Assignment 1
Due date:

1 Work Done by Heart

The heart is the core machinery which pumps blood through the body. In a typical ventricular contraction, about stroke volume of 80 cm^3 of blood is pumped by each of the ventricles. During systole the left ventricular pressure increases from a very low value (that in the left atrium), to $P_{diastole} = 80$ mmHg (at time t_1), and then up to $P_{systole} = 120$ mmHg, and it stays at this value until the end of systole (at time t_2). The aortic valve first opens when this pressure rises above $P_{diastole}$ (at t_1), and blood is pumped out until systole is over (at t_2).

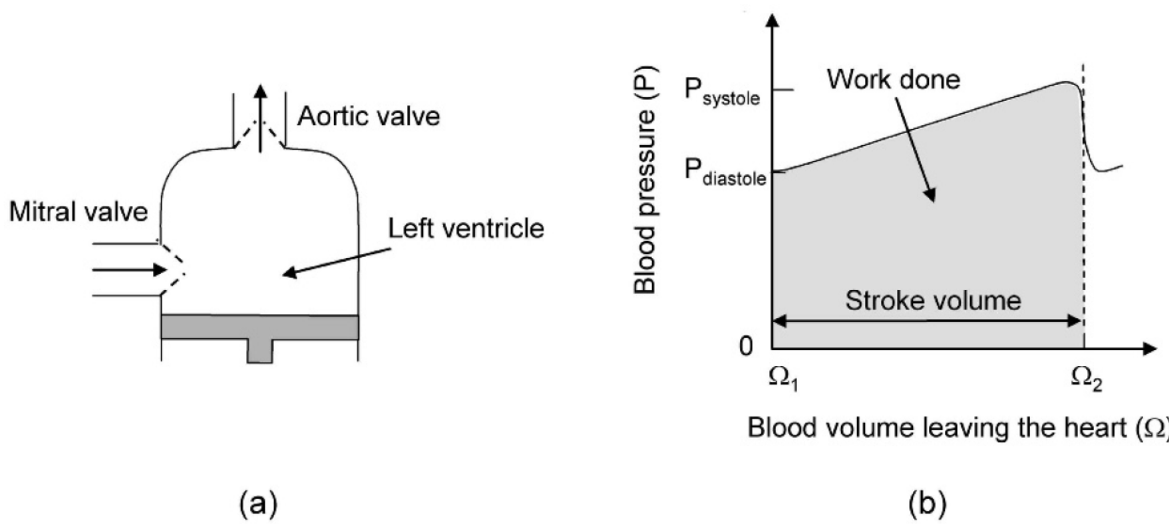

Figure 1: a) Schematic of left ventricle as pump, b) Pressue and volume in left ventricle during systolic contraction.

The average pressure in the heart is about 100 mmHg. The area under the curve is the work done, and is defined by the following equation:

$$W = \int PdV \tag{1}$$

a) With a heart rate of 60/min = 1/s, what is the Power used by the left ventricle? (Power is work done per unit time.) (1 mmHg = 133 Pa)
The pumping action takes place in less than one-third of the cardiac cycle and the heart muscle rests for over two-thirds. Thus the power during pumping phase is more than three times the average value you calculated.

b) The efficiency, ϵ, of converting metabolic energy into this mechanical work is approximately 20%. What is the metabolic power needed to run the ventricles?

c) The right ventricle pumps the same volume per cardiac cycle (to maintain the steady-state flow throughout), but at a pressure 1/5 times that of left ventricle, so the work and all of these powers are smaller by a factor of five. Calculate the total Power input for the heart (both ventricles). How much calories on average are needed per day to run the heart? (1 cal = 4.2 Joules.)

2 Continuity of Blood Flow

The human blood circulatory system consists of blood vessels that undergo branching into smaller vessels. Consider the following case where an artery of diameter 1mm branches into four identical arterioles (each of diameter 0.1mm).

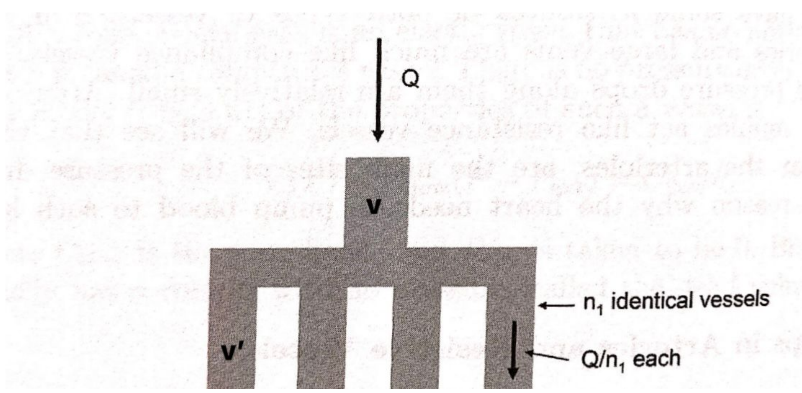

Figure 2: Blood flow in branching vessels. v and v' are velocities of large and small vessels respectively.

