Analyzing reciprocating motion: The geometry of physics

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1 Introduction

Rotating motion is converted into translational motion (or vice-versa) in numerous mechanical devices. The most commonplace is petrol engines. Figure (1) shows a schematic of a 4-stroke petrol engine. The translational kinetic energy of the piston is converted into the rotational kinetic energy of the crankshaft through the connecting rod [1]. The piston compresses the air and fuel mixture where a spark is generated igniting the fuel. The mixtures expands and pushes the piston back such that it can do work on the crankshaft. The cycle then repeats itself, producing a steady flow of work in the form of rotational kinetic energy of crankshaft. The mechanical mechanism of the engine is exactly identical to our experimental setting, however in our setting the conversion of energy is in the other direction.

This experiment is derived from reference [2]. It was originally inspired from the theoretical ideas presented in Paul J. Nahin's *"In Praise of Simple Physics"* [3]. In this experiment we analyze the motion of a similar reciprocating mechanical system made by a simple rotating wheel and a metal shaft. The shaft moves forward as the wheel rotates, and it thus depicts the same dynamical system that is used in the petrol engine in Figure (1). We will also investigate how the length of various components effects the rotatory motion and how the geometry of the system is apparent in the visual representation of the variables of motion of the system. In fact, the observed data in this experiment can only be described by a careful analysis of geometrical structure of the problem.

