Using PhysLogger to Investigate Nonlinear Dynamics with a Magnetic Pendulum

Mohsina Asif, Muhammad Umar Hassan, Muhammad Hamza Humayun and Muhammad Sabieh Anwar
Syed Babar Ali School of Science and Engineering, LUMS

V. 2022-1; July 4, 2022

1 Introduction

Pendulum systems have been commonly studied in mechanics to understand periodic motion but in reality, pendulum exhibits a rich dynamic behavior. In the last couple of decades, pendulum has emerged as a tool to observe non-linear behavior. Non-linearity is a profound concept in the study of physical systems. The characteristics of seemingly very simple systems may turn out to be extremely intricate due to non-linearity. The study of chaos also begins with the study of such simple systems. The magnetic pendulum is one such system. A simple magnetic pendulum system in a repulsive magnetic field starts behaving non-linearly and displays chaotic motion [1, 2]. In this experiment, we will explore the notion of nonlinear and chaotic dynamics using a magnetic pendulum.

KEYWORDS Magnetic Pendulum · Chaos · Phase Portrait · Poincare Map · Repulsive magnetic field · Resonance.

2 Objective

In this experiment, we will discover:

1. how apparently simple systems can be highly non-linear and exhibit a complex behavior under certain conditions,
2. the physical structure of dynamical systems, and

3. the conditions and consequences of the notion of super-sensitivity and its relationship with chaos through simulation and experiment.

3 Theory

We start this experiment through a pre-lab exercise exploring non-linearity. The purpose is to see how a simple pendulum can become nonlinear. In this regime, assertion that the time period is independent of the initial pendulum breaks down. If we ignore friction a simple pendulum's equation of motion can be written as:

$$\ddot{\theta} + \frac{g}{l}\sin(\theta) = 0. \tag{1}$$

where θ is the angular displacement and its second derivative, $\ddot{\theta}$, represents the angular acceleration of the pendulum, l is the length of the pendulum, and g is the acceleration due to gravity. Now if we make the small angle approximation i.e., $\sin(\theta) \approx \theta$ the we obtain:

$$\ddot{\theta} + \frac{g}{l}\theta = 0 \tag{2}$$

Question What factors control the time period of the pendulum in the case of small angles? Does time period depend on the initial amplitude from which the pendulum is displaced? Derive Eq. (1) from Newton's force equation.

For the large amplitude, θ_o, where the small angle approximation breaks down, we can rearrange and integrate Equation (2) to give us the following formula for time period of a pendulum:

$$T(\theta_0) = 4\sqrt{\frac{l}{g}}\, K(k) \tag{3}$$

where,

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}} \tag{4}$$

is called the complete elliptic integral of the first kind and can be easily tabulated through Matlab or looked up in standard references. Here $k = \sin\frac{\theta_o}{2}$. For example, in Matlab[1], this integral can be numerically computed using the function `EllipticK`. Further details about this integral and its usage to determine the time period of a non-linear pendulum can be seen in the reference [5].

[1] These commands require Matlab's Symbolic Math Toolbox. Type `ver` in your Matlab command window to check and see if you have this toolbox.

Exercise Plot time period of non-linear pendulum versus θ_o. Vary values of θ_o from 0^o to 90^o. Choose an appropriate value of the length of the pendulum (that matches your apparatus). At what point could you say the transition from linearity to non-linearity occurs?

Question With the help of the reference [5], derive Eq. (4).

3.1 Temporal Trajectories and Phase Space Portraits

Figure 1 shows the schematic diagram of the magnetic pendulum and the forces present in the system. The distance d is between magnets when the pendulum is in the resting position. The arrows show the magnetic moment vectors. By steadily decreasing the d between the two magnets we can study the dynamic behavior of the magnetic pendulum. This is a two dimensional system with the canonical coordinates being θ and $\omega = \dot{\theta}$. A phase portrait gives the phase space picture, with ω plotted against θ as time progresses.

