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# Experiment 2.12B - Non Linear Dynamics with a Magnetic Pendulum

## A little on chaos:-

Non linear Dynamics :- systems that keep changing with time and are nonlinear OR Dynamical systems for which the principle of superposition does not hold

. A small change in a parameter can lead to sudden and dramatic changes in both qualitative and quantitative behavior of a system.

. Highly sensitive to initial conditions. Even deterministic systems can have widely diverging outcomes. Therefore their deterministic nature does not make these system predictable. Deterministic chaos.

"Chaos: When the present determines the future, but the approximate present does not approximately determine the future"

— Edward Lorenz

. Chaotic behavior does not appear without informing us when it is about to come. Generally starts with 'period-doubling bifurcation'

From given equations -

$$\dot{\theta} = \omega$$

$$\frac{ML}{3}\dot{\omega} = -\frac{L}{2}Mg\sin\theta + T_{driver}\sin(\Omega t) - \gamma\omega + \frac{|\theta|}{\theta}\, L\,\frac{\mu_0}{4\pi}\,\frac{m_1 m_2}{r_\theta^2}\cos\left(|\theta| + \tan^{-1}\left(-\left|\frac{h_\theta}{L\sin\theta}\right|\right)\right)$$

$A = \frac{L}{2}Mg = \frac{3}{2}Lg$ $\quad B = \frac{3T_{driver}}{ML^2}$ $\quad C = \gamma \cdot \frac{3}{ML}$

$D = \frac{L\mu_0}{4\pi}\, m_1 m_2 \cdot \frac{3}{ML}$

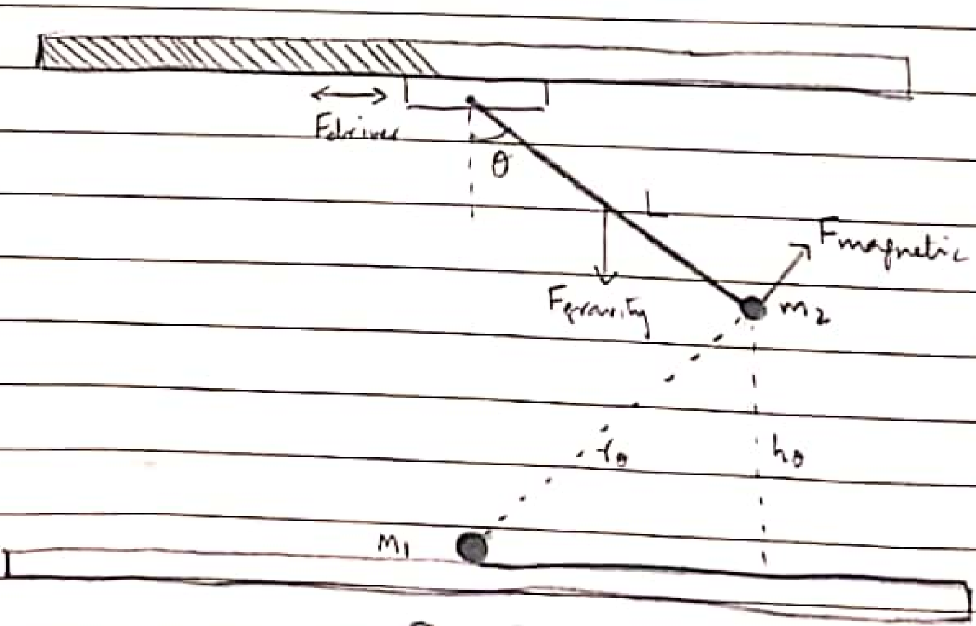
$L = 13\,cm = 13\times10^{-2}\,m$ (This is for our pendulum)

$d = 66.5\times10^{-3}\,m$ (This is when chaos should be observed)

$h_\theta = d + L(1-\cos\theta)$

$r_\theta = \sqrt{(L\sin\theta)^2 + h_\theta^2}$

A schematic of the setup and parameters.



These are values for our pendulum

$M = 5g$ $L = 13\,cm$ $\gamma = 0.001$ (given) $m_1 m_2 = 0.0001$ (given)

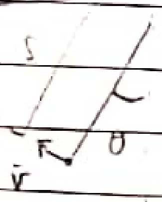
$T_{drive} = 0.001$ (given)

$A = \frac{3Lg}{2}$ $C = \frac{3\gamma}{ML}$

$B = \frac{3T_{drive}}{ML}$ $D = \frac{3\mu_0 m_1 m_2}{4\pi M}$

$\theta$ vs. Time period Plot

Derivation :-



$U = mgs(1-\cos\theta)$

This is the potential energy of the pendulum

Assuming PE to be 0 when pendulum is at equilibrium

$KE = T = \frac{1}{2}mv^2$ $v = s\dot{\theta}$

$$T = \frac{1}{2}ms^2\dot{\theta}^2$$

Since it is an isolated system, the total energy remains a constant.

$$E = T + U.$$

$$E = mgs(1-\cos\theta) + \frac{1}{2}ms^2\dot{\theta}^2$$

let $\theta = \theta_0 \rightarrow$ max angle achieved by pendulum

at $\theta = \theta_0$, $\dot{\theta} = 0$

Energy equation then becomes

$$E = mgs(1-\cos\theta_0)$$

$$mgs(1-\cos\theta) + \frac{1}{2}ms^2\dot\theta^2 = mgs(1-\cos\theta_0)$$

$$g(1-\cos\theta) + \frac{1}{2}s\dot\theta^2 = g(1-\cos\theta_0)$$

$$\dot\theta^2 = \frac{2g}{s}(\cos\theta - \cos\theta_0)$$

differentiating $\quad 2\dot\theta\ddot\theta = -\frac{2g}{s}\sin\theta\dot\theta$

$$\ddot\theta + \frac{g}{s}\sin\theta = 0.$$

Using small angle approximation on

$$\frac{d^2\theta}{dt^2} + \frac{g}{s}\sin\theta = 0.$$

This becomes a regular pendulum equation

$\theta_0 \Rightarrow 80°$

$\dot{\theta} = \omega.$

$$\frac{ML^2}{3}\dot{\omega} = -\frac{L}{2} Mg\sin\theta + T_{deliver}\sin\phi - \gamma\omega + \frac{|\theta|}{\theta}\, L\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r_\theta^2} \times \cos\left(|\theta| + \tan^{-1}\left(-|L\theta/L\sin\theta|\right)\right)$$

$A = \frac{3g}{2L}$ ; $B = \frac{3T_{deliver}}{ML^2}$

$C = \gamma$ $D = \frac{3}{ML}\frac{\mu_0}{4\pi} m_1 m_2$

For a general solution we will have to solve these equations exactly

$$\frac{d\theta}{dt} = \sqrt{\frac{2g}{s}}\sqrt{\cos\theta - \cos\theta_0}$$

T is the time taken to go from $\theta_0$ to $-\theta_0$ and then back to $\theta_0$.