KEYWORDS Reciprocating Motion · Angular Frequency · Kinetic Energy · Engines

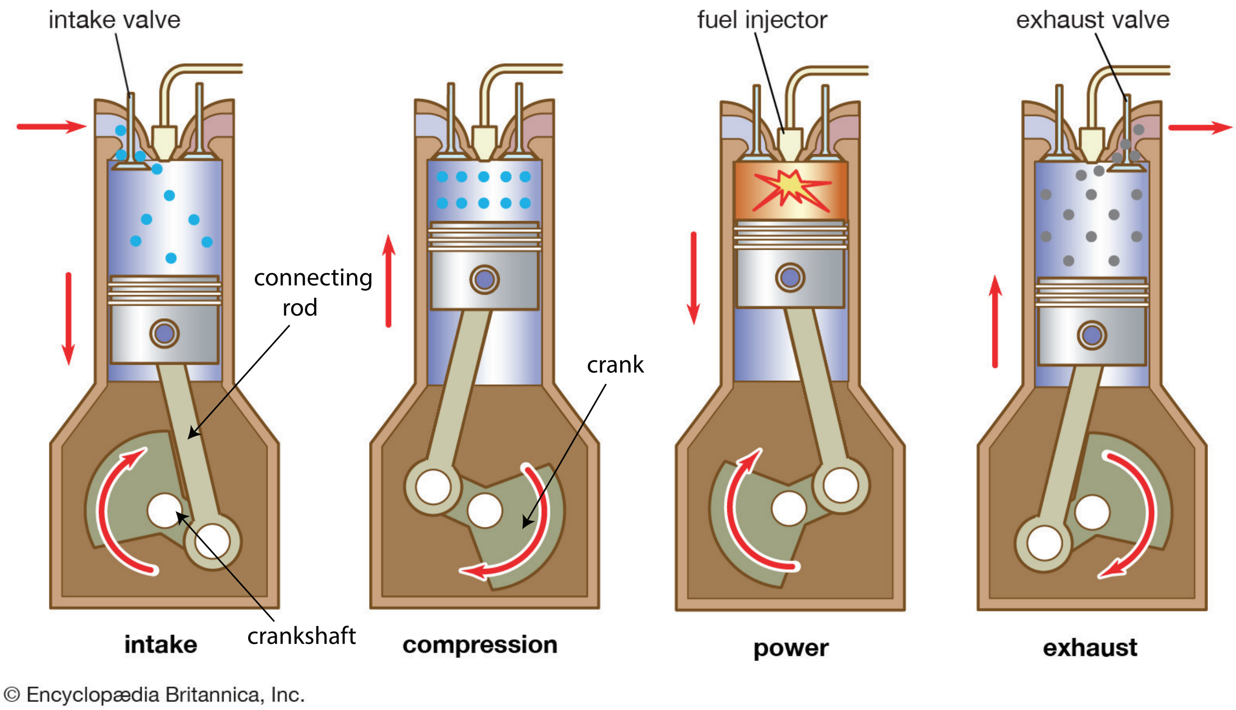

Figure 1: A basic 4-stroke engine cycle, the fuel intake happens as the piston slides down, it is then compressed as the piston moves back up again. The compression is followed by an ignition, the fuel ignites, forcing the piston to go back and the piston therefore does work on the crankshaft.

2 Objectives

In this experiment, we will:

1. observe conversion of rotatory motion into transnational motion,
2. obtain the displacement, velocity and acceleration plots of our system,
3. match our experimental observations to theoretical prediction of displacement, velocity and acceleration for our experimental set up,
4. practice curve fitting to more complicated models and finally,
5. observe how the geometry of the system effects the underlying dynamics of the system.

3 Theory

Figure (2) shows a schematic representation of our experimental apparatus while Figure (3) shows a photograph. A wheel is connected to a slider through the connecting rod. When the wheel rotates, the connecting rod converts its rotational kinetic energy into the translational kinetic energy of the slider. The diagram also shows the variables of interest for our experiment. Herein L is the length of the connecting rod, R is the radius of rotating wheel, x is the distance between the screen and an ultrasonic sensor which records the translational motion

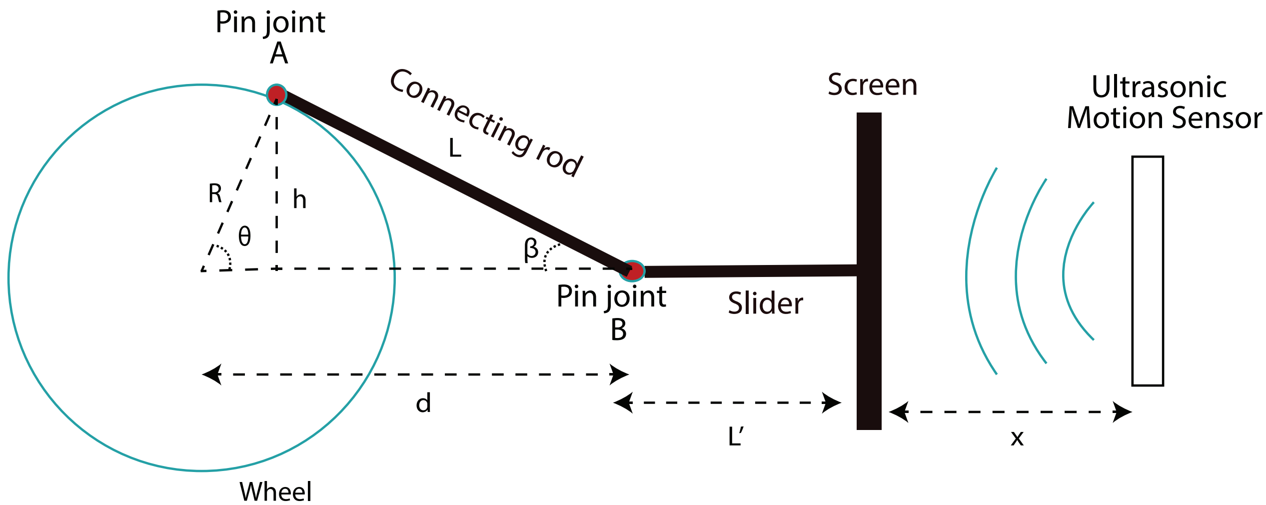

Figure 2: Schematic of experimental set up. The slider and screen are connected to the wheel through the connecting rod at points A and B. The screen moves back and forth as the wheel rotates, and the motion is detected by the ultrasonic sensor.

of the slider (shaft).

