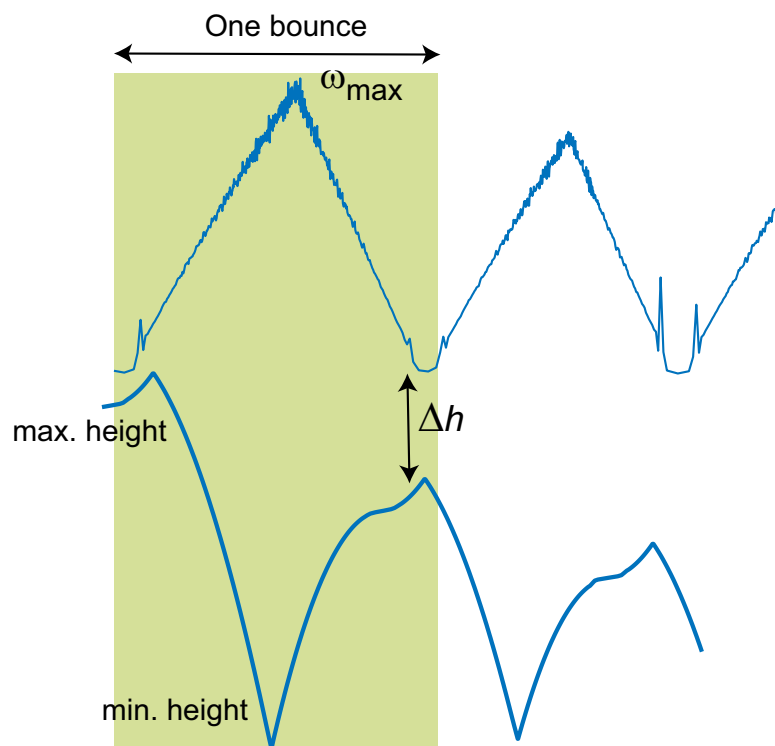


Figure 4: Two bounce cycles of the data are shown. On the top, there is the angular velocity and on the bottom, the corresponding linear height. Compare the points of maximum ω and minimum height and vice versa. The loss in height across the bounce cycle is also labeled as Δh in this diagram.

the position of the masses on the shafts and observe the changes in the angular velocity and angular acceleration.

4.3 The optional part!

Take a longer thread so that the hanging weight will touch the ground in its downward fall. As it touches the ground, the thread becomes lax. At this point compute the angular velocity. Let's call it ω_o . What is your estimate of the total energy lost during the descent of the weight through an overall height H ? Using the energy conservation equation:

$$MgH = \{\text{Total energy lost}\} + I\omega_o^2 \quad (5)$$

can you estimate the moment of inertia of the shaft with its attached masses?

References

- [1] Details on PhysLogger can be accessed from <https://www.physlab.org/facility/PhysLogger/>.