$$dt = \sqrt{\frac{s}{2g}}\frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

$$T = 2\sqrt{\frac{s}{2g}}\int_{-\theta_0}^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

Since $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$$T = \sqrt{\frac{s}{g}} \int_{-\theta_0}^{\theta} \frac{d\theta}{\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}}$$

$$K = \sin\theta_0/2 \qquad \sin\frac{\theta}{2} = K\sin\phi$$

$$d\theta = \frac{2K\cos\phi\, d\phi}{\cos(\theta/2)} = \frac{2K\cos\phi\, d\phi}{\sqrt{1-K^2\sin^2\phi}}$$

$$\therefore \quad T = \sqrt{\frac{s}{g}} \int_{-\theta_0}^{\theta_0} \frac{2k\cos\phi\, d\phi}{\sqrt{1-K^2\sin^2\phi}} \times \frac{1}{\sqrt{K^2 - K^2\sin^2\phi}}$$

$$= 2\sqrt{\frac{s}{g}} \int_{-\theta_0}^{\theta_0} \frac{d\phi}{\sqrt{1-K^2\sin^2\phi}}$$

$$\sin\frac{\theta_0}{2} = \sin\frac{\theta_0}{2}\sin\phi \longrightarrow \phi = \pi/2$$

$$\sin\left(-\frac{\theta_0}{2}\right) = \sin\frac{\theta_0}{2}\sin\phi \qquad \phi = -\pi/2$$

$$T = 2\sqrt{\frac{s}{g}}\int_{-\pi/2}^{\pi/2}\frac{d\phi}{\sqrt{1-K^2\sin^2\phi}}$$

$$T = 4\sqrt{\frac{s}{g}}\int_0^{\pi/2}\frac{d\phi}{\sqrt{1-K^2\sin^2\phi}}$$

if $K(K) = \int_0^{\pi/2}\frac{d\phi}{\sqrt{1-K^2\sin^2\theta}}$

Then $T(\theta_0) = 4\sqrt{\frac{s}{g}}\;K(K)$ $\quad K = \sin(\theta_0/2)$

and $s$ = length of pendulum

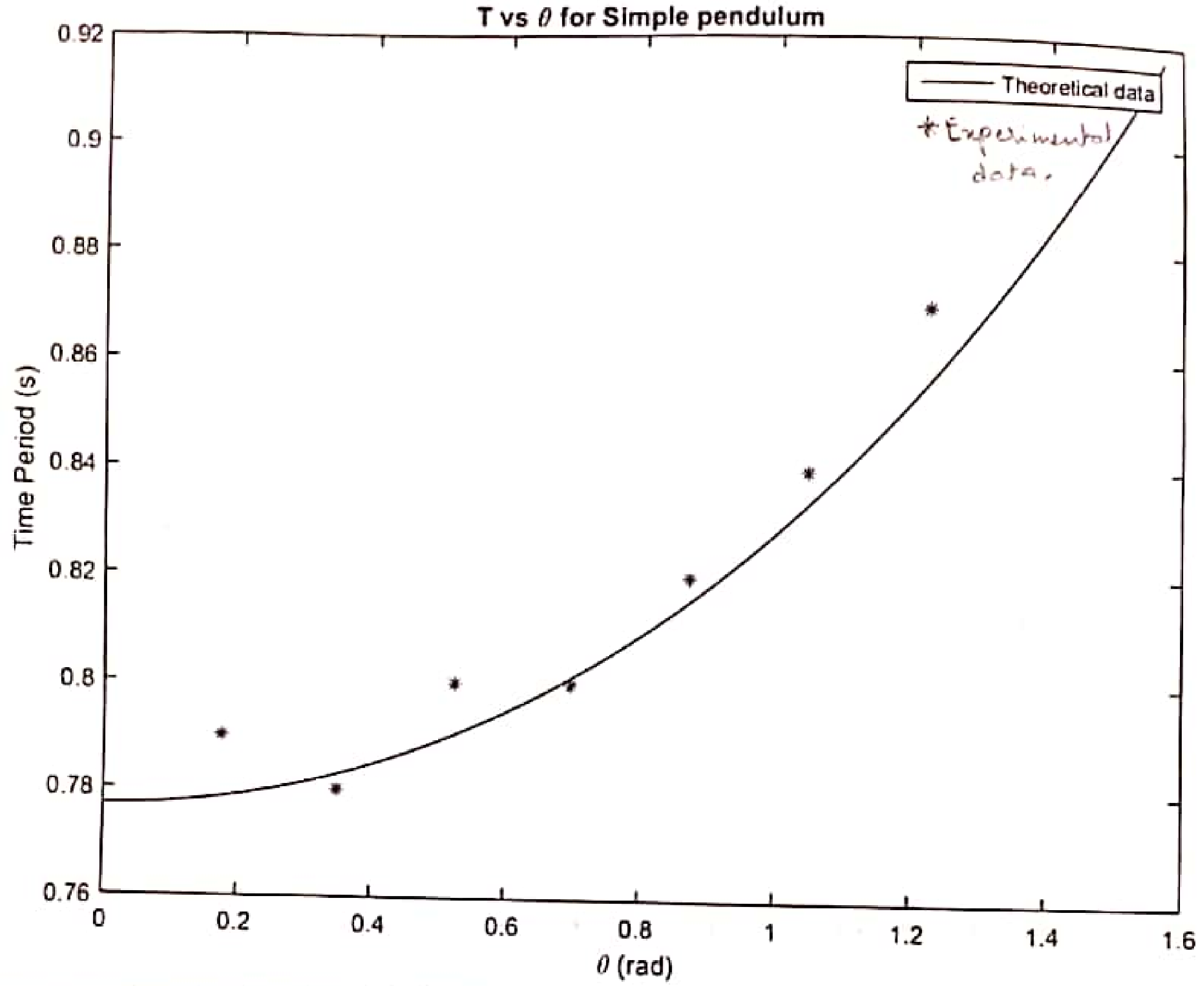



Figure shows a plot of time period $T$ as a function of mean angle $\theta$

- For small angles, $\theta$, $T$ is roughly constant as can be seen on the graph.
- For larger angles $\theta$, $T$ increases dramatically.
- Theoretical and experimental data agree. Length of pendulum found to be 15 cm. This value has been used to generate the theoretical plots

Tasks

1. Simulation (code refinement, data collection)
2. ~~Resonance~~ Resonance frequency plot.
3. Video of experiment
4. Sample results.
5. With magnet data
6. Estimate parameters A B C E of our own setup and then repeat 1, 4 and 5.
7. Peakfinder with uncertainties

