Quantitative Verification of Faraday's Law of Electromagnetic Induction with PhysLogger

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Faraday's law of electromagnetic induction states that an electromotive force (EMF ε) is induced in a loop of copper wire when there is a change of magnetic flux Φ linking the loop. According to this, the induced EMF is equal to the negative of the rate of change of flux,

$$\varepsilon = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{1}$$

This principle can be tested out with an oscillating magnet that goes through a loop of wire fashioned into a solenoid. This is the case being used in the present experiment. The collected data can be used to quantitatively verify equation (1) by computing the integral of the EMF with respect to time t,

$$\Phi(t) = -\int_0^t \varepsilon(t)\mathrm{d}t \tag{2}$$

establishing in this process, a relationship between the EMF, flux, angular amplitude, angular velocity, angular acceleration, time, etc. This experiment is particularly suited to investigate these relationships.

KEYWORDS

Faraday's law of electromagnetic induction · EMF · Flux · Integral · Angular Amplitude · Angular Velocity · Angular Acceleration · Eddy Currents · Magnetic Braking ·

1 Objectives

In this experiment, we will

1. observe the phenomenon of electromagnetic induction,

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2. obtain the integral of the EMF through a solenoid with respect to time, and argue that this is indeed the magnetic flux,

3. observe how the EMF is related to angular amplitude, angular velocity,

4. observe the damping of rotational motion due to eddy currents, and finally,

5. mathematically corroborate Faraday's law of electromagnetic induction.

2   Introducing the Experiment

The experiment is derived from reference [1]. The setup comprises a semi-circle made of metal with weights attached to each end. Small magnets are attached to its centre inside a depression. There is also a loop of insulated copper wire wound around a hollow plastic frame surrounding the space in which the semi-circle oscillates. The data is acquired with the help of a PhysLogger (a data acquisition and control device), PhysCompass (the rotary motion sensor), and V Probe (a voltage sensing device). Further information about PhysLogger can be found online [2]. The rotary sensor is connected to the semi-circle and the voltage sensor is connected to the loop. Both of these are connected to PhysLogger, which is connected to a computer. The software program of PhysLogger, `PhysLogger Desktop App`, is used to collect the data. The setup is shown in Fig. 1.
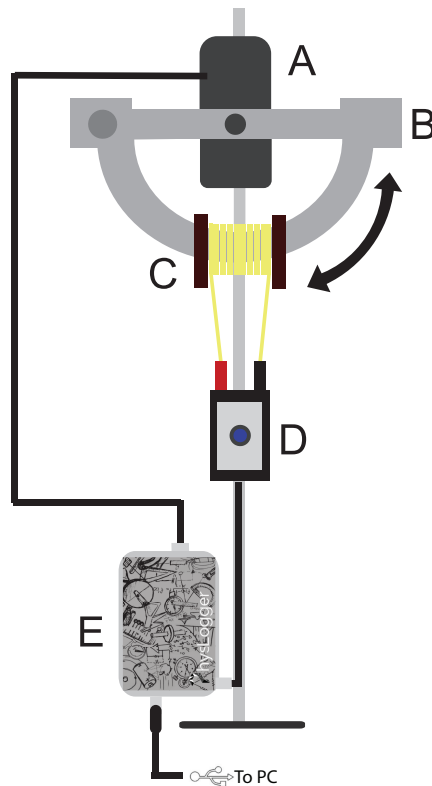

Figure 1: The experimental setup, comprising A: PhysCompass (the rotary motion sensor), B: the metal semi-circle, C: the copper coil loop around a plastic frame, D: voltage sensing V Probe, and E: PhysLogger.
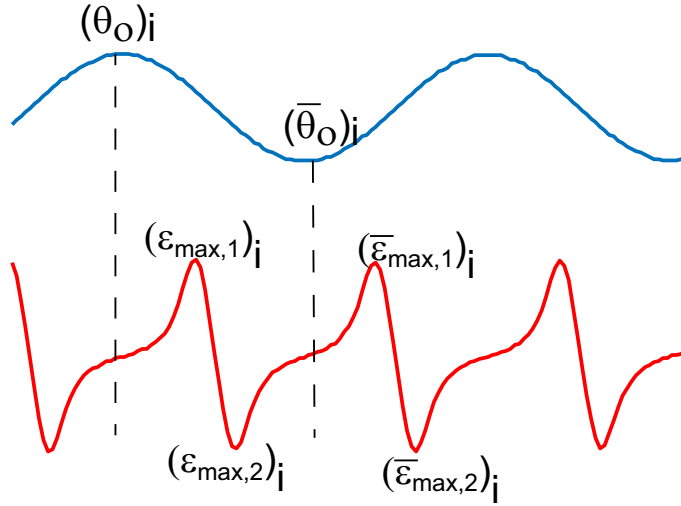
Figure 2: Signals of the angle $\theta(t)$ and EMF $\varepsilon(t)$ for a small interval of time. The ith cycle of the angular displacement is shown. The variables $(\theta_0)_i$ and $(\bar{\theta}_0)_i$ show the positive and negative amplitudes of the angles. The magnet approaching and receding the solenoid after $(\theta_0)_i$ result, respectively in the positive and negative EMF peaks, $(\varepsilon_{max,1})_i$ and $(\varepsilon_{max,2})_i$. A similar argument can be made for $(\bar{\theta}_0)_i$ generating $(\bar{\varepsilon}_{max,1})_i$ and $(\bar{\varepsilon}_{max,2})_i$.