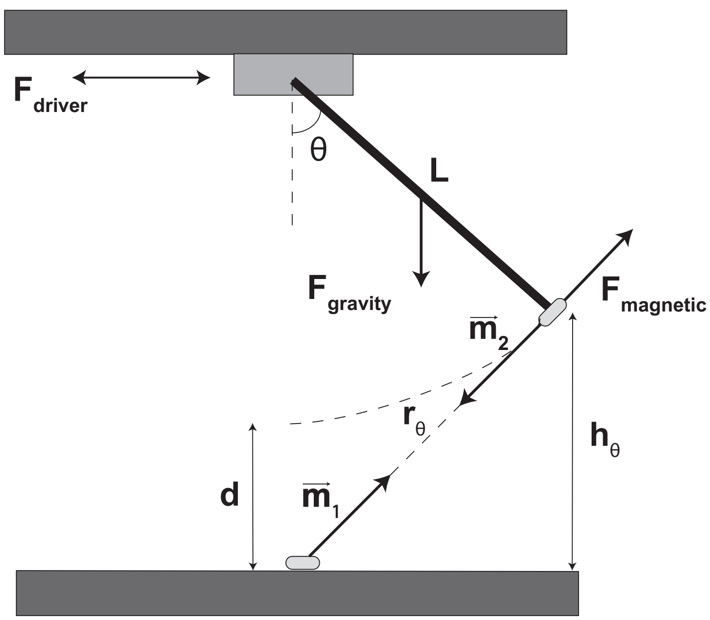


Figure 1: Schematic diagram of the magnetic pendulum and the forces acting on it.

3.2 Modeling of Dynamic Behavior of Magnetic Pendulum

We will first gain insight by simulating the motion of our pendulum in MATLAB. For this purpose, we will need to know the equation of motion of the pendulum. This will then be numerically solved. This, in turn, necessitates the need for a 'model' of the system. The motion exhibited by our pendulum is of the rotational kind. Hence knowing all the torques τ will allow us to write its equation of motion. Figure 1 identifies the various torques acting on the pendulum. Newton's Second Law gives us:

$$I\frac{d^2\theta}{dt^2} = \Sigma\tau_i \tag{5}$$

where I is the moment of inertia of the pendulum and $\Sigma\tau_i$ is the vector sum of all the torques acting on the pendulum:

$$\Sigma\tau_i = \tau_{gravity} + \tau_{driver} + \tau_{damping} + \tau_{magnetic} \tag{6}$$

The torques in this equation are self-explanatory. Substituting the expressions for each of these torques yields the following differential equations [2]:

$$\dot{\theta} = \omega \tag{7}$$

$$\begin{aligned} \frac{ML^2}{3}\dot{\omega} &= -\frac{L}{2}Mg\ \sin\theta + T_{driver}\sin(\Omega t) - \gamma\omega \\ &+ \frac{|\theta|}{\theta}L\frac{\mu_o}{4\pi}\frac{m_1 m_2}{r_\theta^2}\cos\left(|\theta| + \tan^{-1}\left(-\left|\frac{h_\theta}{L\sin\theta}\right|\right)\right). \end{aligned} \tag{8}$$

where M is the mass and L is the length of the pendulum. The second term on the right side is the torque produced by the force exerted by the reciprocating crankshaft, T_{driver} represents the maximum torque and Ω being the frequency of the reciprocating motion. The third term on the R.H.S. is a damping term which is proportional to the angular velocity ω and γ is a damping coefficient. The fourth term arises due to a magnetic dipolar attraction between the two magnets, which are each assumed to be magnetic dipoles of strengths m_1 and m_2. The vectorial magnetic force can be approximated as:

$$\mathbf{F}_{magnetic} = \frac{\mu_o}{4\pi}\frac{m_1 m_2}{r_\theta^2}\hat{\mathbf{r}}. \tag{9}$$

Equation (9) assumes a Coulomb-like inverse square law between two magnetic moments m_1 and m_2 separated by a distance r_θ. The permeability of vacuum is μ_o. The unit vector $\hat{\mathbf{r}}$ points radially away from the line joining the two magnets and indicates the direction of the magnetic force. Furthermore, $r_\theta = \sqrt{(L\sin\theta)^2 + h_\theta^2}$ and $h_\theta = d + L(1 - \cos\theta)$. The variables d, r_θ and h_θ are also shown in Figure 1.

Exercise Interpret each term of Equation (8).