Assuming that the blood remains incompressible, the volumetric flow rate, Q, must remain constant.

a) Find the ratio $\frac{v'}{v}$. Recall the Continuity Equation (Q = Av).

b) Generalise the case for 'n' branches.

Flow can be laminar or turbulent. In laminar flow, a particle moves in a smooth manner along well-defined streamlines. In contrast, the motion is very random locally in turbulent flow. The Reynolds number 'Re' crudely divides the regimes of laminar and turbulent flow. Flow in a rigid tube with Re $<$ 2,000 is laminar and that with Re $>$ 2,000 is turbulent. $Re = \frac{\rho v d}{\eta}$

c) What is the Reynolds number of the larger artery ($d = 1mm$)? Assume a density $\rho = 1g/cm^3 = 1000kg/m^3$, velocity $v = 1m/s$, and viscosity $\eta = 1.5g/(m.s)$. What type of drag dominates? (Beware of SI units)

3 Bernoulli's Principle and Arteriosclerosis

Bernoulli's principle states that as the speed of a moving fluid increases, the pressure within the fluid decreases. Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2. \tag{2}$$

Arteriosclerotic plaque narrows down a section of an artery to 20 % of its normal cross-sectional area.

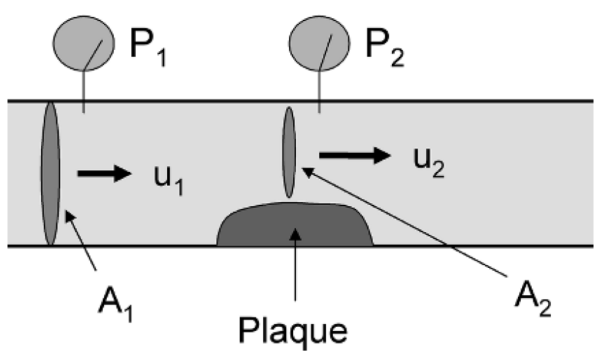

Figure 3: Clogged artery

Use the Continuity and Bernoulli's equation to find out the pressure in that section if immediately before it the pressure is 110 mmHg and flow speed is 0.10 m/s. ($\rho = 1g/cm^3 = 1000kg/m^3$), (1 mmHg = 133 Pa).

4 Hydrostatic pressure while standing

Hydrostatic pressure is exerted by the blood due to force of gravity and is calculated using $P = \rho hg$. In other words, the deeper point in a fluid has higher pressure. When you're standing upright, the blood pressure at the aorta has to be high enough to pump the blood to the top of your brain. This distance is about h = 50 cm for humans.

a) For reference, if the pressure in arteries surrounding the heart is 100 mmHg, what is the pressure in the arteries of the brain? (1 mmHg = 133 Pa)

b) When you stand on your head, why does your head become red and why do your legs become pale?

c) By how much would the pressure in brain increase if you stand on your head?

d) What must the pressure in the aorta in a giraffe be for its brain to receive blood? (How can you estimate the elevation of its brain above its aorta?)

5 Drug dispensation

a) How much force F must be applied to a plunger to inject $10^{-6}m^3$ of the solution in 3.0 s with a hypodermic syringe? For the pressure drop across the needle, apply Poiseuille's Law:

$$\frac{dP}{dx} = \frac{8\eta}{\pi R^4}Q \tag{3}$$

The needle is injected into a vein with a (gauge) pressure of 14mmHg (1,900Pa).

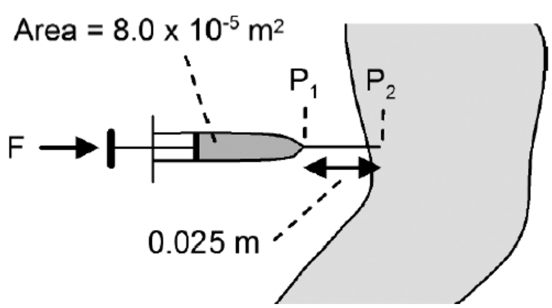

Figure 4: Intravenous injection

Assume the plunger has an area of $8.0 \times 10^{-5}m^2$ and the syringe is filled with a solution with viscosity of 1.5×10^{-3} Pa.s. The needle has an internal radius of 4.0×10^{-4} m and a length of 0.025 m. Remember that you want to apply a (gauge) pressure in excess of the venous pressure to achieve the desired flow rate Q.

b) A drip bottle is hung in a hospital to deliver a drug of density 2 g/cm^3, which reaches down under influence of gravity (Figure 4).

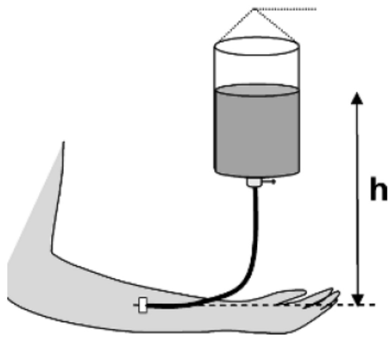

Figure 5: Intravenous infusion under gravity

What should be the height of the bottle so that the drug just only enters the vein? Take the gauge pressure in the vein to be 18 mmHg. Also assume that the needle has a large diameter. Should the bottle be placed higher,lower, or at same height in case the needle has a small diameter? Why?