When the straight edge of the semi-circle is nudged, the magnet oscillates from side to side, inducing an EMF in the loop of wire.

The rotary motion sensor measures the angular amplitude of the semi-circle $\theta(t)$, while the EMF $\varepsilon(t)$ developed across the solenoid is also amplified and measured by the V Probe and PhysLogger. Both of the signals are fed into the computer using PhysLogger and can be analyzed. The angle $\theta(t)$ also decays with time. The EMF $\varepsilon(t)$ will have a unique shape comprising alternate positive and negative peaks. The EMF signal also decays. Both the angular and EMF data are shown in Fig. 2.

2.1 Magnitude of the Induced EMF

Equation (1) can be modified to show that the absolute value of the induced EMF, $\varepsilon(t)$ is the derivative of the magnetic flux $\Phi(t)$:

$$|\varepsilon| = \frac{d\Phi}{dt} \tag{3}$$

This can be rewritten as

$$|\varepsilon_{max}| = \left(\frac{d\Phi}{d\theta}\right)_{max} \left(\frac{d\theta}{dt}\right)_{max} = \left(\frac{d\Phi}{d\theta}\right)_{max} \omega_{max} \tag{4}$$

where ω_{max} is the maximum angular velocity and ε_{max} is the maximum EMF in one cycle.

Therefore, it can be said that the EMF $|\varepsilon_{max}|$ is proportional to the maximum angular velocity ω_{max}.

Therefore, once the angular and EMF data is acquired, we will use a computer program to extract $|\varepsilon_{max}|$ and θ_{max} for each cycle. For example in the ith cycle

$$(\varepsilon_{max})_i = \frac{(\varepsilon_{max,1})_i + (\varepsilon_{max,2})_i + (\overline{\varepsilon}_{max,1})_i + (\overline{\varepsilon}_{max,2})_i}{4} \tag{5a}$$

$$(\theta_{max})_i = \frac{(\theta_0)_i + (\overline{\theta}_0)_i}{2} \tag{5b}$$

Since ω_{max} is proportional to the maximum angular amplitude θ_{max}, for amplitudes that are not too small (see the Appendix), ε_{max} is also proportional to θ_{max}. If ε_{max} is plotted against θ_{max}, a linear increase in the maximum EMF is seen, which constitutes a quantitative verification of Faraday's law of electromagnetic induction

The maximum angular amplitude for each cycle can also be plotted against time to observe a linear decay. This is due to electromagnetic damping, in addition to energy loss through radiation, friction at the pivot, air resistance, etc. We now describe the damping process.

2.2 The Mechanism of Damping

The oscillating semi-circle slows down due to electromagnetic damping. Usually damped systems show an exponential decay, but linear decay for electromagnetically braked systems can be mathematically proven with the help of equation (4). Disregarding other sources of energy loss, the main source is Joule heating due to the production of eddy currents in the loop. This happens in compliance with Lenz's law. The current in the loop flows in a direction that it produces a magnetic field opposing the changing magnet field due to the source itself, which is the oscillating magnet. The power loss is given by

$$P_{loss} = i^2 R \tag{6}$$

where i is the eddy current and R is the resistance of the loop. Integrating this over a cycle, we obtain

$$\Delta E_{one\ cycle} = \int\limits_{one\ cycle} i^2 R\,\mathrm{d}t = \frac{1}{R} \int\limits_{one\ cycle} \varepsilon_{max}^2\,\mathrm{d}t = \frac{1}{R} \int\limits_{one\ cycle} \left(\frac{\mathrm{d}\Phi}{\mathrm{d}\theta}\right)^2_{max} \left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2_{max} \mathrm{d}t \tag{7}$$

