Quiz 2
Time: 1 Hour

1. **Sweetness Revisited (10 Points)**

   In quiz 1, you were introduced to a novel property of electrons, "sweetness." Every electron observed being either sweet or bitter, the state of a sweet electron is represented by $|S\rangle$ whereas that of a bitter electron is represented by $|B\rangle$.

   In this quiz, you would study sweetness even further. The sweetness operator, $\hat{A}$, is a Hermitian operator that is defined in the following way:

   $$\hat{A}|S\rangle = |S\rangle,$$

   and

   $$\hat{A}|B\rangle = -|B\rangle.$$

   (a) **(5 Points)** Write $\hat{A}$ as a linear combination of some outer products. Work in the basis $\{|S\rangle, |B\rangle\}$.

   (b) **(5 Points)** Most of the electrons observed were in the states

   $$\frac{1}{\sqrt{2}}|S\rangle + \frac{1}{\sqrt{2}}|B\rangle$$

   and

   $$\frac{1}{\sqrt{2}}|S\rangle - \frac{1}{\sqrt{2}}|B\rangle.$$

   Therefore, we have decided to represent $\hat{A}$ in the basis $\{\frac{1}{\sqrt{2}}|S\rangle+\frac{1}{\sqrt{2}}|B\rangle, \frac{1}{\sqrt{2}}|S\rangle-\frac{1}{\sqrt{2}}|B\rangle\}$. Using this basis, write $\hat{A}$ in the form of a matrix.

2. **Meeting the Anti-Hermitian (5 Points)**

   An anti-electron is positive, an anit-proton is negative, and an anti-Universe is one that has an anti-you taking anti-quiz 2 in anti-10-302.

   No physicist can deny his or her fascination with anti-things, and in this problem, you would encounter what physicists call an anti-Hermitian operator.

   An operator, $\hat{A}$, is said to be anti-Hermitian if

   $$\hat{A}^\dagger = -\hat{A}.$$

Prove that every eigenvalue of an anti-Hermitian operator, $\hat{A}$, is imaginary. Why could an anti-Hermitian matrix not represent an observable?

3. **An Uncertainty (5 Points)**

   Considering any eigenstate of $\hat{S}_z$ for a spin-$\frac{1}{2}$ particle, calculate ΔS_x.

4. **Jump to the Eigenbasis (5 Points)**

   A Hilbert space is spanned by the orthonormal basis $\{|1\rangle, |2\rangle, |3\rangle\}$, and operator $\hat{A} = i|1\rangle\langle 2| - i|2\rangle\langle 1| + |3\rangle\langle 3|$. Find the eigenvalues of $\hat{A}$, and represent it in its eigenbasis.