2 Resonance Frequency

driving frequency → 3 : max

within sensitive region go in steps of 0.5

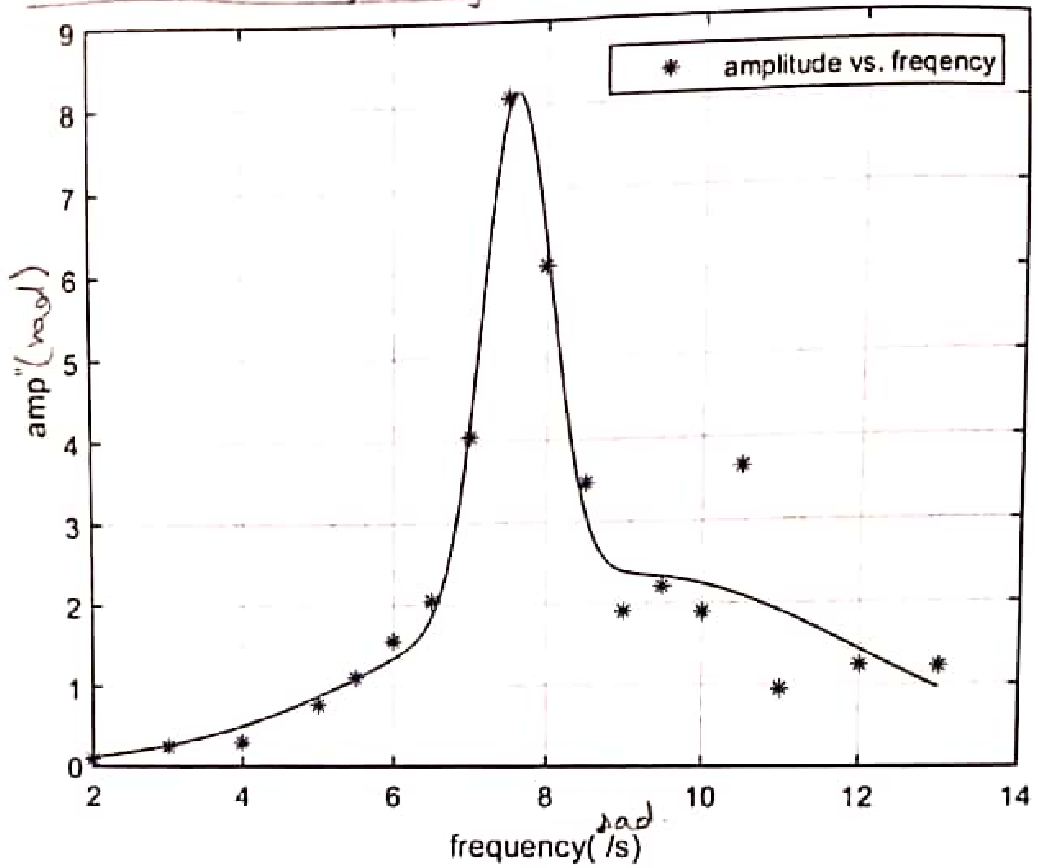
Load dat files into matlab.

This is to find an approximate resonant frequency.

uncert in $\theta$ : ±0.1

- Mag Pendulum shortcomings.
  - Disconnects after recording.
  - Does not show the graph vs time while recording
  - You have to press clear twice before the graph vs time reappear.
  - Does not start recording till after 30s have passed

Resonance Frequency Plot



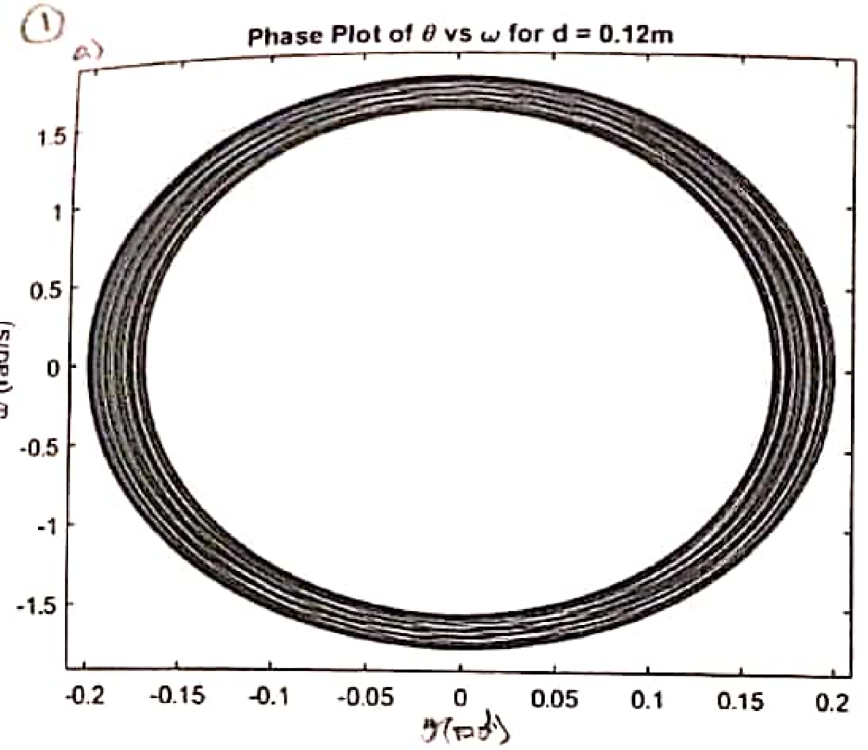

To find the resonance frequency, the driver frequency was raised periodically (from 2 rad $s^{-1}$ till 14 rad $s^{-1}$) and data was collected for 60 seconds of the pendulum's motion. This was then plugged into the code Resonance-freq-plot to get the above graph. The resonant frequency was found to be 7.61 rad $s^{-1}$.

Data was collected in steps of 0.5 rad $s^{-1}$ in sensitive regions and 1 rad $s^{-1}$ in non-sensitive region.