As the wheel rotates, the screen moves forward and backward changing x. We can thus use x to analyze the system. The true target of this experiment is to use the available geometry to predict x, the velocity $v = dx/dt$ and acceleration $a = dv/dt$ for a given R, L and ω.

We will find an expression for x using geometrical arguments and then see if the prediction matches the experimental results. As the wheel rotates, the angle θ changes causing x to change too. For the time being, our goal is to be able to write x as a function of θ, in terms of R and L. This expression will tell us how x changes as θ changes, establishing the link that we want to make between the motion of the wheel and the slider.

First, we need find an expression of d in terms of θ which we will use in our expression for x.

Exercise: Find d in terms of θ and β and show that:

$$R\sin\theta = L\sin\beta \quad \text{and} \quad d = R\cos\theta + L\cos\beta. \tag{1}$$

Solving these two equations simultaneously yields the expression of d as a function of θ

$$\frac{d}{L} = \frac{R}{L}\cos\theta + \sqrt{1 - \left(\frac{R}{L}\sin\theta\right)^2}. \tag{2}$$

Notice that the distance from the center of the wheel to the motion sensor is constant. We will call this constant C, then from the geometry we have

$$d + L' + x = C \implies x = K - d \tag{3}$$

where the new constant K is the sum of L' and constant C and is therefore a constant. We can now express x as a function of θ by substituting equation (2) into equation (3).

Exercise: Use equations (2) and (3) to show that:

$$x = -R\cos\theta - L\sqrt{1 - \left(\frac{R}{L}\sin\theta\right)^2} + K \tag{4}$$

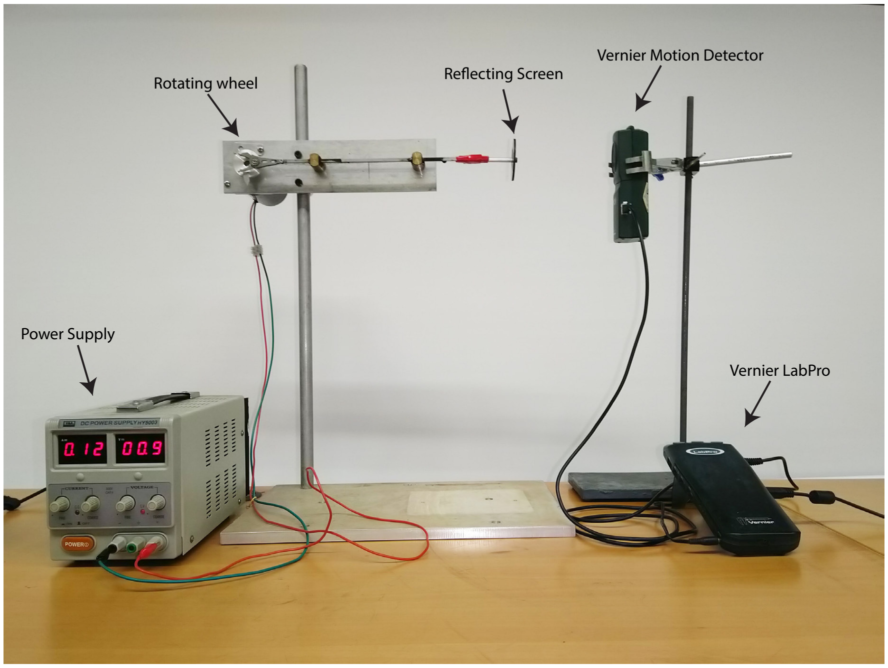

Figure 3: Picture of the provided experimental apparatus.

To analyse the system fully, we will also need to calculate the velocity and acceleration of the screen. The equations of velocity and acceleration of screen can be found by differentiating equation (2) with respect to time.