We now numerically solve the equation 8. For this purpose, Eq. (8) can be simplified and rewritten as:

$$\dot{\omega} = -A\sin\theta + B\sin(\Omega t) - C\omega + \frac{|\theta|}{\theta}\frac{E}{r_\theta^2} \times \cos\left(|\theta| + \tan^{-1}\left(-\left|\frac{h_\theta}{L\sin\theta}\right|\right)\right) \qquad (10)$$

where A, B, C and E are constant coefficients that depend only on the physical construction and parameters associated with the pendulum. For the simulation, we are going to use values of these constants provided in the reference [2]; which are $A = 110\ \mathrm{s}^{-2}$, $B = 0.01\ \mathrm{s}^{-2}$, $C = 0.001\ \mathrm{s}^{-1}$, $E = 0.2\ \mathrm{m}^2\mathrm{s}^{-2}$, $\theta(0) = 0.2$ rad and $\omega(0) = 0$. All these values and conditions will be kept constant throughout all simulation runs. Remember that our pendulum will have *different* values.

Exercise Try to estimate these parameters of the pendulum used in the laboratory.

Download the file named `pendode3.zip` from the Matlab Code link in the software codes section on the experiment's web page. Unzip the file to extract the two Matlab files `pendode3.m` and `pendode3script.m`. Open the m-file `pendode3.m` and you will see the values of A, B, C, g, L, E added in as below.

```
A = 110; B = 0.01; C = 0.001; g = 9.8; L = (3*g)/(2*A); E =
0.5; d = ;
```

You will insert the value of d into the file pendode3.m. For example, if the distance between the magnet and tip of the pendulum is 10 cm, you will type in $d = 100\ e - 3$.

1. You will run the simulation through the script `pendode3.m`. Set the value of d and vary initial conditions for which you want to run the simulation.

2. Here is a brief description of the code inside the script file: `ode113` is a solver, called on to solve the system of differential equations which we have defined in `pendode3.m`. The interval `[0 1000]` indicates the time in seconds (or the values of time vector, T) over which the solver will solve the differential equations defined in Equations (7) and (8). Similarly, the vector `[0.2 0.2]` indicates the initial conditions of the variables `y(1)` and `y(2)`.

   The `options` command sets the relative and absolute tolerance levels of the ode solver for our two parameters: ω, θ. As we are looking at a very sensitive pendulum system therefore the tolerance levels have been set extremely low to give us a high degree of accuracy in our results. These tolerance levels have been adjusted after trials.

   After finishing off its processing the ode solver will return to you two vectors in the Matlab workspace : a time vector `T` and another vector `Y` which comprises two columns, each corresponding to our two variables, ω and θ.

3. Plot the time series of θ.

4. Plot the time series of ω.

5. Make a phase portrait showing the variation of ω versus θ.

A set of sample results for varying distances is shown in Figure 2.