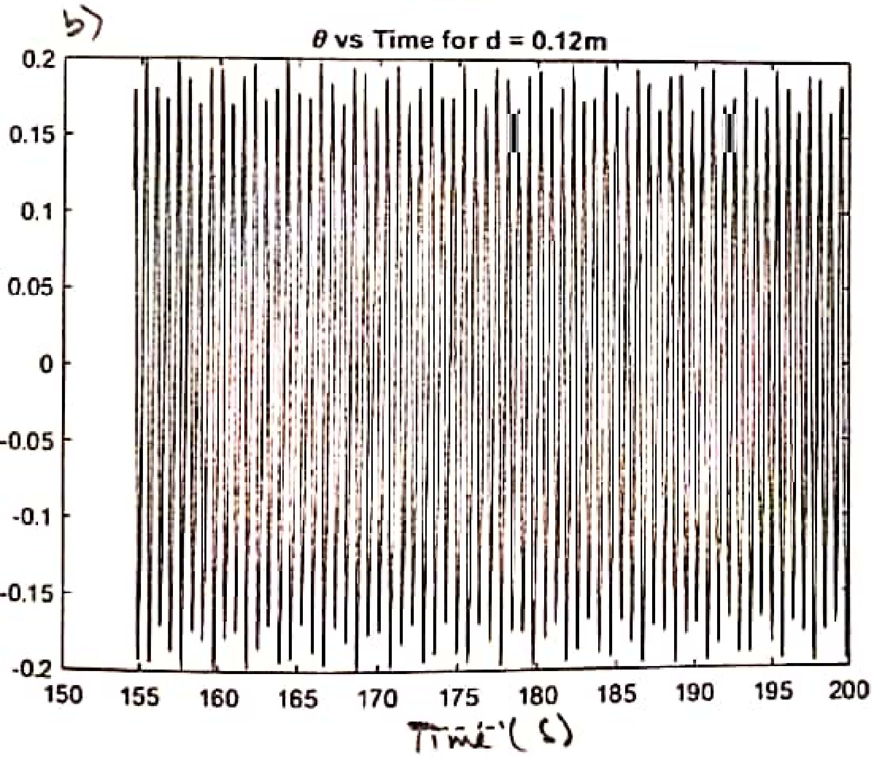
## Simulation Result 1

(1)

a)




b)




d = 120mm or 0.12m

a) Shows the phase plot and periodic motion of the pendulum. At this distance the magnets cannot interact with each other. It is as though no magnet is present.

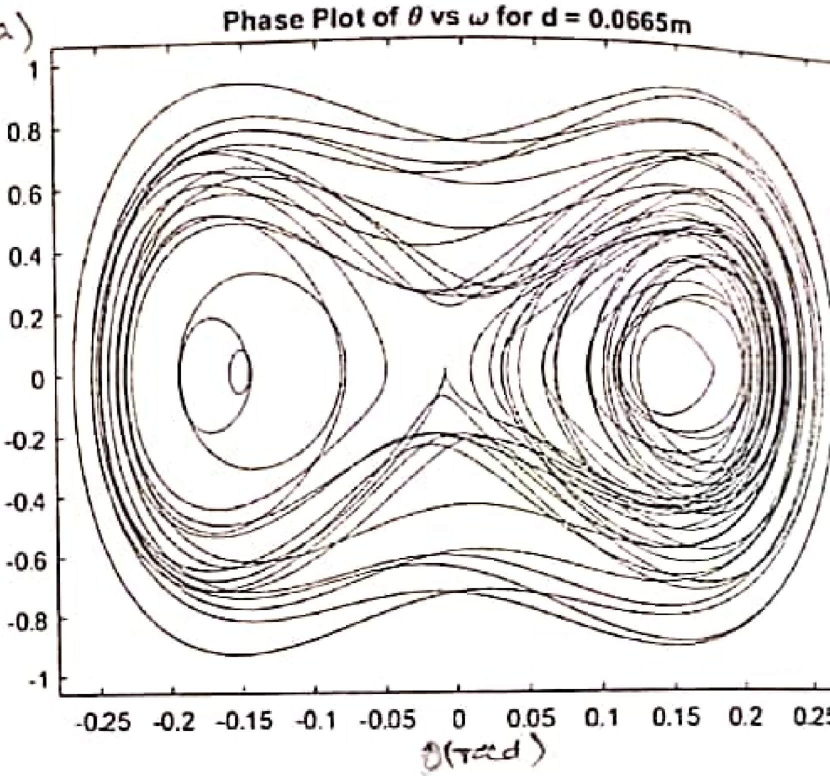
b) Time vs amplitude (θ) plot. Periodic motion can be seen.

Simulation Results (1)

d = 66.5mm or 0.0665m

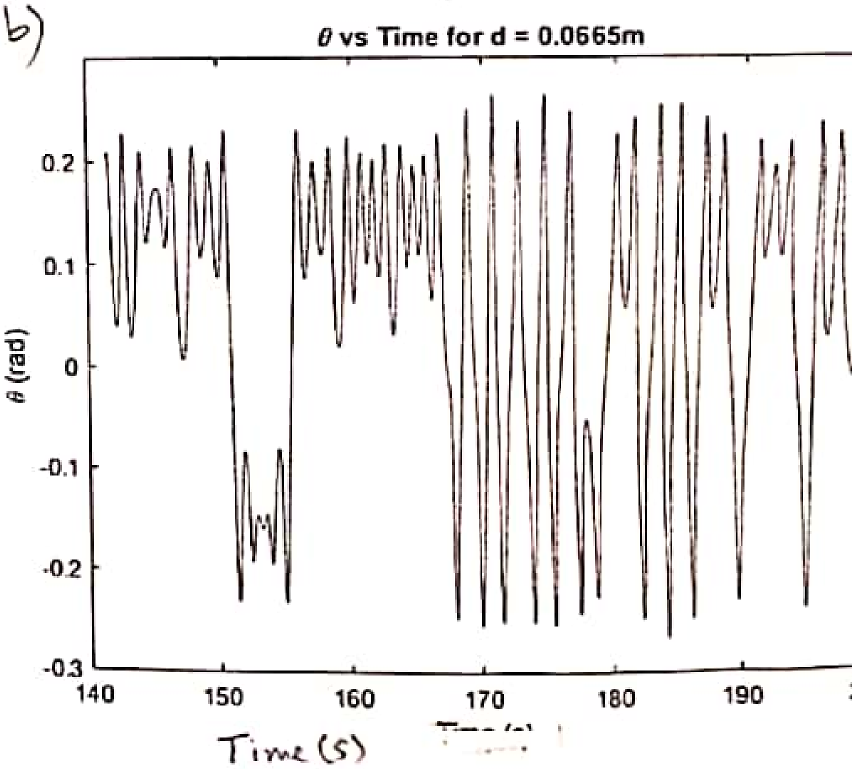
a) The phase plot shows chaotic motion. Period doubling has also occurred.

a)




b) Pendulum has lost all semblance of periodic motion and occasionally oscillates on only one side of the magnet as can be seen by this graph. This is due to the repulsive force of the magnet.

b)