Exercise: By defining ω as

$$\omega = \frac{d\theta}{dt} \tag{5}$$

show that the velocity and acceleration can be written as

$$v = R\omega \sin\theta + \frac{\frac{R^2}{L}\omega \sin 2\theta}{2\sqrt{1-\left(\frac{R}{L}\sin\theta\right)^2}}, \quad \text{and} \tag{6}$$

$$a = R\omega^2 \cos\theta + \frac{4\left(1-\left(\frac{R}{L}\sin\theta\right)^2\right)\left(\frac{R^2}{L}\omega^2\cos 2\theta\right) - \left(\frac{R^4}{L^3}\omega^2\left(\sin 2\theta\right)^2\right)}{4\left(1-\left(\frac{R}{L}\sin\theta\right)^2\right)^{3/2}}. \tag{7}$$

4 The Experiment

We will divide the experiment into five parts and do all of them one by one. These parts will be followed in a chronological order.

1. We will first measure the lengths R and L. These values will be used later as fit parameters for our data.

2. We will also need to get familiar with the equipment that we will be using in our experiment, that will be done in Section (4.2).

3. In Section (4.3), we will set up our experiment.

4. We will discuss in Section (4.4) about optimizing the apparatus so that we are able to gather the best possible results.

5. Finally, you will perform the experiment to acquire the data and analyze.

4.1 Measuring R and L

	1	2	3	4	5	μ	σ_{data}	σ_μ
L (mm)								
R (mm)								

Table 1: A sample table format for recording values of R and L

Use a vernier caliper to measure the radius R of the disk and length L of the connecting rod and fill up Table (4.1). We will measure each variable five times and record their values in the table.The average of these five values is represented by μ while σ_{data} represents the standard deviation of our data. The standard deviation of the mean is σ_μ.

4.2 Vernier's data acquisition system

To detect the motion of screen, we will use Vernier Motion Detector [4]. This system uses ultrasound waves to detect the motion of a moving body. It sends ultrasonic sound waves to the moving body and detects the reflected waves. The time delay between emitted and reflected waves is used to measure the distance to the object.

The motion sensor uses a data acquisition device to send the data to a computer. The data acquisition devices processes raw data from the motion sensor and converts it into digitally presentable data type. For our experiment, we will be using Vernier's LabPro data acquisition device [4]. LabPro is accompanied by LoggerPro, a graphical user interface software, which lets us plot and interpret the data in computer. In fact, LoggerPro will also let us analyze the acquired data in more detail.

4.3 Setting up the experiment

Follow the following steps to set up the experiment.

1. Connect LabPro with the motion sensor using the digital sensor cable through the `DIG` port in Vernier LabPro.

2. Switch on the power supply, and increase the voltage through `COARSE` knob. Adjust this voltage by your own choice between 15 to 25 V. Make sure the power supply does not go above 25 V. This voltage dictates the angular frequency ω of the wheel.

