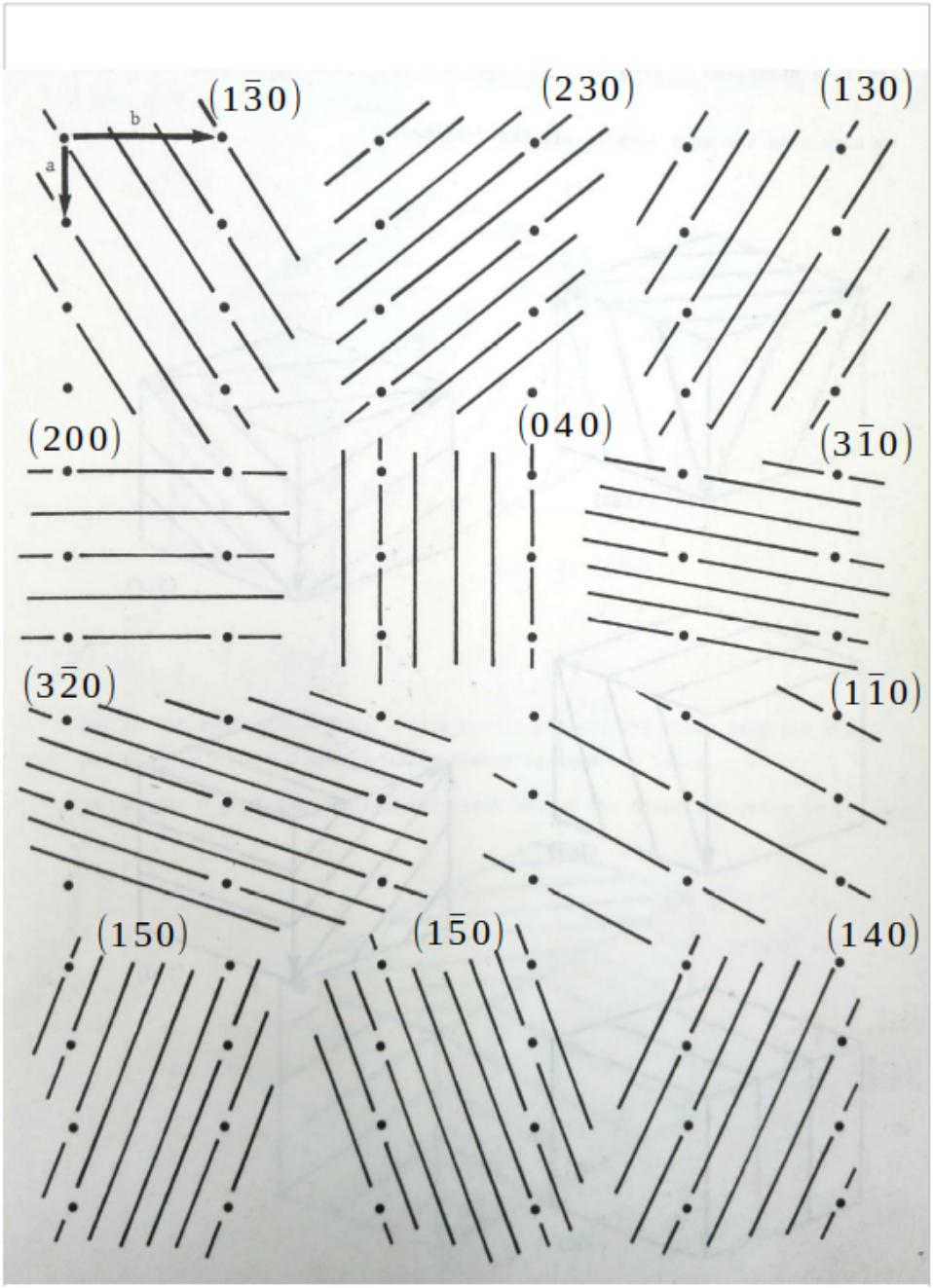