Now, $\mathrm{d}\Phi/\mathrm{d}\theta$ for a cycle is only significant for a narrow range of angles near $\theta = 0$. This is because the magnet is small compared to the larger solenoid, causing the flux to drop to a minimal value when it reaches the geometric centre of the solenoid, and $\mathrm{d}\Phi/\mathrm{d}\theta$ swiftly becomes negligible outside the range. Within this range, $\mathrm{d}\theta/\mathrm{d}t$, the angular velocity is approximately constant ($\approx \omega_{max}$), given that the angular amplitude is not too small either (see the Appendix). Taking this term outside the integral we obtain

$$\Delta E_{one\ cycle} \approx \frac{\omega_{max}}{R} \int \left(\frac{\mathrm{d}\Phi}{\mathrm{d}\theta}\right)^2_{max} \mathrm{d}\theta \tag{8}$$

4

This integral is almost completely independent of the initial angular amplitude (θ_{max}) and thus of ω_{max}. The integral is zero outside a small range of θ. The energy loss per cycle is proportional to ω_{max}

$$\frac{\mathrm{d}E}{\mathrm{d}t} \propto \omega_{max} \tag{9}$$

At this point, the potential energy of the system almost completely converts into kinetic energy, so we obtain the consumed energy

$$E = \frac{1}{2}I\omega_{max}^2 \tag{10a}$$

$$\implies E \propto \omega_{max}^2 \tag{10b}$$

$$\implies \sqrt{E} \propto \omega_{max} \tag{10c}$$

where I is the moment of inertia of the oscillating assembly. Therefore, from equation (9) and (10c) we obtain,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -k\sqrt{E} \tag{11}$$

Given the initial condition $E(0) = E_0$, integrating equation (11) yields

$$\left(\sqrt{E_0} - \sqrt{E}\right) \propto t \tag{12}$$

This implies linear decay of $\sqrt{E}$, therefore of ω_{max}, thus of ε_{max} and also linear decay of θ_{max}. To consider the damping due to friction at the pivot, we cannot consider the frictional force to be proportional to the velocity, as this would yield an exponential decay. Such a model does not fit the empirically collected data. Instead, a constant frictional torque τ at the pivot can be assumed, which is justified by the contact forces at the pivot. This gives the following equation for the additional energy loss in one cycle

$$\Delta E_{one\ cycle} = \int \tau \,\mathrm{d}\theta = \tau 4\theta_{max} \propto \omega_{max} \propto \sqrt{E} \tag{13}$$

once again indicating linear decay of the angular amplitude θ_{max}. In summary, we have $\varepsilon_{max} \propto \omega_{max} \propto \theta max$ and $\mathrm{d}E/\mathrm{d}t \propto \sqrt{E} \propto t$.

3   Preparing for the Experiment

Before starting the experiment, we make sure to verify the following aspects.

1. The semi-circle is tightened to the rotary sensor. Tightening prevents it from the slight to and fro motion. It increases mechanical efficiency and allows for more accurate data collection.