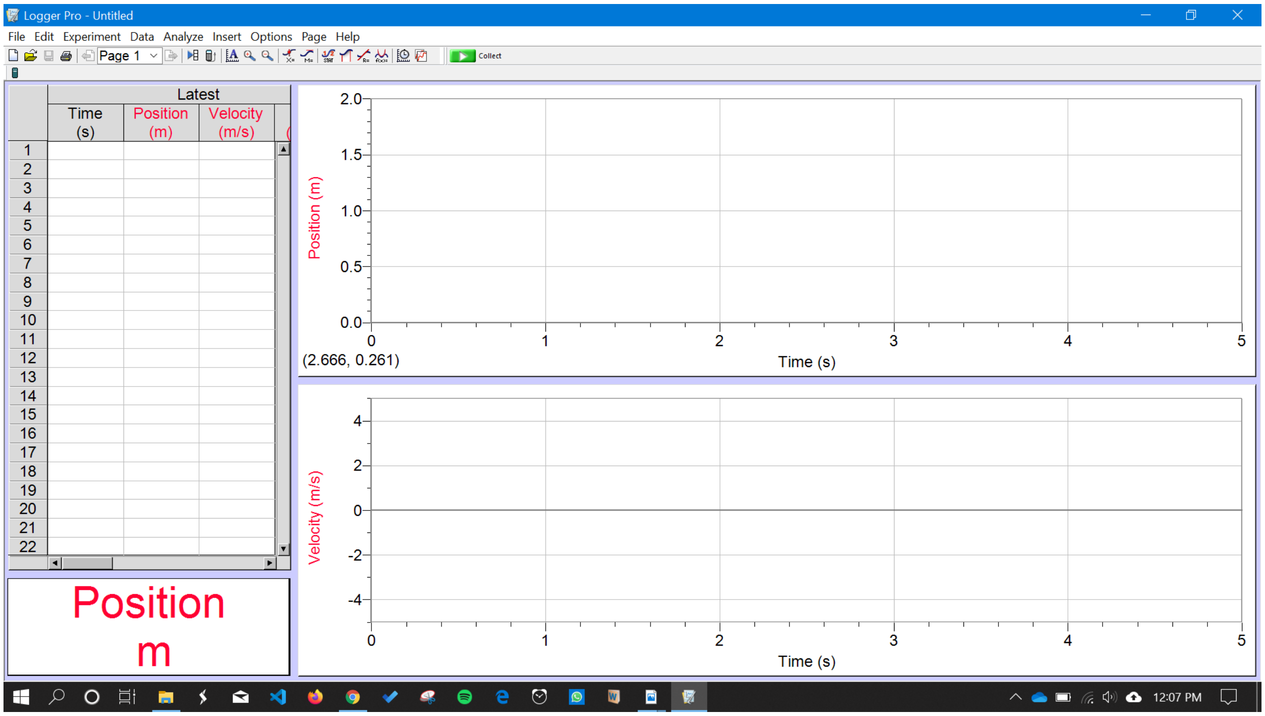

Figure 4: LoggerPro Graphical User Interface. The values of time, position and velocities are visible in the top left columns. The right side shows graphs of those variable.

