Q3.

$$\chi^2 = \sum_i \frac{(y_i - mx_i - c)^2}{\alpha_i^2}$$

after experimentation : x_i, y_i & α_i are fixed.

i) Show that $\frac{1}{2}\frac{\partial^2 \chi^2}{\partial c^2} = \sum_i \frac{1}{\alpha_i^2}$

$$\frac{\partial \chi}{\partial c} = \sum_i \frac{-2c}{\alpha_i^2}$$

$$\frac{\partial^2 \chi}{\partial c^2} = \sum_i \frac{2}{\alpha_i^2}$$

$$\boxed{\frac{1}{2}\frac{\partial^2 \chi}{\partial c^2} = \sum_i \frac{1}{\alpha_i^2}}$$

ii) Show that $\frac{1}{2}\frac{\partial^2 \chi^2}{\partial m \partial c} = \sum_i \frac{x_i}{\alpha_i^2}$

$$\frac{\partial \chi}{\partial m} = \sum_i \frac{2(y_i - mx_i - c).(-x_i)}{\alpha_i^2}$$

$$\boxed{\frac{1}{2}\frac{\partial^2 \chi}{\partial m \partial c} = \sum_i \frac{x_i}{\alpha_i^2}}$$

iii) Show that $\frac{1}{2}\frac{\partial^2 \chi^2}{\partial m^2} = \sum_i \frac{x_i^2}{\alpha_i^2}$

$$\frac{\partial \chi}{\partial m} = \sum_i 2\frac{(-mx_i)(-x_i)}{\alpha_i}$$

$$\boxed{\frac{1}{2}\frac{\partial^2 \chi}{\partial m^2} = \sum_i \frac{x_i^2}{\alpha_i^2}}$$

Q2. $C = \begin{bmatrix} \alpha_1^2 & \rho\alpha_1\alpha_2 \\ \rho\alpha_1\alpha_2 & \alpha_2^2 \end{bmatrix}$

$$C^{-1} = \frac{1}{\alpha_1^2\alpha_2^2 - \rho^2\alpha_1^2\alpha_2^2} \begin{bmatrix} \alpha_2^2 & -\rho\alpha_1\alpha_2 \\ -\rho\alpha_1\alpha_2 & \alpha_1^2 \end{bmatrix}$$

$$C^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} \frac{\alpha_2^2}{\alpha_1^2\alpha_2^2} & \frac{-\rho\alpha_1\alpha_2}{\alpha_1^2\alpha_2^2} \\ \frac{-\rho\alpha_1\alpha_2}{\alpha_1^2\alpha_2^2} & \frac{\alpha_1^2}{\alpha_1^2\alpha_2^2} \end{bmatrix}$$

$$C^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} \frac{1}{\alpha_1^2} & -\frac{\rho}{\alpha_1\alpha_2} \\ \frac{-\rho}{\alpha_1\alpha_2} & \frac{1}{\alpha_2^2} \end{bmatrix}$$

$$C(I-\lambda) = \begin{bmatrix} \alpha_1^2 - \lambda & \rho\alpha_1\alpha_2 \\ \rho\alpha_1\alpha_2 & \alpha_2^2 - \lambda \end{bmatrix}$$

$$0 = \det = \alpha_1^2\alpha_2^2 - \alpha_1^2\lambda - \alpha_2^2\lambda + \lambda^2 - \rho^2\alpha_1^2\alpha_2^2$$

$$\lambda^2 - (\alpha_1^2 + \alpha_2^2)\lambda + \alpha_1^2\alpha_2^2(1-\rho^2) = 0$$

$$\lambda_\pm = \frac{1}{2}(\alpha_1^2 + \alpha_2^2) \pm \sqrt{(\alpha_1^2+\alpha_2^2)^2 - 4\alpha_1^2\alpha_2^2(1-\rho^2)}$$

$$\begin{bmatrix} \alpha_1^2 & \rho\alpha_1\alpha_2 \\ \rho\alpha_1\alpha_2 & \alpha_2^2 \end{bmatrix} \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} = \lambda \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$\alpha_1^2\cos\phi + \rho\alpha_1\alpha_2\sin\phi = \lambda\cos\phi$$