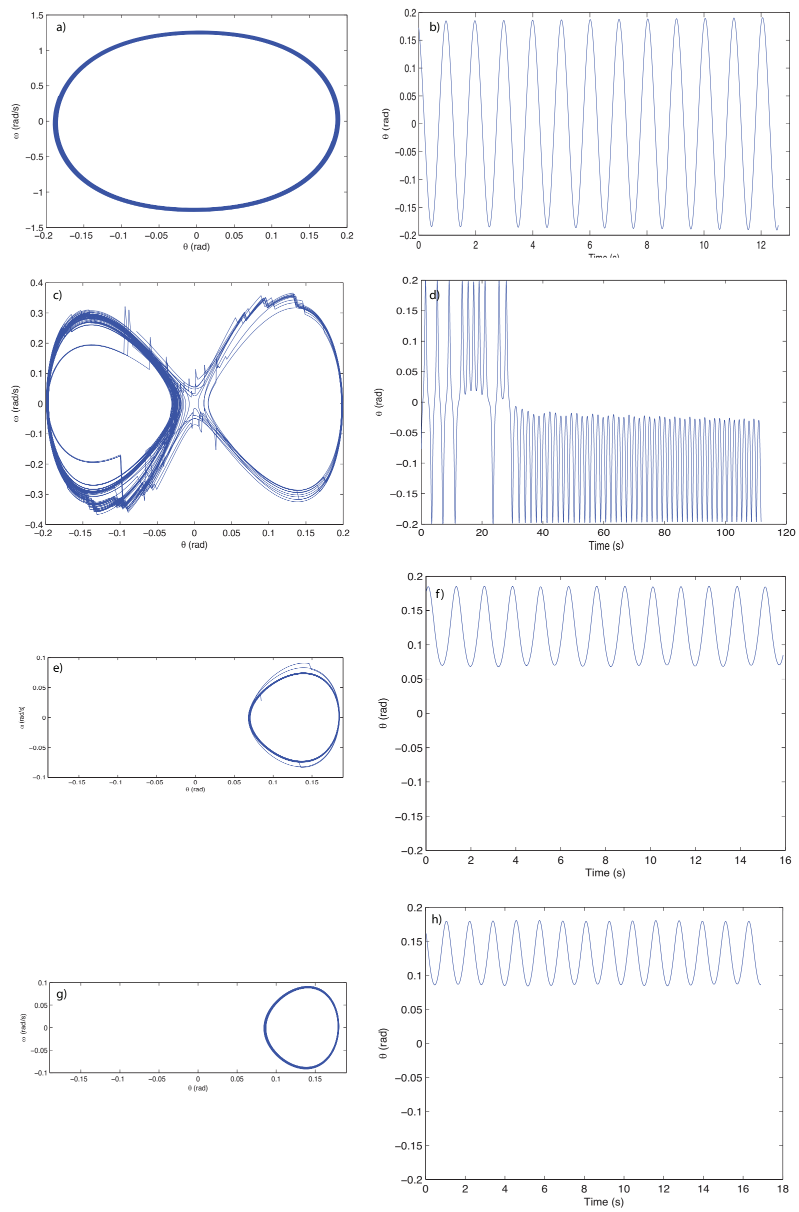


Figure 2: The results of our simulations showing the phase portraits ((a),(c),(e),(g)) and the time series of the angles ((b),(d),(f),(h)). The distance between the magnets d is decreasing as we are going down this panel. In (c) and (d), the pendulum exhibits chaotic motion and then gets stuck towards one side of the magnet. In (e) through (h), the pendulum is unable to overcome the repulsive force and remains glued to the right side.

4 Experimental Setup

To investigate this chaotic behavior experimentally, we drive the pendulum using a stepper motor via a slider-crank mechanism. We attach one magnet at the end of the pendulum and another magnet is fixed to a post positioned vertically below the pendulum's equilibrium position (Figure 3). The poles are oriented such that they repel each other. In our system, we can vary the (i) driving frequency, (ii) separation between magnets in the z-direction, and (iii) separation in the horizontal plane (along the x-direction).

The experimental data is acquired using PhysLogger which is interfaced with the PhysLogger Desktop application on the PC. We call the entire setup PhysMag (details of the experiment can be found at: `https://physlab.org/experiment/physmag-nonlinear-dynamics-with-a-magnetic-pendulum/`).

PhysLogger is connected to two PhysInstruments: Qosain General Stepper Motor Controller (SMC) and PhysCompass. The driver frequency of the pendulum will be varied through the SMC, whereas PhysCompass will be used to collect and plot the θ readings for the pendulum. These readings can be quickly visualized and manipulated in PhysLogger Desktop for a qualitative analysis. Furthermore, the readings can be exported from PhysLogger Desktop App and imported into an advanced software, e.g., MATLAB, for an involved mathematical analysis.

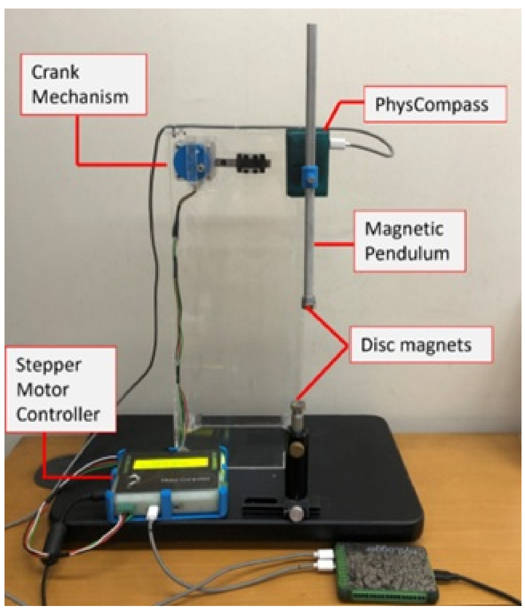


Figure 3: Experimental setup for investigating nonlinear dynamics with a magnetic pendulum (PhysMag).