2. The orientation of the semi-circle is exactly horizontal. It can be verified with the help of a spirit level, and modified through adjusting the position of the weights on the straight edge of the semi-circle with the help of the knobs that hold them in place.
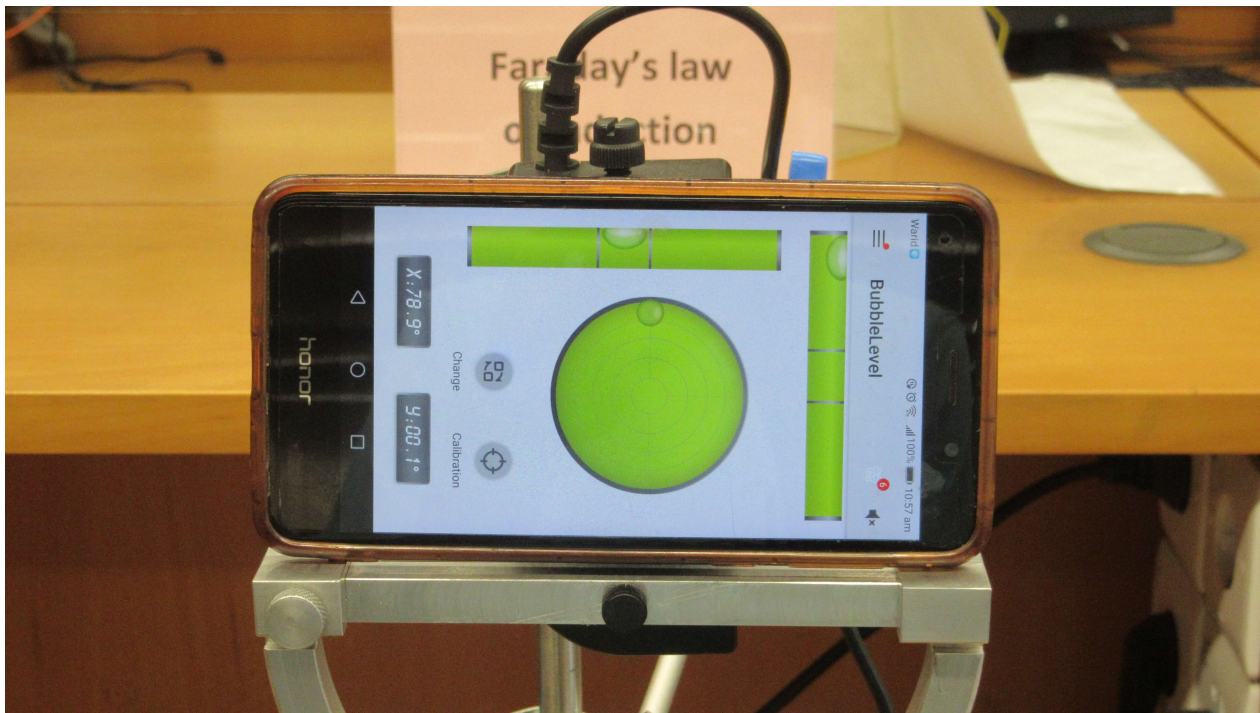
Figure 3: A precise mobile application may be used instead of a physical spirit level as well. An example of such an application is `Bubble Level - Spirit Level`

3. The semi-circle in due motion does not collide with the internal surface of the plastic frame of the loop. Collisions will interfere with data collection as they alter the expected motion of the oscillating semi-circle, giving inaccurate data.

4. All connections are made, as shown in Figure 1.

5. **Notes about configuring PhysLogger:**

   (a) It is connected to the PC via a USB cable.
   (b) It is connected with the PhysCompass via one of its digital channels, and with the V Probe via one of its analog channel.

When all these preparations are made and PhysLogger is responding to changes in the semi-circle's moment, the apparatus is ready for experimentation.

4 Conducting the Experiment

1. In PhysLogger Desktop App, select `Measure > Rotation` and `Measure > Voltage`. Make a LivePlot. (PhysCompass should have been already connected to PhysLogger before starting the App).

2. Go to `Quantities Panel > PhysCompass` and set the units of angular measurements to radians. When the semicircle is stationary at its equilibrium position, `Tare` the readings of PhysCompass.

3. Go to `Extensions Menu > Advanced > Differentiate` and first differentiate `Angle`to compute `Angular velocity` and then differentiate the `Angular velocity` to achieve the `Angular acceleration`.

4. Set a desirable sampling rate, such as 20 `Hz`.

5. Once you are ready to begin the experiment, click on `Clear Plots` and nudge one side of the semi-circle. Make sure not to use excessive force as that may cause a collision between the corners of the semi-circle and the plastic frame of the loop.

6. The magnets will begin to oscillate, the angular data and the resulting EMF detected by PhysLogger are sent to the computer.