$$\rho\alpha_1\alpha_2\cos\phi + \alpha_2^2\sin\phi = \lambda\sin\phi$$

$$\alpha_1^2 \cos\phi + \rho\alpha_1\alpha_2 \sin\phi = \frac{\rho\alpha_1\alpha_2\cos\phi + \alpha_2^2 \sin\phi}{\sin\phi} \cos\phi$$

$$\alpha_1^2 + \rho\alpha_1\alpha_2 \frac{\sin\phi}{\cos\phi} = \rho\alpha_1\alpha_2 \frac{\cos\phi}{\sin\phi} + \alpha_2^2$$

$$\alpha_1^2 + \rho\alpha_1\alpha_2 \tan\phi = \frac{\rho\alpha_1\alpha_2}{\tan\phi} + \alpha_2^2$$

$$\alpha_1^2 - \alpha_2^2 = \frac{\rho\alpha_1\alpha_2}{\tan\phi} - \rho\alpha_1\alpha_2\tan\phi$$

$$\alpha_1^2 - \alpha_2^2 = \rho\alpha_1\alpha_2 \left[\frac{1 - \tan^2\phi}{\tan\phi} \right]$$

$$\frac{(\alpha_1^2 - \alpha_2^2)}{2\rho\alpha_1\alpha_2} = \frac{1}{\tan 2\phi}$$

$$\boxed{\tan 2\phi = \frac{2\rho\alpha_1\alpha_2}{\alpha_1^2 - \alpha_2^2}}$$

~~7.8~~ Q4.

↓ Comes from 7.7

$$\frac{1}{2}A = \begin{bmatrix} \frac{1}{2}\frac{\partial^2\chi}{\partial c^2} & \frac{1}{2}\frac{\partial^2\chi}{\partial m\partial c} \\ \frac{1}{2}\frac{\partial^2\chi}{\partial c\partial m} & \frac{1}{2}\frac{\partial^2\chi}{\partial m^2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sum_i \frac{1}{\alpha_i^2} & \sum_i \frac{x_i}{\alpha_i^2} \\ \sum_i \frac{x_i}{\alpha_i^2} & \sum_i \frac{x_i^2}{\alpha_i^2} \end{bmatrix} = \begin{bmatrix} 700 & 70 \\ 70 & 8.75 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(700)(8.75) - (70)(70)} \begin{bmatrix} 8.75 & -70 \\ -70 & 700 \end{bmatrix} = \begin{bmatrix} 7.14\times10^{-3} & -0.05712 \\ -0.05712 & 0.5712 \end{bmatrix}$$

Correlation matrix:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A\sigma_B} = \frac{C_{AB}}{\sqrt{C_{AA}C_{BB}}} = \begin{bmatrix} \frac{7.14\times10^{-3}}{\sqrt{(7.14\times10^{-3})^2}} & \frac{0.05712}{\sqrt{(7.14\times10^{-3})(0.5712)}} \\ \frac{0.05712}{\sqrt{(7.14\times10^{-3})(0.5712)}} & \frac{0.5712}{\sqrt{(0.5712)^2}} \end{bmatrix}$$