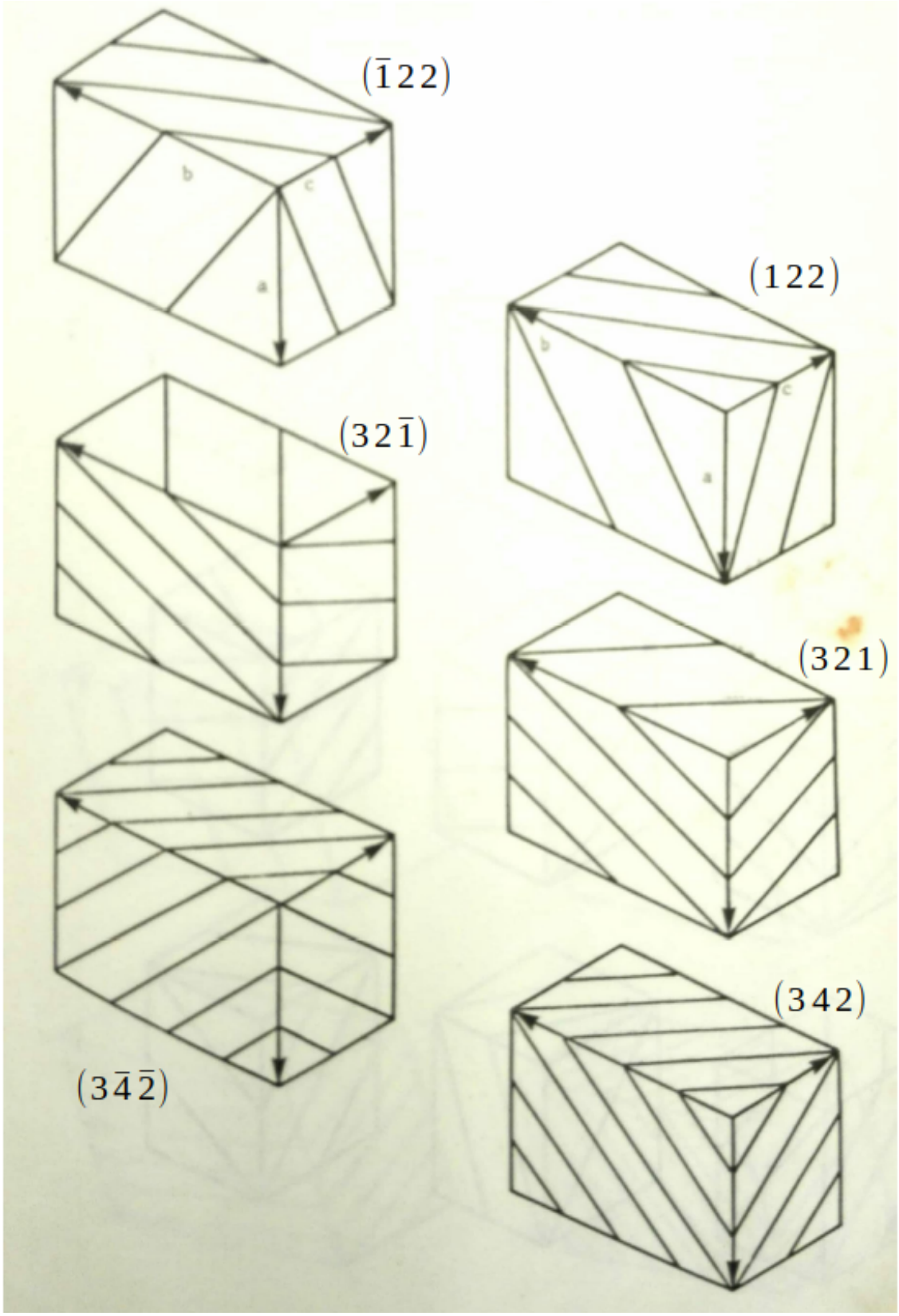
Assignment 3: Solution