5 Experimental Procedure

5.1 Setting the Apparatus

1. Set up the apparatus, as shown in Figure 3.
2. Connect the stepper motor of the crank mechanism to SMC via the motor terminal block on SMC, as shown in Figure 4a.
3. Connect a 12 V power adapter to SMC via the DC power jack on SMC as shown in Figure 4b.
4. Plug the DC power adapter to a power source and press the ON button on the side of SMC. The LCD screen of SMC should light up.
5. Using a USB C to C cable, connect SMC to a digital channel of PhysLogger.
6. Connect PhysCompass to another digital channel of PhysLogger.
7. Connect PhysLogger to your PC by inserting the provided USB cable to the USB port of PhysLogger.

(a) (b)

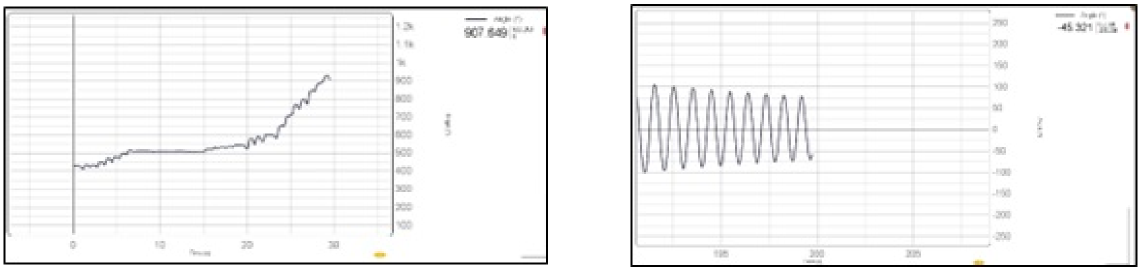
Figure 4: (a) The stepper motor is connected to the motor controller via the motor terminal block on SMC and (b) the motor controller needs to be connected to a DC power adapter to be switched on.

5.2 Calibration of PhysCompass

1. PhysCompass is powered on as soon as the PhysLogger device is connected to PC. This is signified by a red LED on PhysCompass being switched on.
2. Moments after being switched on, PhysCompass automatically calibrates itself.
3. Locate and open `PhysLogger2` application on your PC.
4. Click on `Measure` > `Rotation` > `Make a LivePlot now`.

5. Initially, it is ensured that the distance between both magnets is sufficiently large (>10 cm) so there is minimal magnetic force on the pendulum.

6. To confirm that PhysCompass has been properly calibrated, slightly swing the pendulum and observe how the θ varies. Figure 5 shows a comparison between an uncalibrated and calibrated PhysCompass.



(a) (b)

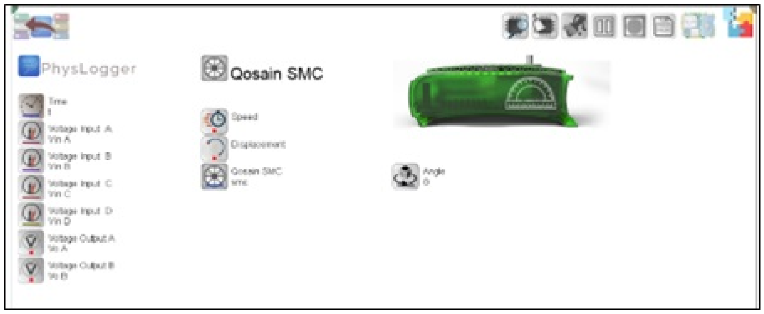
Figure 5: Response of (a) an uncalibrated vs. (b) calibrated PhysCompass.