7. You see a graphed collection of the data in real time. A sample is shown in Fig. 4.
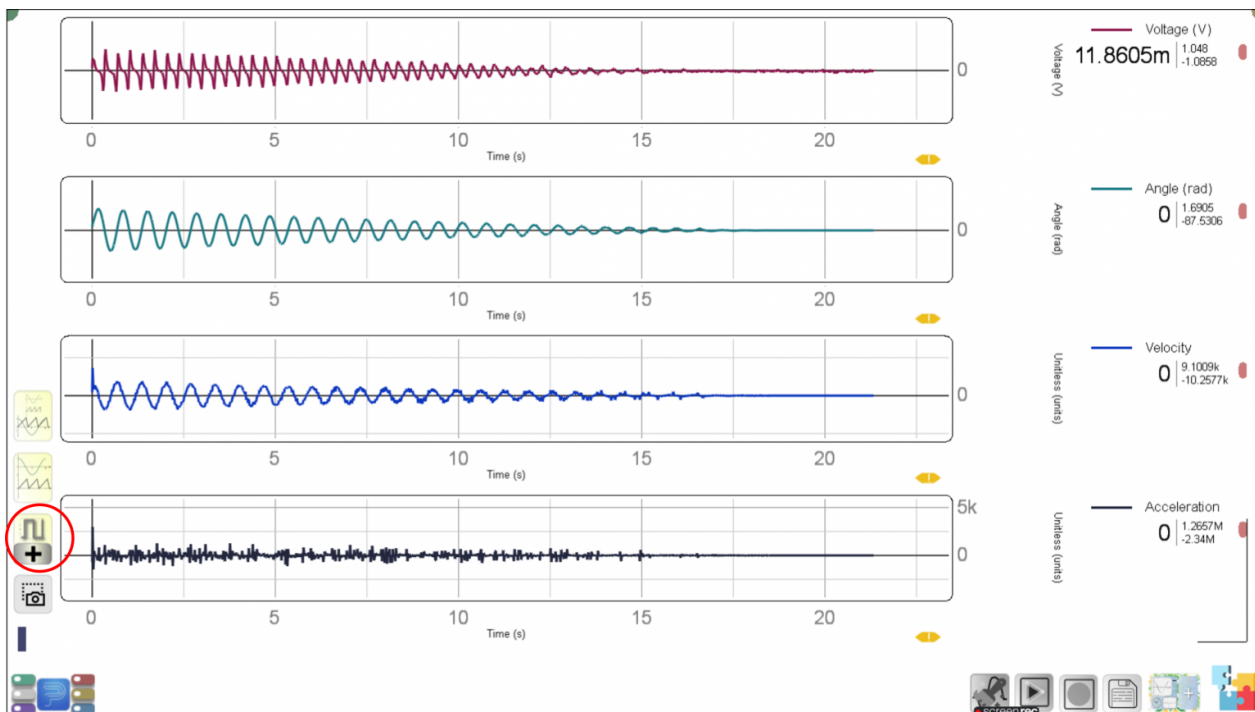

Figure 4: Representation of data acquired shown in `PhysLogger Desktop App`. Multiple plots can be made on the same page by using the `Add LivePlot icon`.

Once the data has been collected, `Save all quantities > Show in Explorer`. Convert the saved file to .txt file for further analysis in MATLAB. (Delete the first row of serial numbers.) *Note that the file includes data with the respective names and measurement units of the measured quantities.*

Now let's analyze the data in MATLAB.

5   Analyzing the Data

1. Calculate the integral of the EMF potential to obtain the flux. You could use MAT-LAB's command `cumtrapz` for this purpose. Explain the shape of the signal and connect it with the physics matter of the swinging magnet.

2. Plot the EMF, flux, angle, angular velocity, and angular acceleration against time. Compare them with each other describe and how they vary as time progresses. Are the shapes of these variables interrelated?

3. Calculate the maximum EMF and the maximum angle for each cycle and plot them against each other. Fit this data with a straight line.

4. Calculate the maximum angle versus time and plot these variables against each other. Fit this data with a straight line.

Could you write a computer program to automatically take your data from `PhysLogger` and output these processed outputs? We encourage you to try this, but if you do not want to try, you can use the collection of MATLAB scripts from our website. Make sure your data *and* your code is in the same folder. Simply run the script and input your data with the input interface. Discuss your findings and answers with your instructor.

Repeat the experiment with the loop left open circuited and short circuited. Processing the data collected with the rotary sensor will allow you to produce plots of the maximum angle for each oscillation against time, showing differences due to electromagnetic damping.

Appendix

The maximum angular velocity ω_{max} is connected to the maximum angle θ_{max} by

$$\omega_{max} = \frac{4\pi}{T} \sin\left(\theta_{max}/2\right) \tag{14}$$

where T is the period of (small) oscillations and $\theta_{max}/2$ is in radians. If $\theta_{max}/2$ is smaller than 1, then ω_{max} is approximately proportional to θ_0. Can you derive this relationship using the principle of energy conservation?

References

[1] Electromagnetic induction and damping: Quantitative experiments using a PC interface, Avinash Singh, Y. N. Mohapatra, and Satyendra Kumar - American Journal of Physics, 424

[2] https://physlab.org/facility/physlogger/