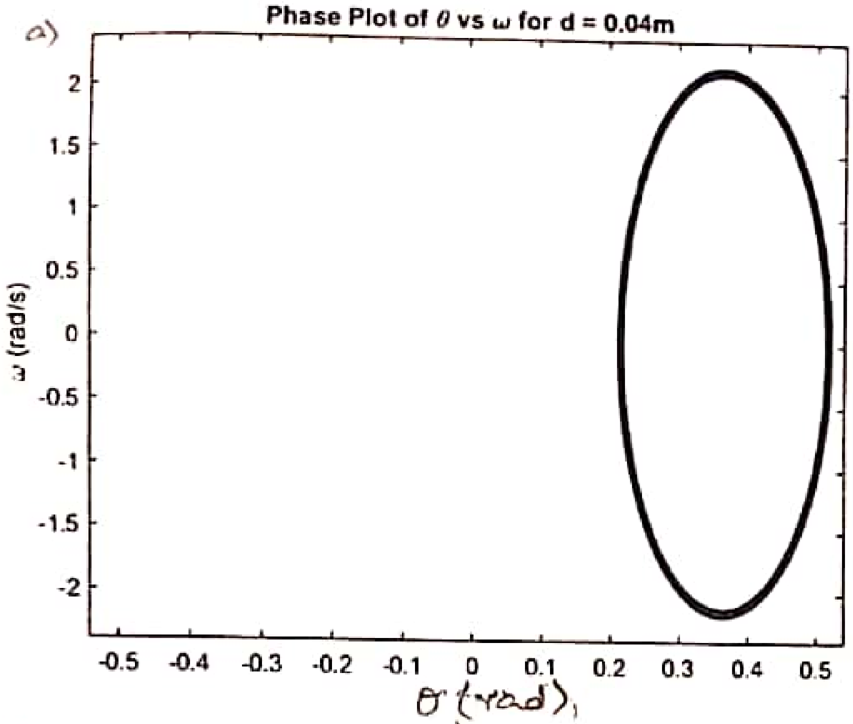



There were problems with the simulation, so parameter values were varied until the graphs attached could be observed. New parameters were: A = 110, B = 0.65 ; C = 0.001 ; E = 0.2

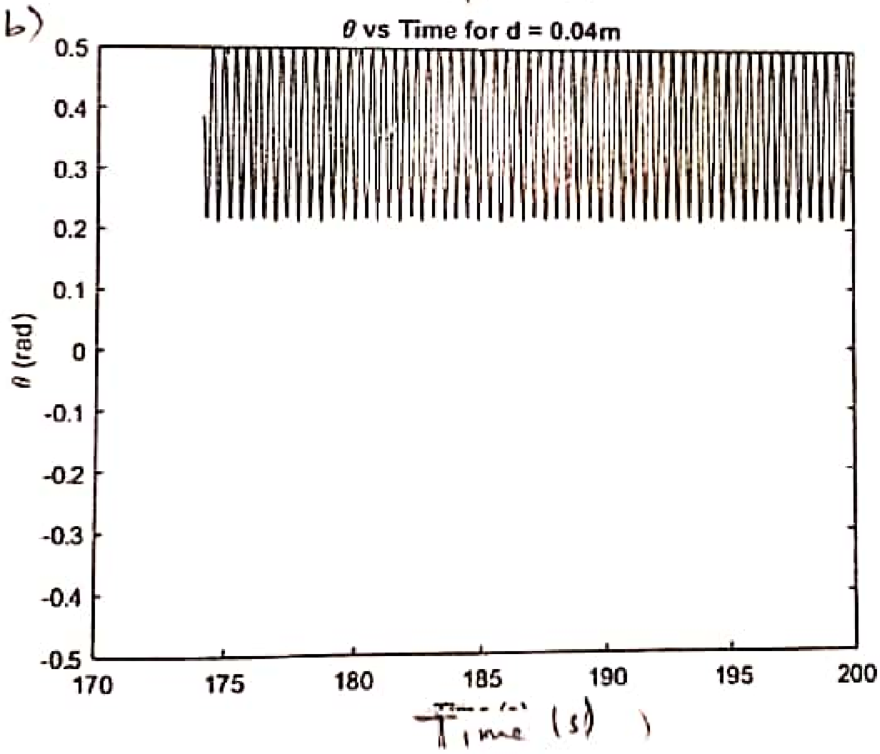
$L = \frac{3g}{2A}$

(3)

a)




b)




Simulation Result (3)

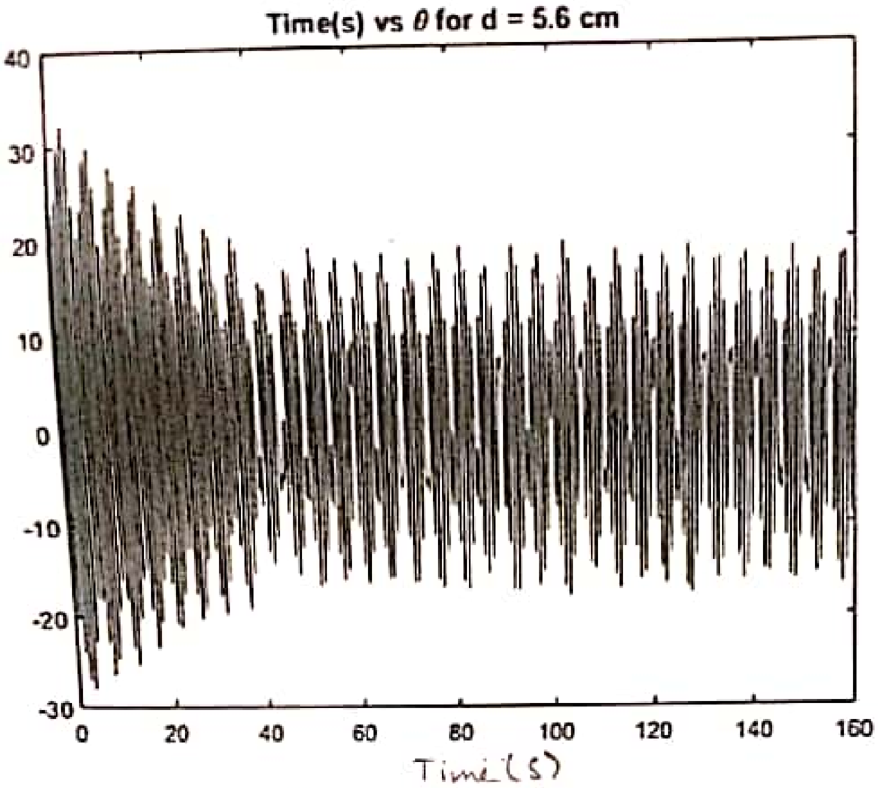
d = 0.04 m.

a) Pendulum is now only oscillating on one side of the magnet as the separation between the two magnets is too great at this d to let the pendulum oscillate freely.