7. In case your PhysCompass was not automatically calibrated and shows an abnormal behavior such as that shown in Figure 5a, calibrate PhysCompass using any of the following methods:

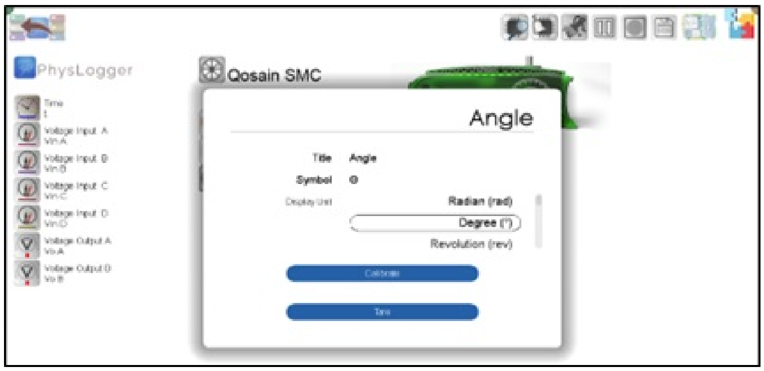
   (a) Unplug PhysCompass from PhysLogger and plug it back (ignore any warnings given by the application). Repeat Step 1.

   (b) Go to `Quantities Panel` > `PhysCompass` > `Calibrate`. Swing the pendulum.

8. When the pendulum is at its stationery mean position, tare PhysCompass readings. This can be done by pressing and holding the hardware `Tare` button on PhysCompass (for at least 3 seconds) or using the software `Tare` control, as shown in Figure 6b. The θ readings are zeroed once PhysCompass has been tared.



(a)



(b)

Figure 6: (a) PhysInstruments (SMC and PhysCompass) icons appearance on the `Quantities Panel` after proper connection to the PhysLogger. (b) Configuration of PhysCompass can be done by clicking on its icon in the `Quantities Panel`.

5.3 Configuring the Stepper Motor Controller (SMC)

1. From the `Quantities Panel`, go to `Qosain SMC` > `Working Mode` > `Speed`.
2. In `Speed` mode, select a display unit for motor parameters (rad/s in this case).
3. In `Source`, create a graphical control widget, such as a horizontal slider. This will allow you to vary the driver frequency i.e., speed of the motor in run time from the workspace.
4. You may rename your motor speed controller slider (we named it Motor Speed). Figure 7 shows how the SMC is set up in the application.
5. Specify the range of the horizontal slider by clicking on the minimum and maximum value of the slider. By default, the slider has a maximum and minimum value of -1 and 1, respectively. We changed it to 0 rad/s to 12 rad/s (Figure).
6. To operate the SMC from PhysLogger Desktop, enable Power (its value will be set as 1).

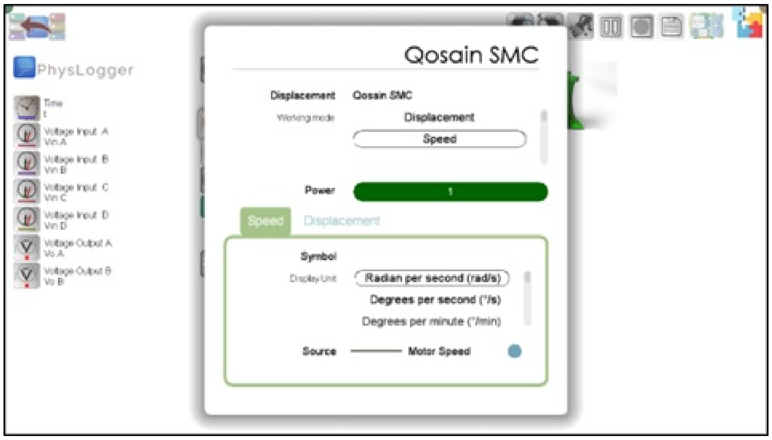


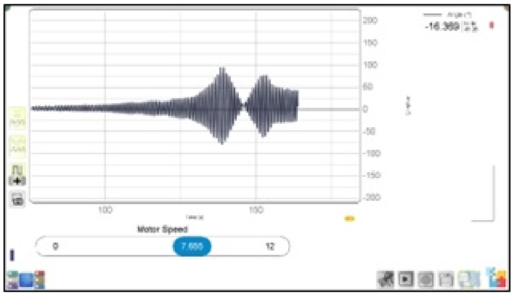
Figure 7: Setting up an adjustable slider (named `Motor Speed`) and other parameters of SMC in PhysLogger Desktop to control the speed of the stepper motor.