$$\rho = \begin{bmatrix} 1 & -0.8944 \\ -0.8944 & 1 \end{bmatrix}$$

i) $u_{y\text{-int}} = \sqrt{7.14 \times 10^{-3}} = 0.0845$

$u_{grad} = \sqrt{0.5712} = 0.7558$

ii) $y = mx + c$

$y = (431.7)(0.08) + (-0.03)$

$y = 34.506$

In[30]:= Question 1

```
Clear["Global`*"]

noise = {};
values = Table[5 Exp[-0.2 var], {var, 0, 10, 0.2}];
var = Range[0, 10, 0.2];


For[i = 1, i < 52, i++, AppendTo[noise, RandomVariate[NormalDistribution[0, 0.2]]]]
noisydata = values + noise;



idealdata = a Exp[-b var];
chisquare1 = Expand[Sum[(idealdata[[n]] - noisydata[[n]])^2/0.2^2, {n, 1, 51}]];
gradχsq = -Grad[chisquare1, {a, b}];
H = {{D[D[chisquare1, a], a], D[D[chisquare1, a], b]},
   {D[D[chisquare1, b], a], D[D[chisquare1, b], b]}};
diagH = DiagonalMatrix[Diagonal[H]];

arraychisquare1 = {};
arraychisquare2 = {};
steparray = {};

vec = {8, 3};
vec2 = {};
step = 0.01;




For[i = 0, i < 50, i++,
 chisquare1 = Expand[Sum[(idealdata[[n]] - noisydata[[n]])^2/0.2^2, {n, 1, 51}]];
 H = {{D[D[chisquare1, a], a], D[D[chisquare1, a], b]},
   {D[D[chisquare1, b], a], D[D[chisquare1, b], b]}};
 diagH = DiagonalMatrix[Diagonal[H]];
 gradχsq = -Grad[chisquare1, {a, b}];



 a = vec[[1]];
 b = vec[[2]];
 chisquare2 = chisquare1;
 AppendTo[arraychisquare2, chisquare2];


 variation = -Inverse[H + (step diagH)].gradχsq;
```

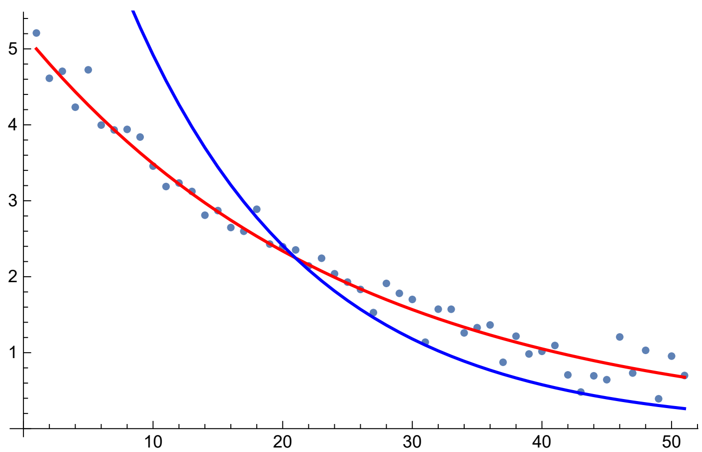
```
 vec = vec + variation;

 Clear[a, b, chisquare1];
 chisquare1 = Expand[Sum[(idealdata[[n]] - noisydata[[n]])^2 / 0.2^2, {n, 1, 51}]];
 a = vec[[1]];
 b = vec[[2]];

 AppendTo[arraychisquare1, chisquare1];
 AppendTo[steparray, step];
 AppendTo[vec2, vec];

 a = vec[[1]];
 b = vec[[2]];

 If[chisquare1 > chisquare2, step = step * 10; vec = vec - variation, step = step/10; ];

 Clear[a, b, chisquare1, chisquare2];
]


vec2;
steparray;
arraychisquare1;
arraychisquare2;
a = vec[[1]]
b = vec[[2]]
fit = Table[a Exp[-b var], {var, 0, 10, 0.2}];

plot1 = ListPlot[noisydata];
plot2 = ListLinePlot[values, PlotStyle → Red];
plot3 = ListLinePlot[fit, PlotStyle → Blue];
Show[plot1, plot2, plot3]
```