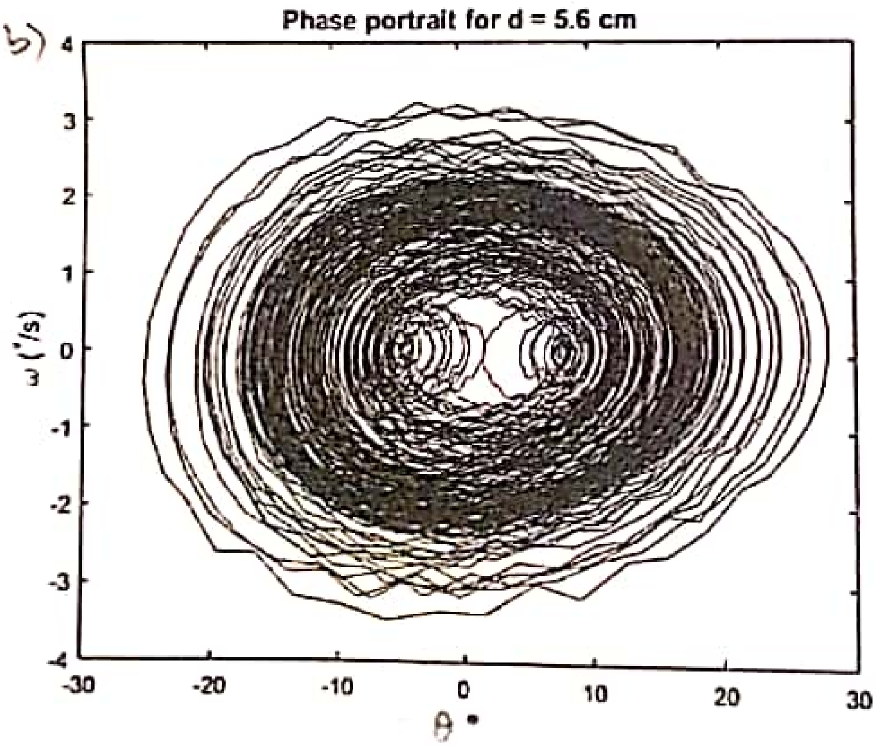
b) Time vs amplitude plot show oscillation on only one side.

1

a)




b)




Experimental Results and Analysis (with magnet)

Due to constraints in the apparatus we could not go above d = 5.6 cm. Therefore, the plots begin with d = 5.6 cm.

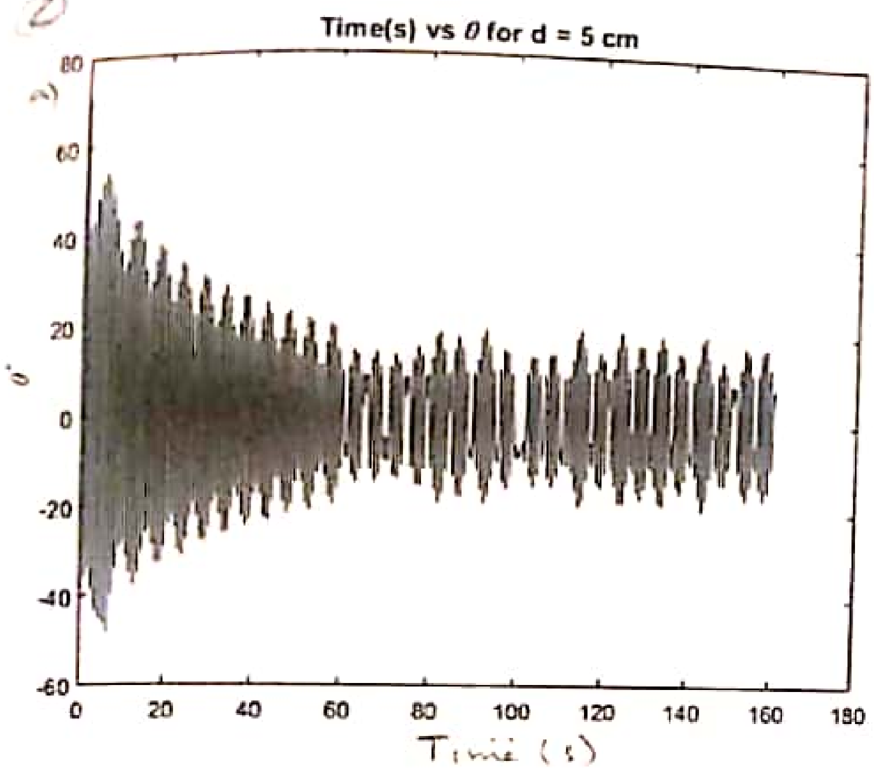
Period doubling has occurred here as can be seen.

Initial conditions for experiment were
$\theta = 70°$
$\Omega = 6.1\ rad\,s^{-1}$.

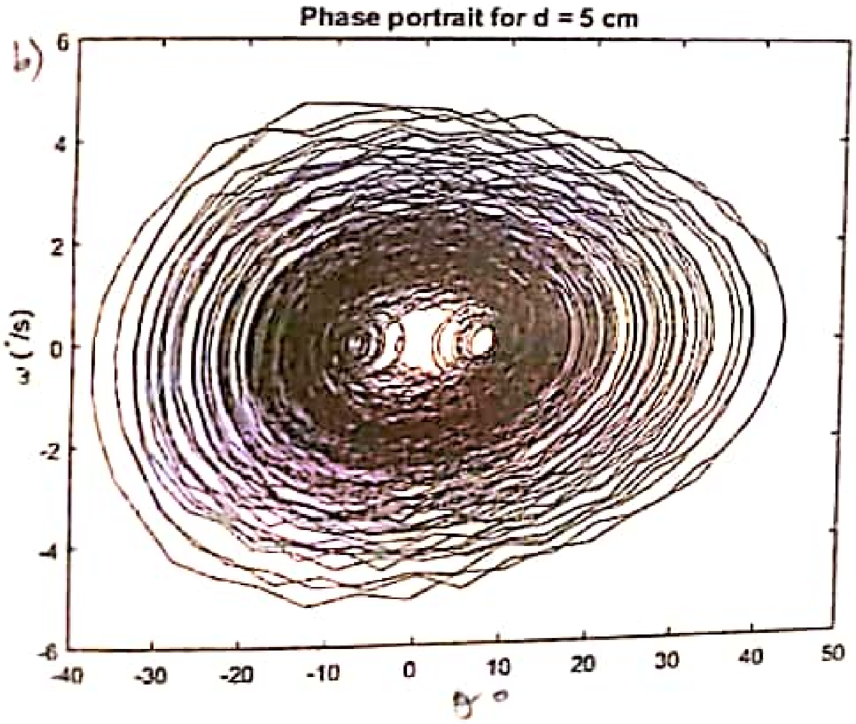
Data has been collected for d = 5.6 cm; 5 cm; 4.5 cm, 3.7 cm, 2.1 cm

②

a)




b)