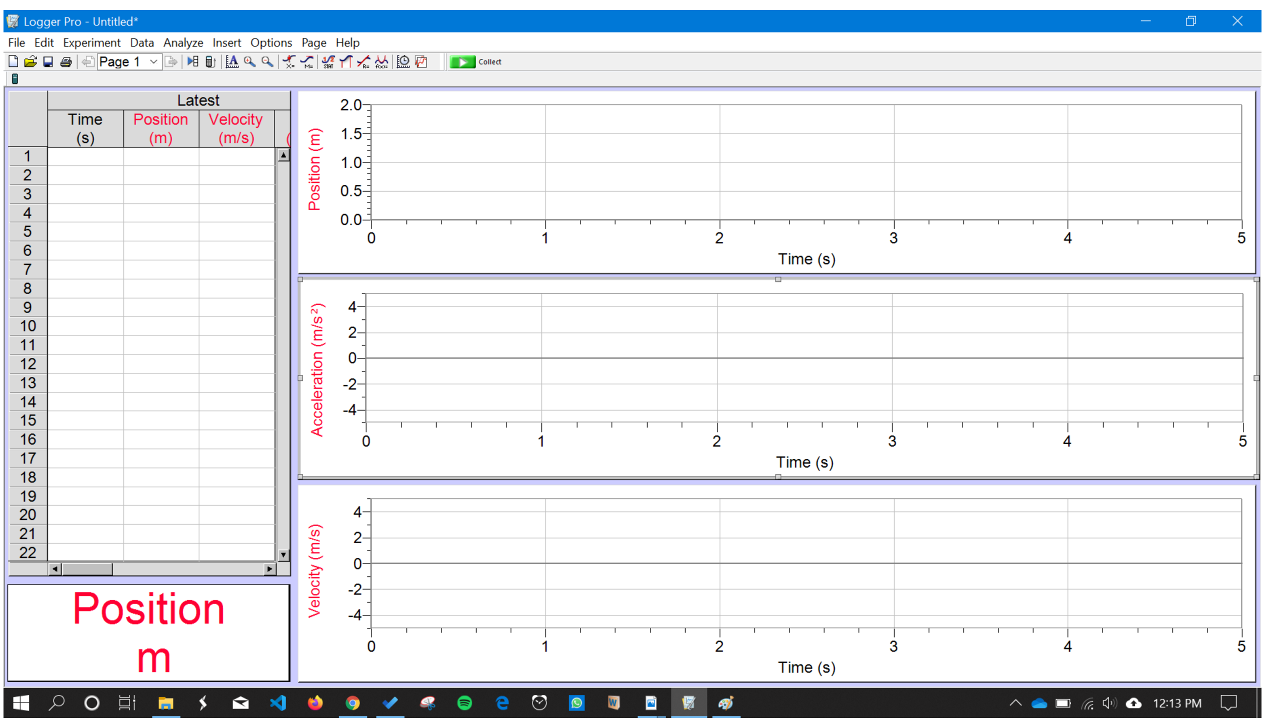

Figure 5: The graphs are adjusted to fit the frame, the graphs can float so you can arrange them anyway you want.

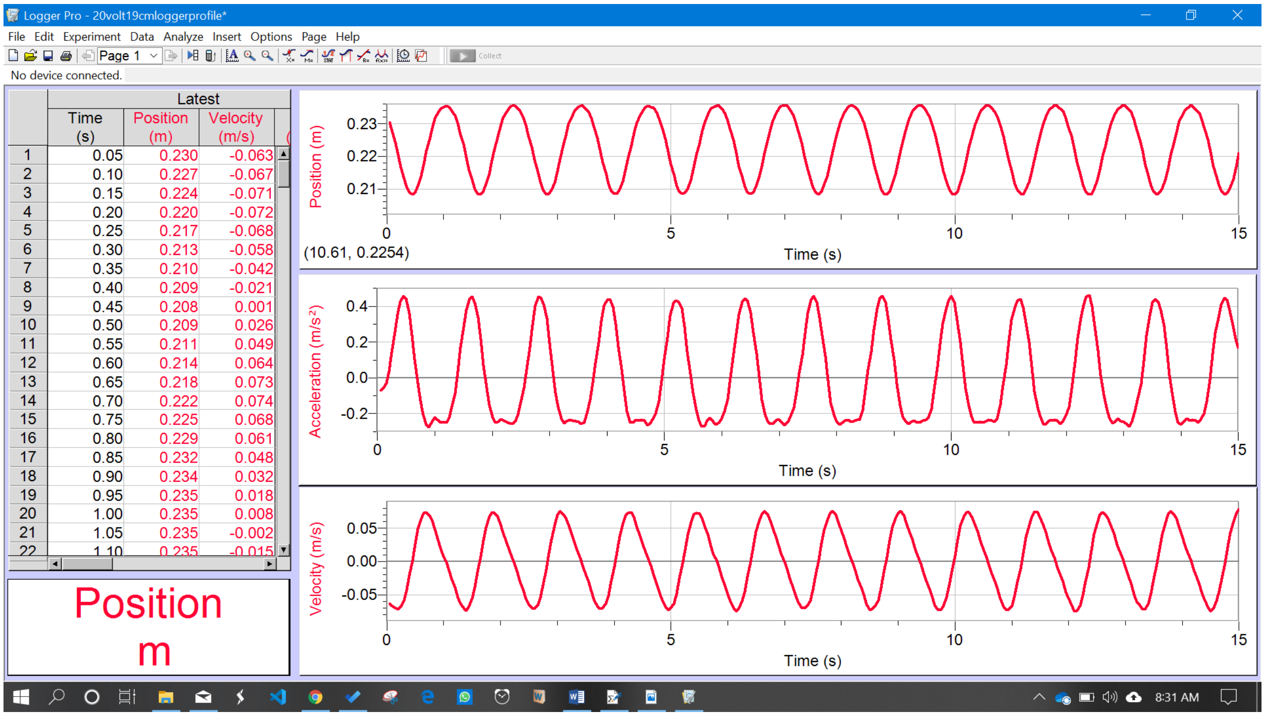

Figure 6: Sample plot with correctly adjusted distance. The dips in acceleration plots represent the geometry of the apparatus. Adjust your distance until you get something like this.