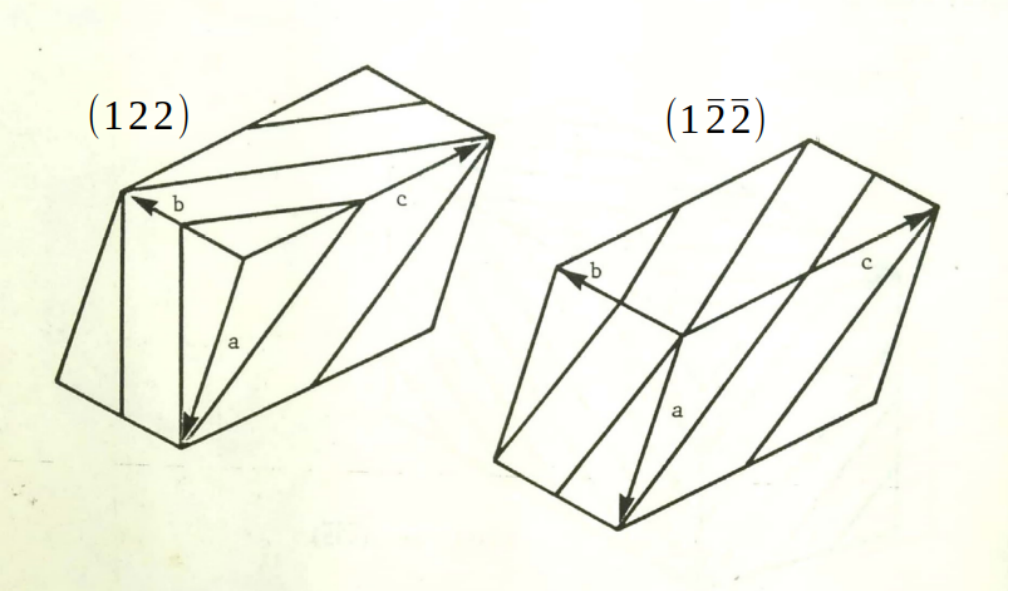
March 4, 2021

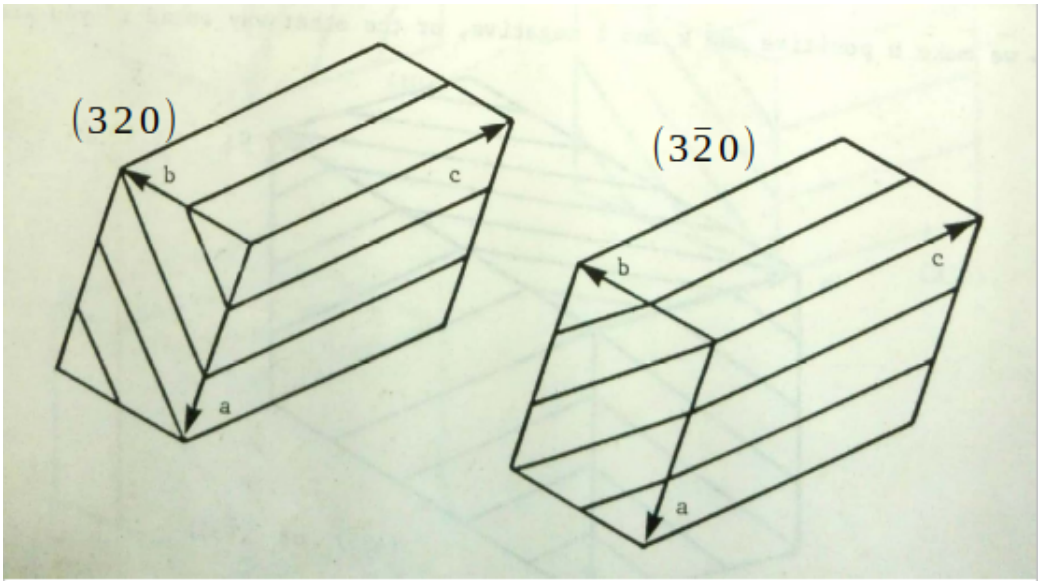
1. Index all the sets of planes below. Be sure of vectors **a** and **b** before attempting to index the planes. If we were to use different axes the indices (hkl) would change too.