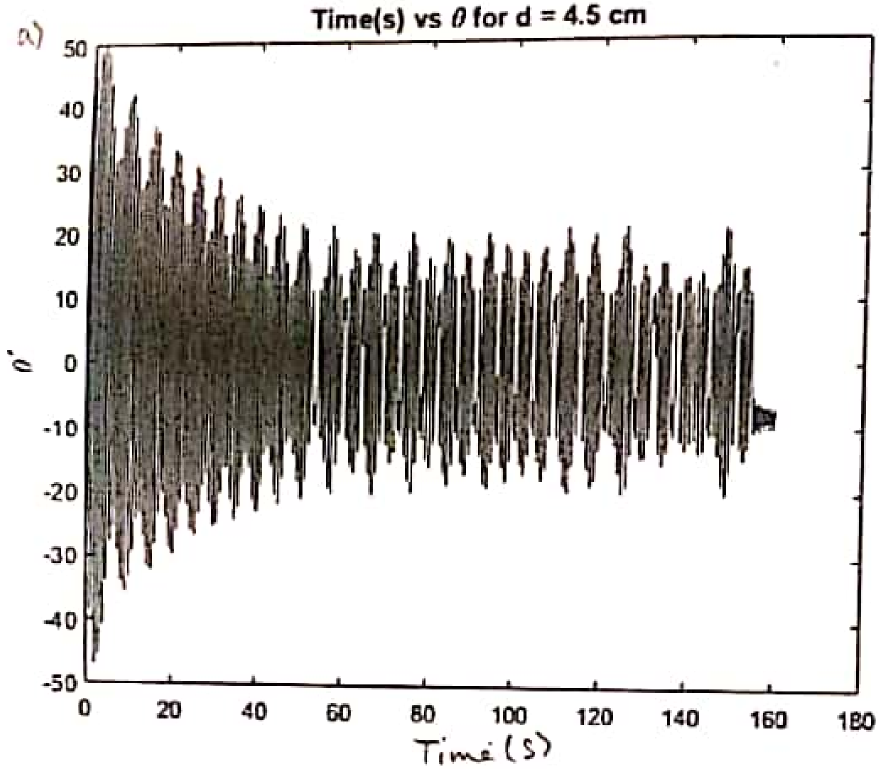



2 a) In this graph, compared to 1c), we can see that the closer the pendulum gets to the magnet, the greater the repulsive force and the greater the damping.

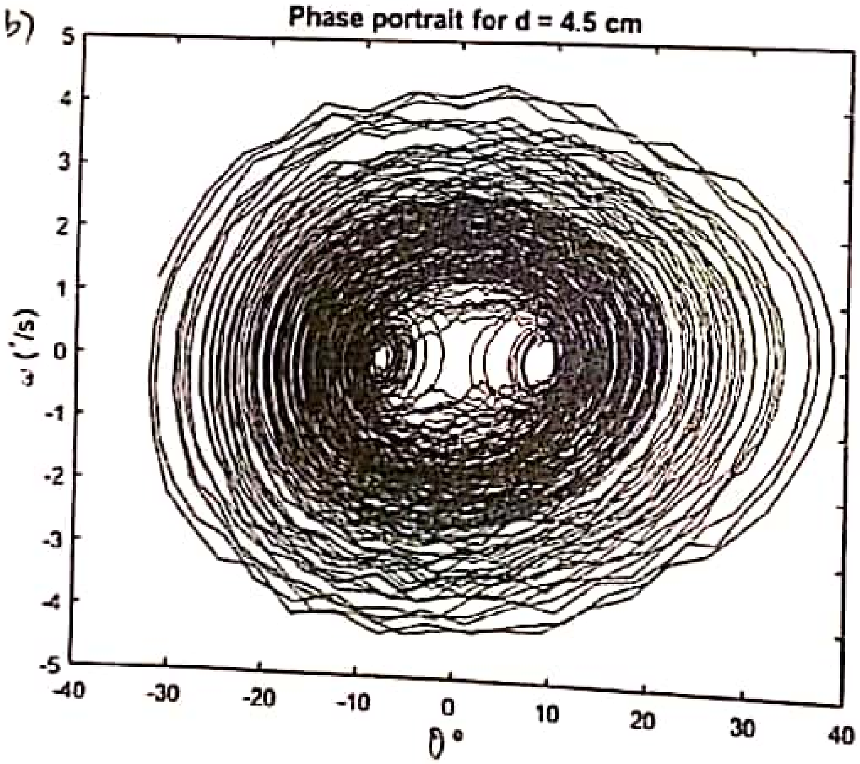
As d is being decreased, the likelihood of the pendulum getting stuck on one side of the magnet becomes greater.

(3)

a)




b)




(3) and (4) show plots for d: 4.5 cm and d: 3.1 cm respectively.

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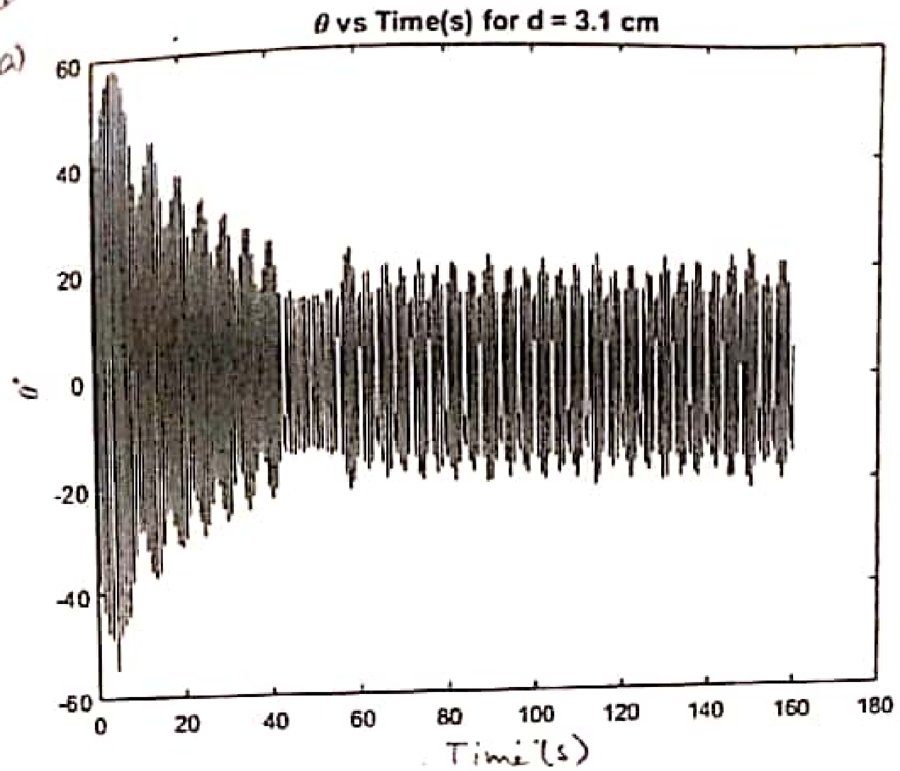
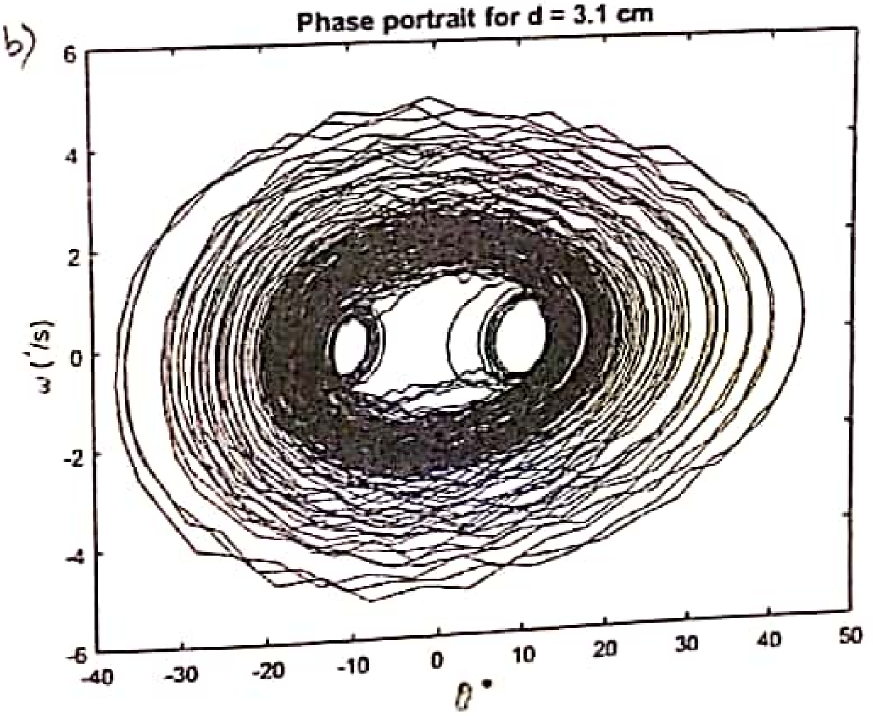
4