Out[30]= Question

Out[53]= 9.3508

Out[54]= 0.356726

Out[59]=



************Question 1. (Alternate) [MATLAB]****************

```
A1=5;
B1=2e-1;
xdata=0:0.1:10;
randstrength=0.1;
ydata=A1*exp(-B1*xdata)+randstrength*randn(1,length(xdata));
figure; plot(xdata,ydata);
kmax=1000;
u=0.1; %uncertainity in each measurement

x=[1;0.02]; %initial value of the parameters

chi2_initial=sum((ydata-x(1)*exp(-x(2)*xdata)).^2/u.^2);
chimatrix=chi2_initial;
xmatrix=x;

slopextemp=zeros(2,1);
M=zeros(2,2); %M is our Hessian matrix
beta=0.00001;

%implementing the Levenberg-Marquart method

k=0;
deltachiabs=5;

while deltachiabs>1e-10
   k=k+1;
%computing the gradient
slopex1=2/u^2*sum(exp(-2*x(2)*xdata).*(x(1)-ydata.*exp(x(2)*xdata)));
slopex2=2/u^2*sum(x(1)*exp(-2*x(2)*xdata).*xdata.*(-x(1)+ydata.*exp(x(2)*xdata)));
slopex=[slopex1;slopex2];

%computing the Hessian matrix
M(1,1)=2/u^2*sum(exp(-2*x(2)*xdata));
M(1,2)=2/u^2*sum(exp(-2*x(2)*xdata).*xdata.*(-2*x(1)+ydata.*exp(x(2)*xdata)));
M(2,1)=M(1,2);
M(2,2)=-2/u^2*sum(x(1)*exp(-2*x(2)*xdata).*xdata.^2.*(-2*x(1)+ydata.*exp(x(2)*xdata)));
Mtemp=(M+beta*diag(diag(M)));
Mtempi=1/(Mtemp(1,1)*Mtemp(2,2)-Mtemp(1,2)*Mtemp(2,2))*[Mtemp(2,2) -Mtemp(1,2); -Mtemp(2,1) Mtemp(1,1)];

xa=x; %current parameters
xb=x-Mtempi*slopex; %proposed paramaters

chia=sum((ydata-xa(1)*exp(-xa(2)*xdata)).^2/u.^2);
chib=sum((ydata-xb(1)*exp(-xb(2)*xdata)).^2/u.^2); %proposed chi^square

deltachitest=chib-chia;
deltachiabs=abs(chib-chia);

if deltachitest >= 0
beta=beta*10;
xmatrix=[xmatrix xa];
```

```
chimatrix=[chimatrix chia];
x=xa;
chi=chia;

else
   beta=beta/10;
   xmatrix=[xmatrix xb];
   chimatrix=[chimatrix chib];
   x=xb;
   chi=chib;
end

fprintf('i=%d, A=%f, B=%f, chi=%.3f dchitest=%e beta=%f \n', ...
k, x(1),x(2),chi, deltachiabs, beta);
end
```

****************QUESTION 5****************

```
angled=0:10:270;
angle=angled*pi/180;
lengthdata=length(angle);
intensity=0.5*cos(0.9*angle+0.6).^2-0.3+0.03*randn(1,lengthdata);
figure; plot(angle,intensity,'o');
u=[0.0005*ones(1,lengthdata)];

x=[1; 2; -1; 0]; %initial guess of the parameters
chi=sum((intensity-(x(1)+x(2)*cos(x(3)*angle+x(4)).^2)).^2/u.^2);
T=1000; %initial temperature

for k=1:1000
   xa=x;
   Ea=sum((intensity-(xa(1)+xa(2)*cos(xa(3)*angle+xa(4)).^2)).^2/u.^2);
   xb=x+0.1*randn(4,1);
   Eb=sum((intensity-(xb(1)+xb(2)*cos(xb(3)*angle+xb(4)).^2)).^2/u.^2);
   DE=Eb-Ea;
   if Eb<Ea
   x=xb;
   else
      R=rand;
      P=exp(-DE/T);
      if P>R
      x=xb;
      else
         x=xa;
      end

   end
   T=T-0.01*T;
fprintf('i=%d, T=%.7f, chi=%f, A=%f, B=%f, C=%f, D=%f,  \n', ...
k, T, Ea, x(1), x(2), x(3), x(4) );

end

figure; plot(angle,intensity,'o');
```

```
hold on;
plot(angle,x(1)+x(2)*cos(x(3)*angle+x(4)).^2);
hold off;
```