2. Index all the planes shown in the following **10** diagrams.





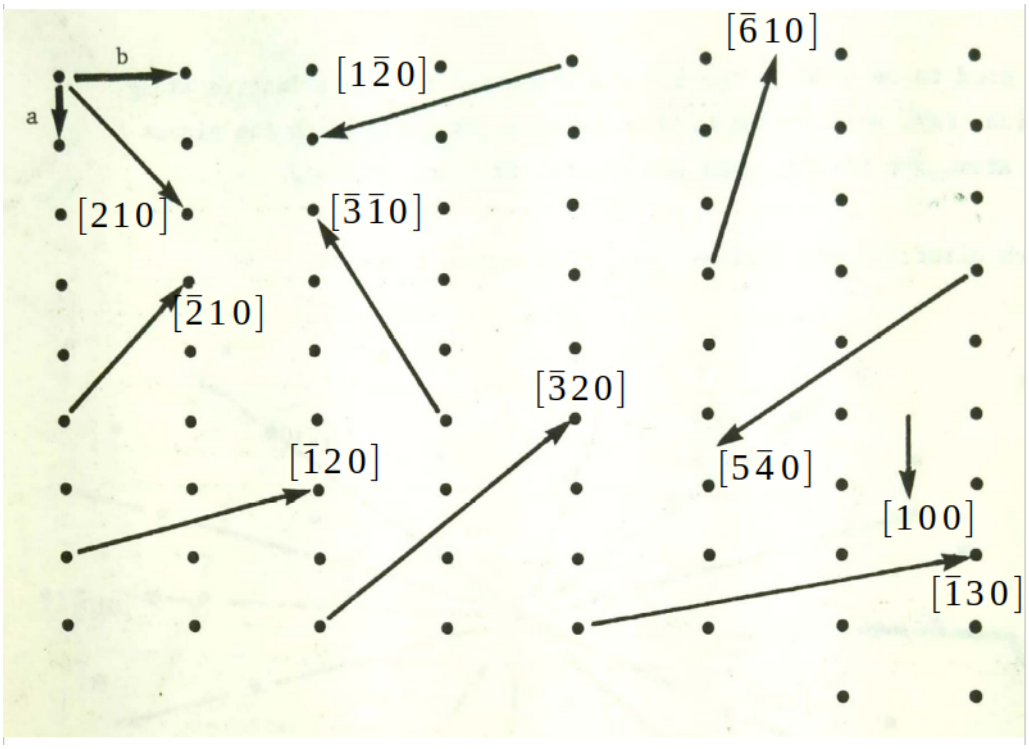


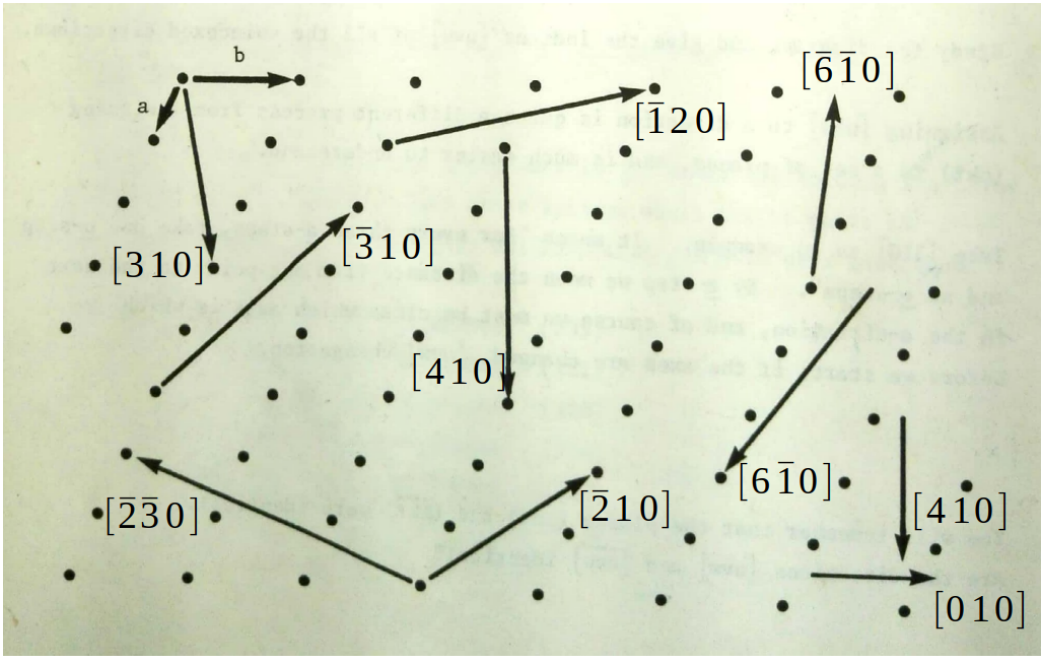
3. Give the indices $[uvw]$ of all the directions shown in the orthorhombic lattice and the monoclinic lattice, respectively on page 4 of this assignment. The method is the same, whatever the lattice, but be sure which axis is which before starting.