6 Measurements

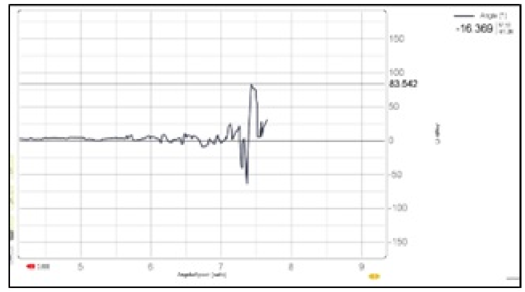
6.1 Determining the Resonance Frequency

In the experiment, the motor is driven at about 70–80% of the pendulum's resonance frequency Ω_0. Thus Ω_0 needs to be determined first.

1. To find Ω_0 of the system, the magnets are positioned sufficiently further apart such that there is negligible force between them.
2. Using the horizontal slider created earlier, gradually vary the speed of motor and observe how θ of the pendulum varies.
3. You may create a LivePlot of θ against motor speed to identify the resonance frequency. (Add a new LivePlot by navigating to `Extensions Menu` > `Build from scratch` > `Widgets` > `LivePlot`.)
4. The Ω_0 is the motor speed at which the θ is the maximum. Refer to Figure 8 to see how we identified the resonance frequency.



(a)



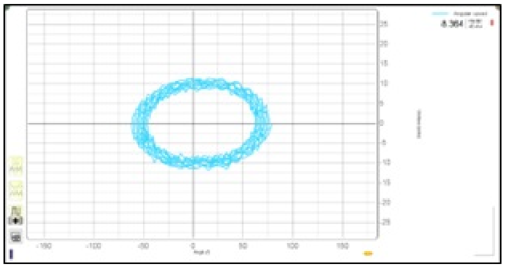
(b)

Figure 8: To identify the resonance frequency Ω_0 of the system, vary the `Motor Speed` using a horizontal slider you created. (a) Identify the speed (position of horizontal slider) at which the amplitude of oscillations is the maximum. For better precision you may also (b) plot angular displacement θ against `Motor Speed` and note down the motor speed at which the amplitude of the pendulum is the maximum (in this case the resonance frequency Ω_0 was found to be 7.437 rad/s).

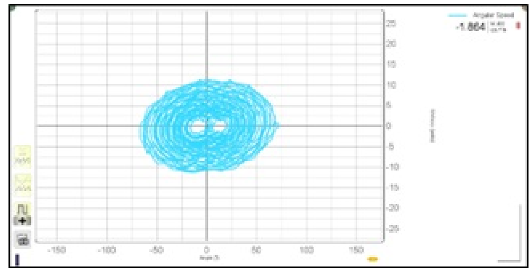
6.2 Experimental Observation of the Nonlinear Motion

After setting $\Omega \approx 0.7\ \Omega_0$, the distance d between the two magnets is varied by adjusting the height of the post on which the magnet is perched [3]. As the distance is changed, and different phase portraits are obtained, remember to keep the initial conditions of the pendulum constant. The initial conditions include the motor speed and the initial angle from which the pendulum is displaced.

1. To calculate the $\dot{\theta}$ of the magnetic pendulum, you need to numerically differentiate the θ of the pendulum.
2. Go to `Extensions Menu` > `Build from scratch` > `Advanced` > `Differentiate`.
3. In the `Differentiate` window, specify the angle θ of the magnetic pendulum as the source. `Differentiate` utilizes four-point differentiation algorithm to compute the $\dot{\theta}$ of the pendulum.
4. In a new LivePlot, plot this differentiated quantity ($\dot{\theta}$) on the vertical axis against θ on the horizontal axis.
5. Gradually decrease the distance d between the two magnets.
6. For each value of d, observe the changes that occur in the time course plots and the phase portraits of the pendulum.
7. For an advanced quantitative analysis of the data, save and export the readings.



(a)



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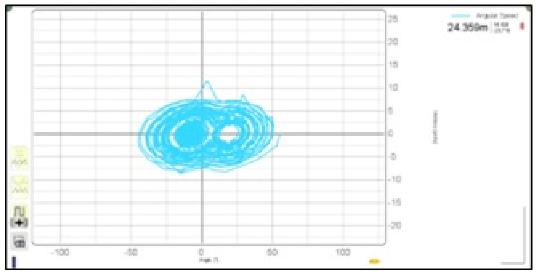

(c)

Figure 9: Some phase portraits generated in PhysLogger Desktop App. The distance d between the magnets was decreased from (a) to (c).