a)

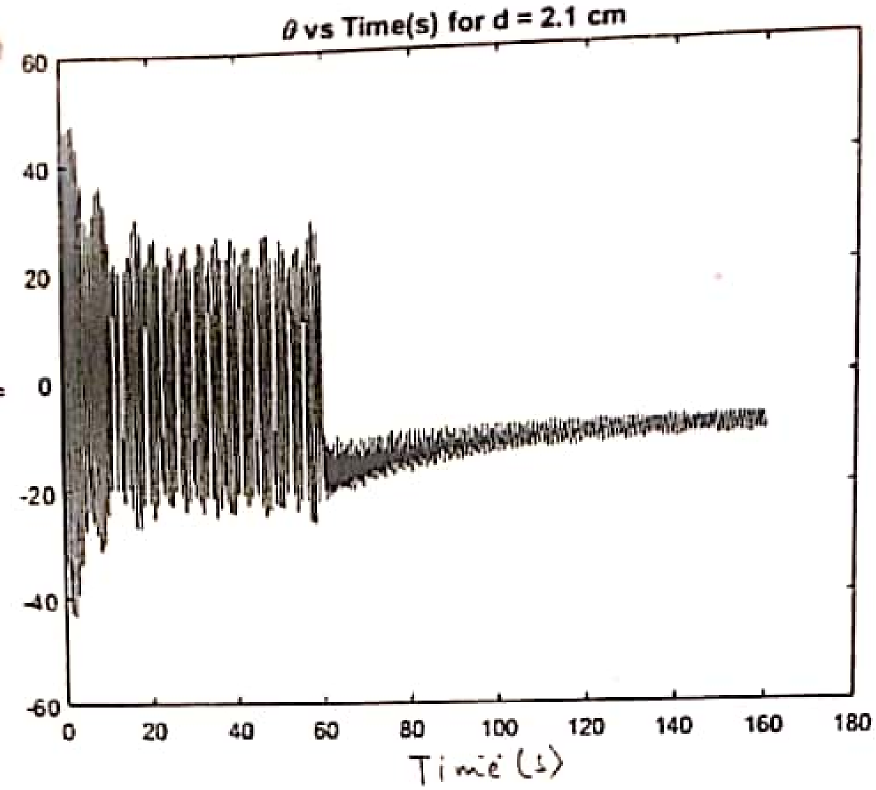

θ vs Time(s) for d = 3.1 cm
60
40
20
0
-20
-40
-60
0
20
40
60
80
100
120
140
160
180
θ°
Time (s)

b)

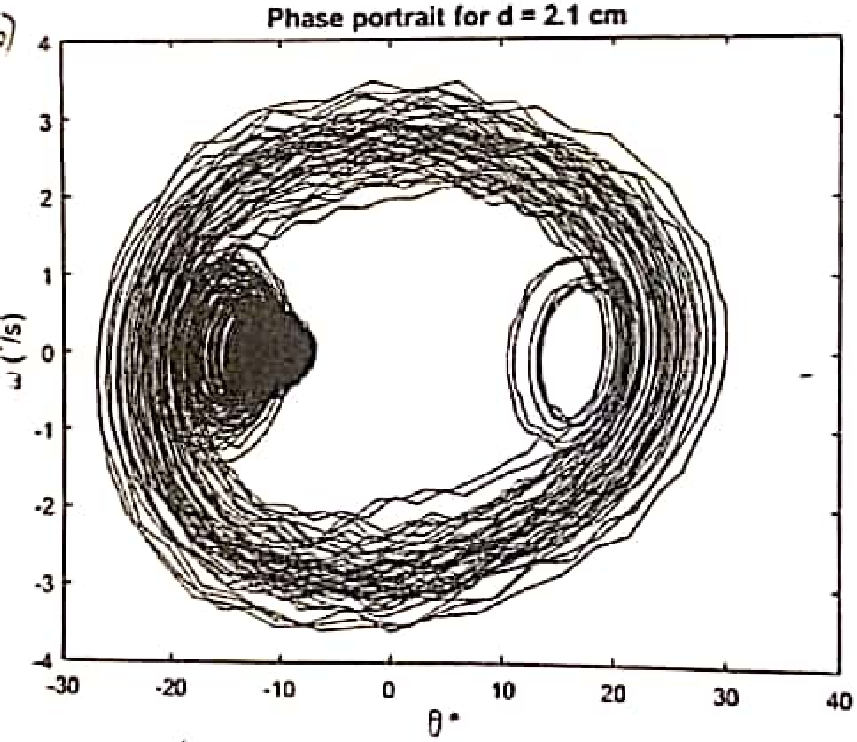

Phase portrait for d = 3.1 cm
6
4
2
0
-2
-4
-6
-40
-30
-20
-10
0
10
20
30
40
50
ω (°/s)
θ°

5

a)




b)




5) As d becomes very small eg. d: 2.1cm, the separatrix force has increased to a point where the pendulum gets permanently stuck on one side of the magnet and continues to oscillate there. This can be seen very clearly in 5a as all the oscillations are below 0° on the graph.

All the experimental results agree with the simulated plots. Any discrepancies that occurs are due to the difference in the parameters of our experimental setup