   For simplicity here, all directions shown are in the plane of the paper, so w is zero in all cases.

4. Give the indices $[uvw]$ of the each of the fourteen directions which are shown here radiating from two points in this orthorhombic P lattice. This is on page 5.

   Before starting, establish clearly in your mind the direction of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ which are given in the far top left corner. The c-axis is intended to appear to be pointing towards you.

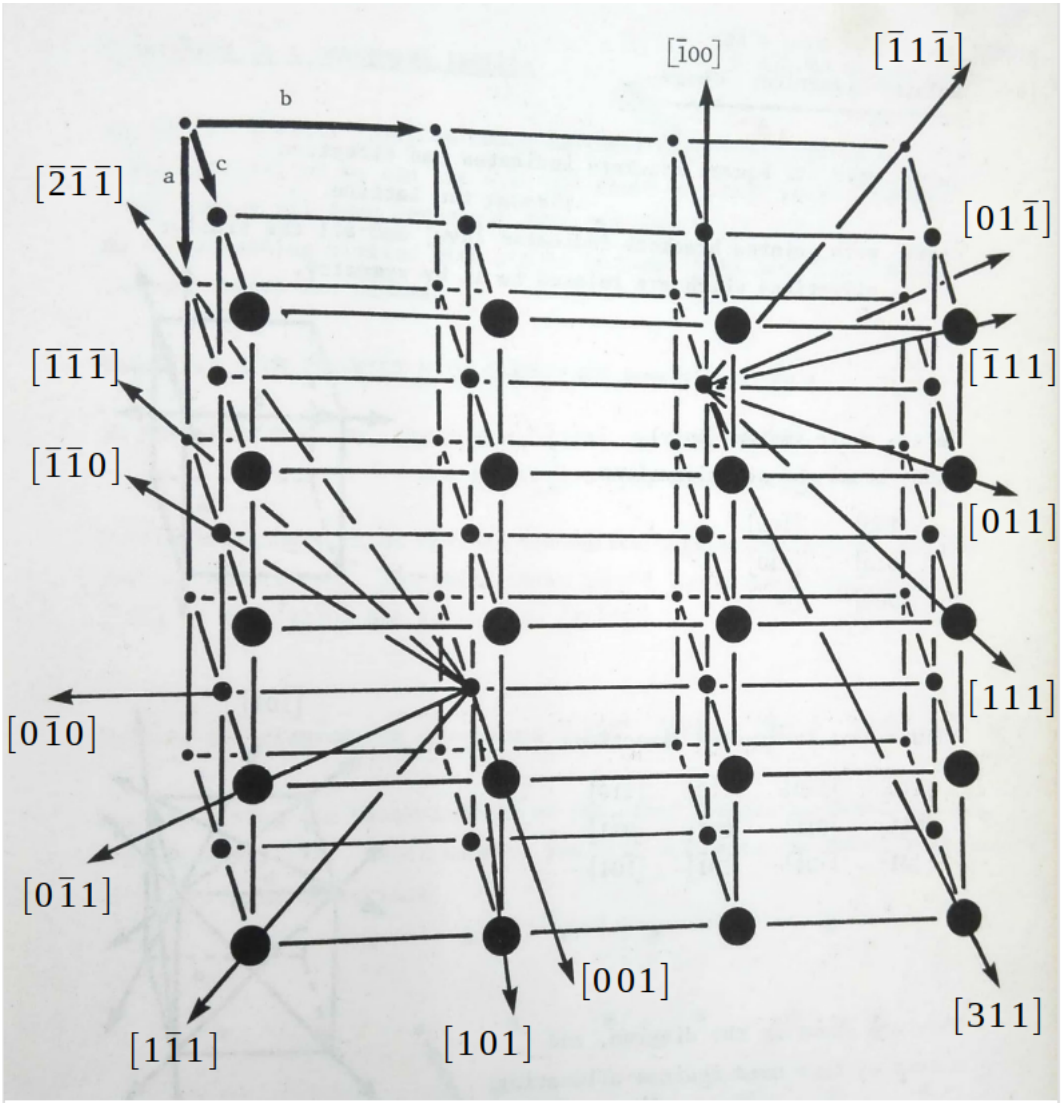




5. a) Find two directions that lie in the (212) plane.
Solution:
As we know, $[uvw]$ lies on plane (hkl) if following condition satisfied:

$$uh + vk + lw = 0. \tag{1}$$

So, two directions that satisfy this are $[12\bar{2}]$ and $[10\bar{1}]$. There can be many more.



b) Along what direction do the planes $(\bar{2}10)$ and (011) intersect?
Solution:
We can write a matrix,