3. Open the LoggerPro software on the computer. The LoggerPro interface should look somewhat like Figure (4).

4. Go to `INSERT` on the top and select `GRAPH` to insert the acceleration graph. Drag and adjust all the graphs until your LoggerPro interface looks like Figure (5).

4.4 Optimizing the apparatus

The mean distance of the screen from motion sensor effects the readings that we get. If the sensor is too close, it will miss out most of the motion of the body and if it is too far away it will again miss out on the motion of the body. We will need to adjust the distance of motion sensor from the mean to produce the best possible results. This distance will depend upon the choice of your voltage value that you used in setting up the experiment. Click `Collect` in icon bar to start collecting the data.

Question: What is the voltage of your power supply? What is the ideal distance for your selected voltage? What happens when the sensor is too close or too far?
Adjust the distance until your graphs look similar to figure (6).

The scaling of graphs can be adjusted by using the auto-scale option icon on icon toolbar and the data acquisition time and sampling frequency can be adjusted by using the data-collection icon in the icon toolbar.

4.5 Acquiring the data

We are now ready to acquire the data for our analysis. For your selected value of voltage, record the data for distance, velocity and acceleration for 10 seconds and save them as a LoggerPro file.

Go to `INSERT` and click `STATISTICS` to get the statistics of your graphs.

Question: What are the maximum and minimum values of your position, velocity and acceleration graphs? What is the mean and standard deviation of all three variables?

5 Analyzing the Data

5.1 Geometry of the apparatus

You will notice that you have dips in your acceleration curves, which signifies the fact this is not a simple harmonic motion. **This is the most important assessment part of the experiment.**

Question: Explain the origin of this dip? What is happening physically when the dip occurs? Use the geometry of the apparatus to answer your question. Refer to Figure (2) while coming up with an explanation.

Question: Refer to the lab reading [2] and explain how the ratio R/L describes these curves. For what value of R/L does the dip disappear?

5.2 Curve fitting

We will now fit equations (4) and (7) to the acquired data, by using the built in curve fitting options of LoggerPro.

Once you have taken the data, click `ANALYZE` and then click `CURVE FIT`. A screen similar to Figure (7) will show up. Click `DEFINE` function and fill it up with following equations. Each equation should be fitted to their respective plots. Use the values of R, L and K found in section (4.1). Please take care of brackets and do not put empty spaces in your equation when you input it into LoggerPro. Curve fitting should return the values of parameters ω and P.

$$x = -R\cos(\omega t - P) - L\sqrt{1 - \left(\frac{R}{L}\sin(\omega(t-P))\right)^2} + K \tag{8}$$

Question: What are the values of parameters P, and ω? What do they represent?

$$v = R\omega\sin(\omega t - P) + \frac{\frac{R^2}{L}\omega\sin(2\omega t - 2P)}{2\sqrt{1 - \left(\frac{R}{L}\sin(\omega(t-P))\right)^2}} \tag{9}$$