A few sample phase portraits generated in PhysLogger are shown in Figure 9 for different values of d. Figure 9a shows the linear response when the separation between the two magnets is large and the repulsive magnetic force has little damping effect on the swing of the pendulum. As d is reduced, we see more complicated orbits. Figures 9(b–c) display the phase portraits when the pendulum is in chaotic motion. Interestingly when the repulsive force is large enough, the pendulum swing is trapped on one side of the pendulum [4].

6.3 Additional tasks

1. Repeat the experiment for various values of d.

2. Remember that chaos is highly sensitive to the initial conditions. Vary the θ and see the effect.

3. Check if there is hysteresis in the system.

4. If curious and motivated, you may explore several controls and features of PhysLogger through this experiment. Tables can be made in the application, on-screen widgets attached to quantities, meaningful LivePlots with different readings generated, and an intuitive understanding developed of the many underlying concepts.

5. Try developing a theoretical model using the reference papers.

6.3.1 Minimizing noise in the readings

For instance, you can try to minimize the noise in readings. If you spot erratic behavior in the angular speed readings of magnetic pendulum (i.e., the differentiated angular displacement), try minimizing this noise by applying a low pass filter to the readings (determine an appropriate cut off frequency) or experiment with various sampling frequencies. The **low pass filter** can be found in the `Extensions Menu` > `Build from scratch` > `Advanced`. **Sampling frequency** can be adjusted by clicking on the `Time` quantity.

Further, identify the possible reason behind these fluctuations. Is it inherent or due to some connected instruments? Can the four-point differentiation algorithm be a reason? You may also test if there is any change in the noise when motor is on and off.

Figure 10 shows an instance where we applied a low pass filter on the angular velocity of the magnetic pendulum. A noticeable reduction in noise can be observed. You may perform similar tests on your readings.

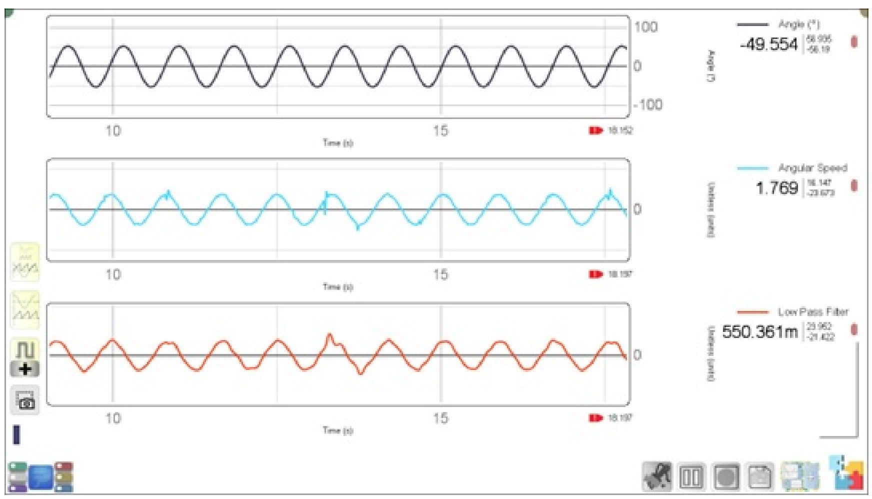

Figure 10: An example of how one can reduce noise in readings (angular speed of magnetic pendulum) by applying a low pass filter on the readings. This filter is present in the widgets menu of PhysLogger Desktop.

References

[1] G.L. Baker "Control of the chaotic driven pendulum", Am. J. Phys. **63**, 832 (1995).

[2] A. Siahmakoun, V. A. French, J. Patterson "Nonlinear dynamics of a sinusoidally driven pendulum in a repulsive magnetic field", Am. J. Phys. **65**, 393 (1997).

[3] P. W. Laws "A unit on oscillations, determinism and chaos for introductory physics students", Am. J. Phys. **72**, 446 (2004)

[4] Y. Kraftmakher "Experiments with a magnetically controlled pendulum", Eur. J. Phys. **28**, 5 (2007)

[5] Loius A. Pipes, Lawrence R. Harvill, "Applied Mathematics for Engineers and Physicists", Third Edition, International Student Edition, McGraw-Hill Kogakusha Ltd. pp. 598–601, (1970)