$$\begin{bmatrix} u & v & w \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$u = (1 \times 1) - (1 \times 0) = 1$$
$$v = -((-2 \times 1) - (0 \times 0)) = 2$$
$$w = (-2 \times 1) - (0 \times 1) = -2$$

So, planes $(\bar{2}10)$ and (011) intersect at $[uvw] = [12\bar{2}]$

c) Find the plane in which the directions $[132]$ and $[21\bar{2}]$ lie.
Solution:
We can write a matrix,

$$\begin{bmatrix} h & k & l \\ 1 & 3 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

$$\begin{aligned} h &= (3 \times -2) - (1 \times 2) = -8 \\ k &= -((1 \times -2) - (2 \times 2)) = -6 \\ l &= (1 \times 1) - (2 \times 3) = -5 \end{aligned}$$

So, the directions [132] and [21$\bar{2}$] lie on the plane $(hkl) = (865)$

d) Are the planes $(102), (\bar{1}2\bar{2})$ and (122) members of a zone?
Solution:
If these planes are the members of a zone then they should have a common zone axis and, their miller indices satisfy the following relation:

$$\begin{vmatrix} h_1 & k_1 & l_1 \\ h_2 & k_2 & l_2 \\ h_3 & k_3 & l_3 \end{vmatrix} = 0$$