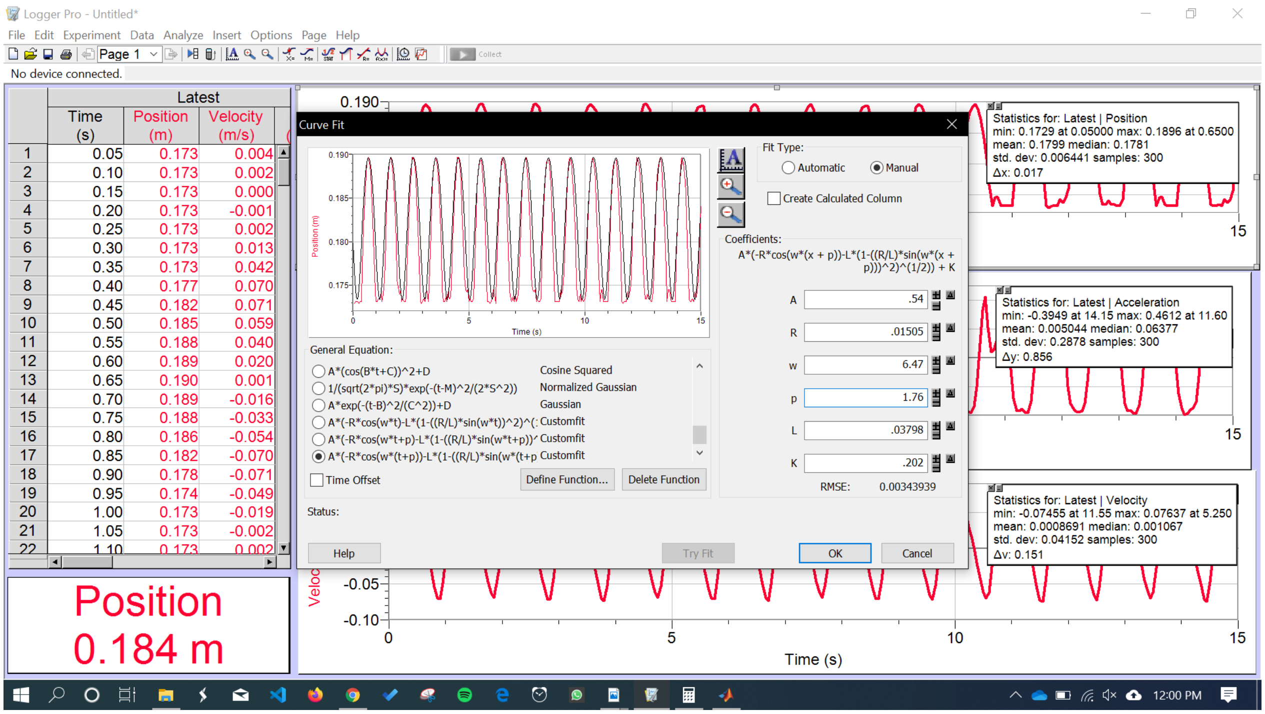

Figure 7: LoggerPro curve-fitting setting. The bottom left side shows the equations that you can use to fit your data. The right side shows the values of fit parameters generated by the fitting algorithm when a curve is fitted. You can change the values by yourself too to make your fit better.

$$a = R\omega^2\cos(\omega t - P) + \frac{4\left(1-\left(\frac{R}{L}\sin(\omega t - P)\right)^2\right)\left(\frac{R^2}{L}\omega^2\cos(2\omega t - 2P)\right) - \left(\frac{R^4}{L^3}\omega^2(\sin(2\omega t - 2P))^2\right)}{4\left(1-\left(\frac{R}{L}\sin(\omega t - P)\right)^2\right)^{3/2}} \tag{10}$$

Question: Do the values of ω and P match for all the fits?

You will readily observe how the geometry of manifests in the deviations from harmonic motion and are able to explain extra, albeit minute features in the velocity and acceleration waveforms.

Do your theoretical plots match with your experimental plots? If no, why not?
Happy exploring!

References

[1] Orville C. Cromer and Charles Lafayette Proctor, *Gasoline engine* , Encyclopedia Britannica , https://www.britannica.com/technology/gasoline-engine.

[2] Najeha Rashid and Muhammad Sabieh Anwar, *Analyzing reciprocating motion: Shaft attached to a rotating*, www.physlab.org.

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[4] https://www.vernier.com/.