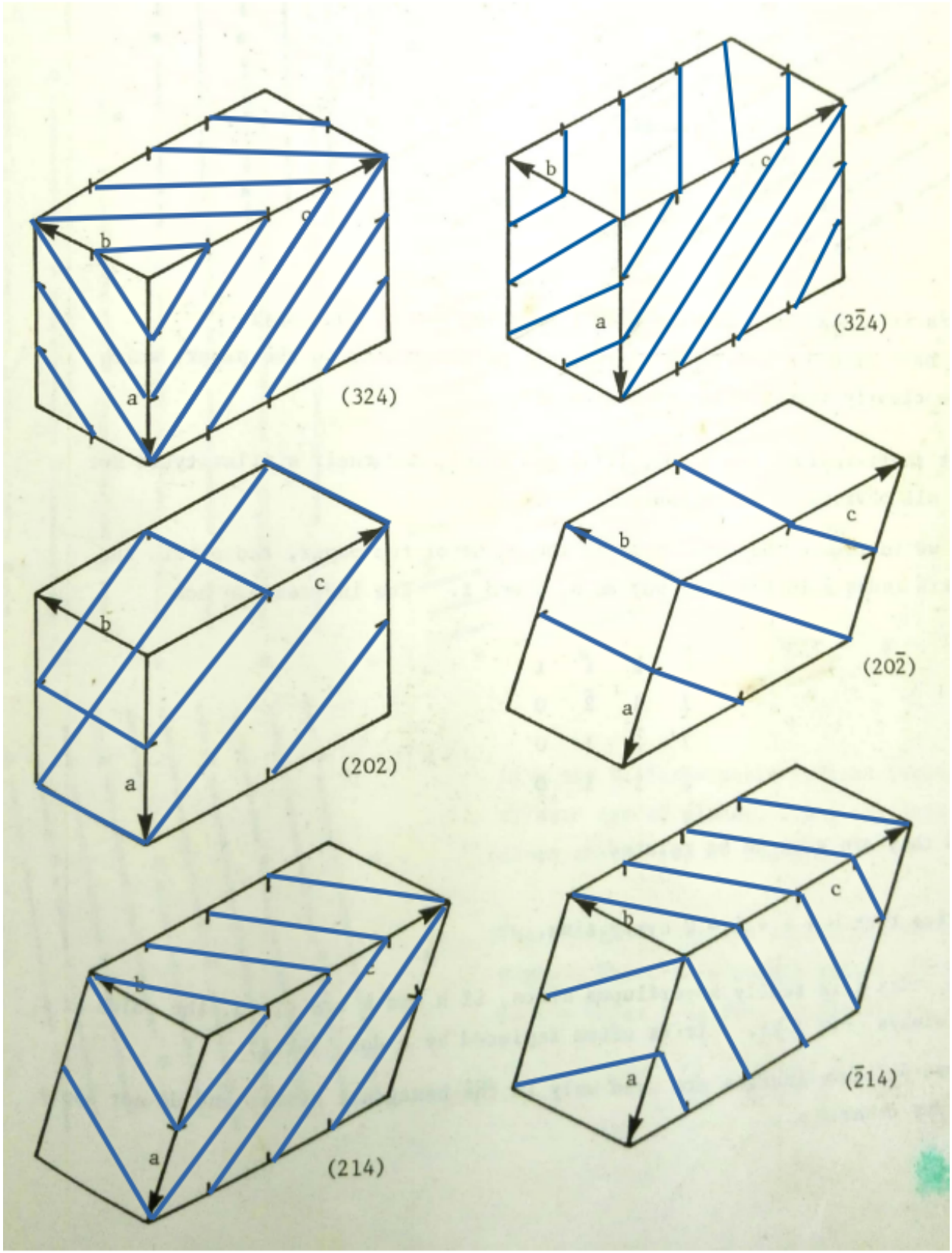
$$\begin{aligned} \begin{vmatrix} 1 & 0 & 2 \\ -1 & 2 & -2 \\ 1 & 2 & 2 \end{vmatrix} &= 1(4+4) + 2(-2-2) \\ &= 0 \end{aligned}$$

So, the given planes are indeed members of a zone.

e) What is the angle between the directions [210] and [1$\bar{2}$1]?
Solution:

$$\begin{aligned} \cos\theta &= \frac{(u_1u_2 + v_1v_2 + w_1w_2)}{\sqrt{u_1^2 + v_1^2 + w_1^2}\sqrt{u_2^2 + v_2^2 + w_2^2}} \\ \cos\theta &= \frac{2 - 2 + 0}{\sqrt{4+1+0}\sqrt{1+4+1}} \\ \cos\theta &= 0 \\ \theta &= \cos^{-1} 0 \\ \theta &= 90^o. \end{aligned}$$

6. Try sketching the indicated sets of planes in the unit cells on the last page of this assignment; the edges are marked out for the purpose. It's a tricky job, particularly with negative indices, and usually requires two or three attempts.


(324)
(3$\bar{2}$4)
(202)
(20$\bar{2}$)
(214)
($\bar{2}$14)